1). The value of $\int \frac{x^2+1}{x^2-3x-10}dx$ is (for some constants A, B, c):

A. $A\ln|x-5| + B\ln|x-2| + c$

B. $1 + A\ln|x-5| + B\ln|x-2| + c$

C. $A\ln|x+5| + B\ln|x-2| + c$

D. $x + A\ln|x-5| + B\ln|x+2| + c$

E. $\frac{A}{x+2} + \frac{B}{x-5} + c$

F. $x^2 + A\ln|x-5| + B\ln|x-2| + c$

2). Assume that $\int f(x)dx = h(x)$, and $\int h(x)dx = g(x)$. Evaluate $\int x^5 f(x^3)dx$.

A. $\frac{1}{3}(xh(x) - g(x)) + c$

B. $\frac{1}{3}(x^3h(x^3) - g(x^3)) + c$

C. $\frac{1}{5}x^5h(x^3) + c$

D. $\frac{1}{3}(h(x^3) - g(x^3)) + c$

E. $x^3g(x^3) - h(x^3) + c$

F. $\frac{1}{3}g(x) - h(x) + c$

3). Using the appropriate trigonometric substitution, the integral $\int \frac{x}{(4-3x^2)^{3/2}}dx$ is equivalent to

A. $\int \frac{1}{6}\sec^2(\theta)\,d\theta$

B. $\int \sqrt{3}\tan(\theta)\,d\theta$

C. $\int \frac{1}{4\sqrt{3}}\frac{\sin(\theta)}{\cos^2(\theta)}\,d\theta$

D. $\int \sec(\theta)\,d\theta$

E. $\int \cos(\theta)\sin(\theta)\,d\theta$

F. $\int \frac{1}{6}\tan(\theta)\sec(\theta)\,d\theta$

4). Using the substitution $x = \ln(2\tan(\theta))$, the integral $\int \frac{dx}{e^{3x}+16e^{-x}+8e^{x}}$ is equivalent to which of the following

A. $\int \frac{1}{2}\cos^2(\theta)\,d\theta$

B. $\int \frac{1}{2}\sin^2(\theta)\,d\theta$

C. $\int \frac{1}{8}\cos^2(\theta)\,d\theta$

D. $\int \frac{1}{16\sec^4(\theta)}\,d\theta$

E. $\int (\tan^3(\theta) + 16(\tan(\theta))^{-1} + 8\tan(\theta))\,d\theta$

F. $\int \ln(\tan^3(\theta) + 16(\tan(\theta))^{-1} + 8\tan(\theta))\,d\theta$

5). Suppose we used trigonometric substitution method to evaluate $\int f(x)\,dx$, with the substitution $x = 2\tan(\theta)$, $\theta \in (\frac{-\pi}{2}, \frac{\pi}{2})$, and we got $\int f(x)\,dx = \int(\cos(2\theta) + 2)\,d\theta$. Find the value of the integral with respect to x

A. $\frac{2x}{x^2+4} + 2\tan^{-1}(x/2) + c$

B. $\frac{2\sqrt{x^2-4}}{x^2} + 2\tan^{-1}(2/x) + c$

C. $\frac{1}{2}\sin(2x) + 2x + c$

D. $\frac{2x}{\sqrt{4+x^2}} + 2\tan^{-1}(x/2) + c$

E. $\frac{1}{4}x\sqrt{4+x^2} + 2\tan^{-1}(x/2) + c$

F. $\frac{1}{4}x\sqrt{4-x^2} + 2\tan^{-1}(x) + c$

Part(2): Fill out with the correct FINAL answer:(2 points each)

7

(1) Write down the partial fraction decomposition of the function $f(x) = \frac{x-6}{x^5-8x^2}$. (do not evaluate the constants):

$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} + \frac{Dx+M}{x^2+2x+4}$ 2

(2) Set up the integral which represents the length of the curve $y = \cos(\sqrt{3x})$,

between $x = a$ and $x = b$, where $0 < a < b$. $\int_a^b \sqrt{1+((\cos\sqrt{3x})')^2}$ 1

(3) The values of p for which the improper integral $\int_a^b \frac{1}{(b-x)^{2p+1}}dx$ for $a < b$, converges are: 0

(4) The value of $\int \cos(3x)\sin(7x)dx = \frac{-\cos(4x)}{8} - \frac{\cos(10x)}{20}$ 2

(5) The best trigonometric substitution to evaluate the integral $\int \frac{\sqrt{x^2-2x+17}}{2x}dx$ is given by: $x = 4\tan\theta + 1$

المسوحة ضوئيا بـ CamScanner

Part 3):
Problem (1): (5 points) Evaluate $\int \frac{\cot^5(3x)}{\sin^3(3x)}\,dx$ (show all steps of solution).

$y^2 = \csc^2(3x)$

$$\int \cot^5(3x)\csc^3(3x)$$

$$\int \cot^5(3x) * y^3 \frac{dy}{-\csc(3x)\cot(3x)}$$

$$-\int \cot^4(3x)\, y^2\, dy$$

$$-\int (\csc^2(3x)-1)^2\, y^2\, dy$$

$$-\int (y^2-1)^2 y^2\, dy$$

$$-\int (y^4-2y^2+1)\, y^2\, dy$$

$$-\int y^6 - 2y^4 + y^2$$

$$-\frac{y^7}{7} + \frac{2y^5}{5} - \frac{y^3}{3} + C$$

$$-\frac{\csc^7(3x)}{7} - \frac{2}{5}\csc^5(3x) - \frac{\csc^3(3x)}{3} + C$$

y

$\frac{dy}{dx}$

$dx = \frac{dy}{\csc(3x)\cot(3x)}$

8

Problem (2):(5 points) Use the Comparison test to determine whether the following improper integral converge or diverge. (show all steps of solution).

$$\int_0^\infty \frac{dx}{3\sqrt{x} + 2x^4}$$

Q4 $\int \cos(3x)\cdot\sin(7x)\,dx$

Sol

$$\int \sin(7x)\cdot\cos(3x)\,dx$$

$$\Rightarrow \int \frac{1}{2}\left[\sin(7-3)+\sin(7+3)\right]dx$$

$$\Rightarrow \frac{1}{2}\int \sin 4x+\sin 10x\,dx$$

$$\Rightarrow \frac{1}{2}\left[\frac{-1}{4}\cos 4x-\frac{1}{10}\cos 10x\right]+C$$

$$\Rightarrow \frac{-\cos 4x}{8}-\frac{\cos 10x}{20}+C$$

② find the setup of Integral of the length the curve :-

$y = \cos(\sqrt{3x})$

between $(x = a)$ and $(x = b)$

Sol $L = \int_a^b \sqrt{1 + (f'(x))^2}$

$f(x) = \cos(\sqrt{3} * \sqrt{x})$

$f'(x) = -\sqrt{3} \sin(\sqrt{3} * \sqrt{x}) * \frac{1}{2\sqrt{x}}$

$f'(x) = \frac{-\sqrt{3} \sin(\sqrt{3x})}{2\sqrt{x}}$

تم حوضها بالقانون

$(f'(x))^2 = \frac{3 \sin^2(\sqrt{3x})}{4x}$

$$\boxed{L = \int_a^b \sqrt{1 + \left(\frac{3 \sin^2(\sqrt{3x})}{4x}\right)} \, dx}$$

5 the best trig. sub.

to evaluate the Integral:-

$$\int \frac{\sqrt{x^2-2x+17}}{2x}\,dx$$

Eq $x^2-2x+17$ بإكمال المربع

$x^2-2x+1-1+17$ $\left(\frac{-2}{2}\right)^2=1$

$\Rightarrow (x^2-2x+1)+16$:

$\Rightarrow (x-1)^2+16$

$$\Rightarrow \int \frac{\sqrt{(x-1)^2+16}}{2x}\,dx$$

let $\boxed{x-1=4\tan\theta}$

Part 3 : سؤال الحل

$$\int \frac{\cot^5(3x)}{\sin^3(3x)} dx = \int \cot^5(3x) \cdot \csc^3(3x) dx$$

$\Rightarrow$ Sol let $y = \csc(3x)$

$$dy = -3\csc(3x) \cdot \cot(3x)\, dx$$

$$dx = \frac{dy}{-3y \cdot \cot(3x)}$$

$$\Rightarrow \int \cot^5(3x) * y^3 * \frac{dy}{-3y \cdot \cancel{\cot(3x)}}$$

$$* \cot^2 3x = \csc^2 3(x) - 1$$

$$\frac{-1}{3}\int \frac{(y^2-1)^2 * y^{\cancel{3}\,2}}{\cancel{y}} * dy = \frac{-1}{3}\int (y^4 - 2y^2 + 1) * y^2 dy$$

$$\Rightarrow \frac{-1}{3}\int y^6 - 2y^4 + y^2 dy$$

$$\Rightarrow \frac{-1}{3}\left[\frac{y^7}{7} - \frac{2}{5}y^5 + \frac{1}{3}y^3\right] + c$$

$$\Rightarrow \frac{-1}{3}\left[\frac{\csc^7(3x)}{7} - \frac{2}{5}\csc^5(3x) + \frac{1}{3}\csc^3(3x)\right]$$

Q4 use $x = \ln(2\tan\theta)$

$$\int \frac{dx}{e^{3x} + 16e^{-x} + 8e^{x}}$$

Sol $dx = \frac{2\sec^2\theta}{2\tan\theta}\, d\theta \Rightarrow dx = \frac{\sec^2\theta}{\tan\theta}\, d\theta$

$x = \ln(2\tan\theta) \Rightarrow e^{x} = 2\tan\theta$

$$\int \frac{dx}{e^{3x} + \frac{16}{e^{x}} + 8e^{x}} = \int \frac{e^{x}}{e^{4x} + 8e^{2x} + 16}\, dx$$

$$\Rightarrow \int \frac{\cancel{2\tan\theta}}{16\tan^4\theta + 8*4\tan^2\theta + 16} * \frac{\sec^2\theta}{\cancel{\tan\theta}}\, d\theta$$

$$\Rightarrow \frac{2}{16} \int \frac{\sec^2\theta}{(\tan^4\theta + 2\tan^2\theta + 1)}\, d\theta \rightarrow (\tan^2+1)(\tan^2+1) \Rightarrow (\tan^2+1)^2 \rightarrow (\sec^2\theta)^2$$

$$\frac{1}{8} \int \frac{\cancel{\sec^2\theta}}{\sec^4\theta}\, d\theta$$

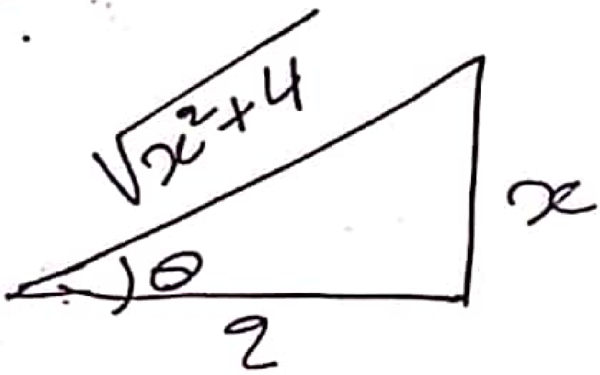
$$\frac{1}{8} \int \frac{1}{\sec^2\theta}\, d\theta = \boxed{\frac{1}{8}\int \cos^2\theta\, d\theta}$$

C الاجابة فرع

Q5 : $x = 2\tan\theta$

$$\int \cos(2\theta) + 2 \, d\theta$$

Sol $\frac{x}{2} = \tan\theta \Rightarrow$



$$\Rightarrow \frac{1}{2}\sin 2\theta + 2\theta + C$$

$$\frac{1}{\cancel{2}} * \cancel{2}\sin\theta * \cos\theta + 2\theta + C$$

$$\Rightarrow \frac{x}{\sqrt{x^2+4}} * \frac{2}{\sqrt{x^2+4}} + 2\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$\Rightarrow \boxed{\frac{2x}{x^2+4} + 2\tan^{-1}\left(\frac{x}{2}\right) + C}$$

A الاجابة فرع

Part $\underline{\underline{2}}$:- فاينل آنسر

① $f(x) = \dfrac{x-8}{x^5 - 8x^2}$

$\underline{Sol}$ $f(x) = \dfrac{x-8}{x^2(x^3-8)} = \dfrac{x-8}{x^2(x-2)(x^2+2x+4)}$

$\Rightarrow$ by P.F.D :-

$\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{(x-2)} + \dfrac{Dx+E}{x^2+2x+4}$

(Q3) $\int \frac{x}{(4-3x^2)^{\frac{3}{2}}} dx = \int \frac{x}{\sqrt{(4-3x^2)^3}} dx$

Sol let $\sqrt{3}x = 2\sin\theta \Rightarrow dx = \frac{2\cos\theta}{\sqrt{3}} d\theta$

$x^2 = \frac{4\sin^2\theta}{3}$

$\Rightarrow \int \frac{2\sin\theta}{\sqrt{3}} * \frac{1}{\sqrt{(4-\cancel{3}*\frac{4\sin^2\theta}{\cancel{3}})^3}} * \frac{2\cos\theta}{\sqrt{3}} d\theta$

$\frac{4}{3*8}\int \frac{\sin\theta \cdot \cos\theta}{\sqrt{(\underbrace{1-\sin^2\theta}_{\cos^2\theta})^3}} d\theta$

$\frac{1}{6}\int \frac{\sin\theta \cdot \cancel{\cos\theta}}{\cos^{\cancel{3}}\theta} d\theta$

$\frac{1}{6}\int \frac{\sin\theta}{\cos\theta} * \frac{1}{\cos\theta} d\theta$

$\Rightarrow \boxed{\frac{1}{6}\int \tan\theta \cdot \sec\theta \, d\theta}$

(F) الجزء ٢

حل امتحان "Calculus 2"

للمهندس الدكتور سعود السعدي

P1 $$\int \frac{x^2+1}{x^2-3x-10}\,dx$$

Sol

$$\begin{array}{r|l} & 1 \\ x^2-3x-10 & x^2+1 \\ & \mp x^2 \pm 3x \pm 10 \\ \hline & 3x+11 \end{array}$$

$$\Rightarrow \int 1 + \frac{3x+11}{x^2-3x-10}\,dx$$

$$\Rightarrow \int 1\,dx + \int \frac{3x+11}{(x-5)(x+2)}\,dx$$

by

$$\Rightarrow \int 1\,dx + \int \frac{A}{x-5} + \frac{B}{x+2}\,dx \quad \text{P.F.D}$$

$$\Rightarrow \boxed{x + A\ln|x-5| + B\ln|x+2| + C}$$

D ع م

Q2 $\int f(x)\,dx = h(x)$, $\int h(x)\,dx = g(x)$

Evaluate $\int x^5 f(x^3)\,dx$??

Sol let $y = x^3 \Rightarrow dy = 3x^2\,dx$

$$dx = \frac{dy}{3x^2}$$

$\Rightarrow \frac{1}{3}\int y\, f(y)\,dy \Rightarrow$ by parts

$u = y$ $\quad \int dv = \int f(y)\,dy$ $\quad$ * $\int f(x)\cdot dx = \int f(y)\cdot dy$ Rule

$du = dy$ $\quad v = h(y)$

$\frac{1}{3}\left[y\,h(y) - \underbrace{\int h(y)\,dy}\right]$ من السؤال $\to g(y)$

$\Rightarrow \frac{1}{3}\left[y\,h(y) - g(y)\right] \Rightarrow$ رجع الفرض

$\Rightarrow \frac{1}{3}\left[x^3 h(x^3) - g(x^3)\right] + C$

B الإجابة فرع

The Hashemite University
Second Semester 2018/2019
March 28, 2019
Time: 1 hour

30 Excellent

Calculus (2)
Second Exam

Student Name: F
Student Number
Serial Number:
Lecturer Name:

30-11

1	2	3	4	5	6	7	8	9	10
A	A	A	A	~~A~~	A	~~A~~	A	A	A
B	B	B	B	B	~~B~~	B	B	B	B
~~C~~	C	~~C~~	C	C	C	C	C	C	C
D	D	D	D	D	D	~~D~~	D	D	D
E	E	E	E	E	E	E	E	~~E~~	~~E~~
F	~~F~~	F	~~F~~	F	F	F	~~F~~	F	F

20

$\boxed{D}$ ← الاجابة الصحيحة

Problem(1): **Choose the correct answer and mark it with an (X) in the above table. (2 points each)**

1). The center of the circle $r = -4\sin(\theta) + 3\cos(\theta)$ is:

A. $(-3, 4)$ B. $(3, -4)$ C. $(\frac{3}{2}, -2)$ D. $(\frac{-3}{2}, -2)$ E. $(-3, -4)$ F. $(\frac{-3}{2}, 2)$

2). Find the limit of the sequence $a_n = \ln(2n^2 + 3n) - 2\ln(2n + 1)$

A. $\frac{1}{2}$ B. $\frac{1}{4}$ C. $\ln(4)$ D. $-\ln(4)$ E. $\ln(2)$ F. $-\ln(2)$

3). The integral that represents the area of the region inclosed between the curves $r_1 = 2\sin(\theta)$ and $r_2 = \sqrt{3}$ is given by

A. $\int_{\pi/3}^{\pi/2} (r_1^2 - r_2^2)\,d\theta$

B. $\int_{\pi/3}^{\pi/2} r_1^2\,d\theta + \int_0^{\pi/3} r_2^2\,d\theta$ n

C. $\int_{\pi/3}^{\pi/2} r_2^2\,d\theta + \int_0^{\pi/3} r_1^2\,d\theta$

D. $\int_{\pi/6}^{\pi/2} r_1^2\,d\theta + \int_0^{\pi/6} r_2^2\,d\theta$

E. $\int_{\pi/6}^{\pi/2} r_1^2\,d\theta + \int_0^{\pi/2} r_2^2\,d\theta$

F. $\int_{\pi/6}^{\pi/2} r_2^2\,d\theta + \int_0^{\pi/6} r_1^2\,d\theta$

4). Find the first term of the sequence in which $a_n = 2\,a_{n-1}$ if $a_6 = 8$

A. 1
B. 3
C. 4
D. 2
E. $\frac{1}{2}$
F. $\frac{1}{4}$

5). The sequence $\left\{(4n + e^n)^{\frac{-1}{2n}}\right\}$ is

A. convergent to $e^{-1/2}$
B. convergent to e
C. convergent to e^2
D. convergent to $\frac{1}{e^2}$
E. convergent to $2e$
F. divergent

6). Let $a_1 = 2e$, $a_{n+1} = 2ea_n$, $n \geq 1$, then the n^{th} term of the sequence is

A. ne
B. $(2e)^n$
C. e^{2n}
D. $2e^{2n}$
E. e^{n+1}
F. e^{n-1}

7). The parametric curve $x = e^t$, $y = te^{-3t}$ is concave downward for

A. $t > \frac{5}{6}$
B. $t < \frac{5}{6}$
C. $t > \frac{7}{12}$
D. $t < \frac{7}{12}$
E. $t > \frac{-7}{12}$
F. $t < \frac{-7}{12}$

). The sequence $\{\frac{\sin(n)}{6^n}\}$ is

A. increasing for all $n \geq 1$
B. increasing for all $n \geq 6$
C. decreasing for all $n \geq 1$
D. decreasing for all $n \geq 6$
E. increasing for all $n \geq 36$
F. not monotone

9). The Cartesian equation corresponding to the parametric curve $x = \sec t$, $y = \sin^2 t - 2$ is

A. $y = 1 - \frac{1}{x^2}$ C. $y = 2 - \frac{1}{x^2}$ E. $y = -1 - \frac{1}{x^2}$

B. $y = 3 - \frac{1}{x^2}$ D. $y = \frac{-1}{x^2}$ F. $y = \frac{1}{x^2}$

10). The parametric curve $x = t^3 - 3t$, $y = t^2 + 4t$ has horizontal tangent at the point

A. $(-2, 4)$ C. $(-2, 5)$ E. $(-2, -4)$

B. $(18, -9)$ D. $(2, -5)$ F. $(-18, -9)$

Problem(2): Fill out with the correct FINAL answer: (2 points each)

(1) Find the polar coordinates (r, θ) of the point $(-4\sqrt{3}, 4)$, where $r < 0$, and $0 \le \theta \le 2\pi$. $\boxed{\left(-8, \frac{11\pi}{6}\right)}$ 2

(2) Find the cartesian coordinates (x, y) of the point $(5, \frac{2\pi}{3})$. $\boxed{\left(\frac{-5}{2}, \frac{5\sqrt{3}}{2}\right)}$ 2

(3) Find the cartesian equation corresponding to the polar equation

$r = 4\tan(\theta)$, $\theta \in (0, \pi)$ in the form $y = f(x)$. $\boxed{y = \frac{x^2}{\sqrt{16 - x^2}}}$ 2

(4) Find the value(s) of t where the tangent line to the parametric curve

$C:\ x = e^t,\ y = e^{-t}$ is parallel to the line $5x + y = 1$. $\boxed{t = \frac{-\ln 5}{2}}$ 2

(5) Sketch the region that lie inside both curves $r = 2 + 2\cos(\theta)$ and $r = \sin(\theta) - \cos(\theta)$.

cardiod circle

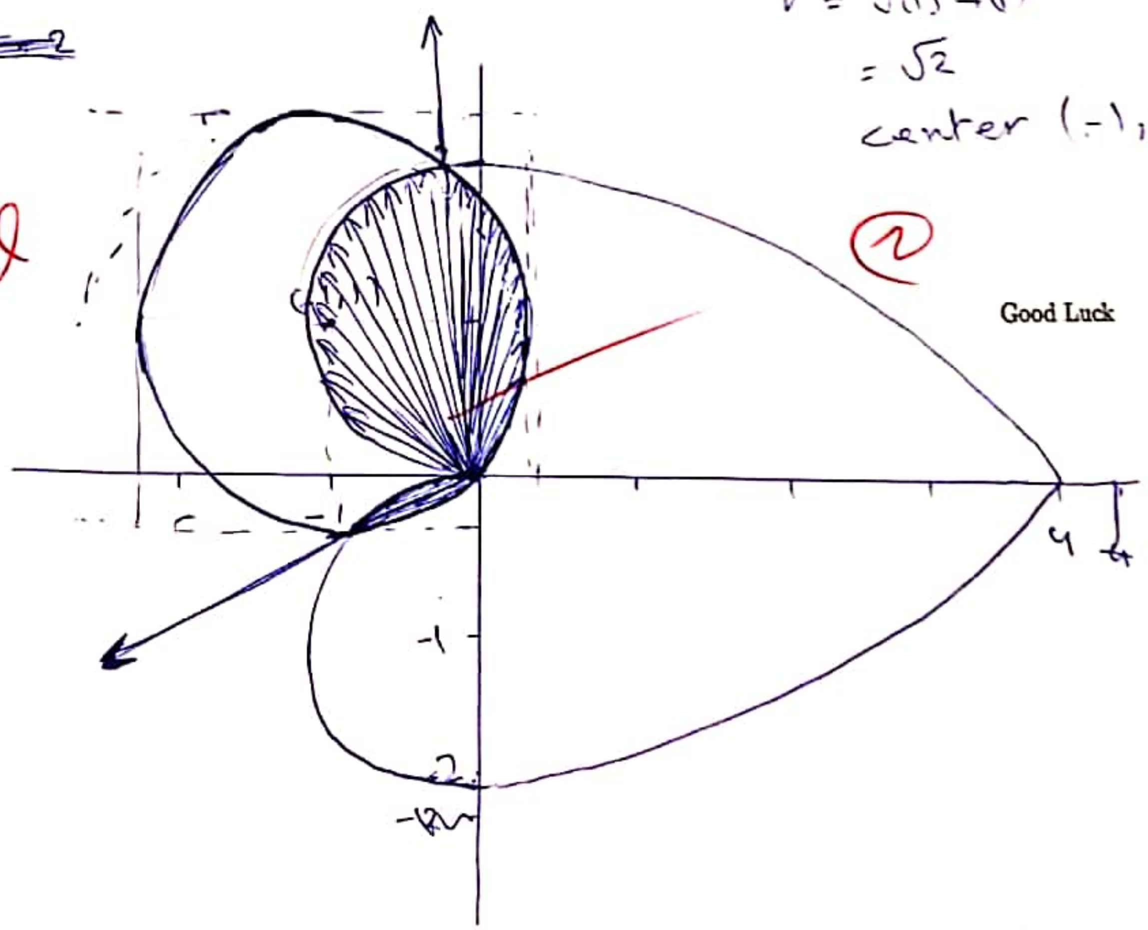
~~$2 + 2\cos\theta = \sin\theta - \cos\theta$~~

~~$\sin\theta - 3\cos\theta = 2$~~

$r = \sqrt{(1)^2 + (1)^2}$

$= \sqrt{2}$

center $(-1, 1)$



v. good

2

Good Luck

Calculus 2. Final Exam [redacted] 1

وقت المحاضرة:

[redacted] ment of Mathematics

الاسم/باللغة العربية

مدرس المادة:

Q1: Determine, by whatever method you wish, whether the following series are convergent or divergent. Circle "C" if the series is convergent or "D" if the series is divergent. (9 points)

1. C D $\sum_{n=1}^{\infty} e^{\frac{1}{n}}$

2. C D $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$

3. C D $\sum_{n=4}^{\infty} \frac{1}{n\sqrt[3]{\ln(n)}}$

4. C D $\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$

5. C D $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

6. C D $\sum_{n=1}^{\infty} (e^{\frac{1}{n}} - e^{\frac{1}{n+1}})$

Q2: Select the sum for each of the following series. (6 points)

1. $\sum_{n=1}^{\infty} (\frac{2}{e})^n =$ $\frac{2}{e-2}$ e^2 $\sin(\frac{1}{2})$ [illegible]

2. $\sum_{n=0}^{\infty} \frac{(-1)^n (2)^{2n+1}}{e^{2n+1}(2n+1)!} =$ [illegible] e^2 $\sin(\frac{2}{e})$ $\frac{1}{2}$

3. $\sum_{n=0}^{\infty} \frac{2^n}{n!} =$ [illegible] e^2 $\sin(\frac{2}{e})$ $\frac{1}{2}$

4. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{3^{2n}(2n)!} =$ [illegible] e^2 $\sin(\frac{2}{e})$ $\frac{1}{2}$

Q3: Select the best correct answer for each of the following. b (25 points)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a	a
b	b	b	b	b	b	b	b	b	b	b	b	b	b	b	b	b
c	c	c	c	c	c	c	c	c	c	c	c	c	c	c	c	c
d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d
e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e	e
f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f	f

1. $\int_{2}^{4} \frac{4x^2}{x-1} dx =$

a. $33+\ln(9)$. b. $6+\ln(3)$. c. $32+\ln(81)$. d. $15+\ln(9)$. e. $15+\ln(27)$. f. $7+\ln(81)$.

2. $\int \sec^2 x \sqrt[3]{\tan x}\, dx =$

a. $-\frac{3}{4}\sqrt[3]{\sec^4 x}+C$. b. $3\sqrt[3]{\sec x}+C$. c. $-\frac{3}{4}\sqrt[3]{\cos^4 x}+C$. d. $\frac{-1}{\sqrt[3]{\tan x}}+C$. e. $\frac{3}{4}\sqrt[3]{\tan^2 x}+C$. f. $\frac{3}{4}\sqrt[3]{\tan^4 x}+C$.

3. According to the method of partial fractions, there is an equation of the form $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$ for some numbers A, B, and C. What is the number C.

a. 1. b. 2. c. −1. d. $\frac{1}{2}$. e. $\frac{3}{2}$. f. −2.

4. Convert the polar equation $r^2 = \sec\theta \csc\theta$ to rectangular form

a. $2y - 3x = 2$.
b. $y = x(x^2 + y^2)$.
c. $x^2 + y^2 - 3x - 2y = 0$.
d. $xy = 1$.
e. $3x - 2y = 2$.
f. $x = y(x^2 + y^2)$.

Mhsen_otaibi

5. Which one of the following sequences is divergent.

a. $a_n = 5^n$. b. $a_n = (0.7)^n$. c. $a_n = \frac{3n-1}{2n+3}$. d. $a_n = \sqrt[n]{2}$. e. $a_n = (0.5)^n$. f. $a_n = \frac{\sin n}{n}$.

6. Given that the sequence a_n defined by the recurrence relation $a_1 = 2$, $a_{n+1} = \frac{1}{2}(a_n + 8)$ for $n = 1, 2, 3, \cdots$ is convergent, find its limit.

a. 8. b. 1. c. 4. d. 5. e. 2. f. 0.

7. Determine which one of the following geometric series is divergent.

a. $\sum_{k=1}^{\infty} (\cos 1)^k$. b. $\sum_{k=1}^{\infty} \frac{1}{(\sqrt{2})^k}$. c. $\sum_{k=1}^{\infty} \frac{e^k}{\pi^k}$. d. $\sum_{k=1}^{\infty} (0.2)^k$. e. $\sum_{k=1}^{\infty} e^k$. f. $\sum_{k=1}^{\infty} \frac{1}{e^k}$.

8. Find the limit of the sequence $a_n = n e^{-n}$.

a. 1. b. $\frac{1}{2}$. c. $-\pi$. d. π. e. 2. f. 0.

9. If the nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $S_n = \frac{n-1}{2n+1}$. Find $\sum_{n=1}^{\infty} a_n$.

a. 1. b. -2. c. -1. d. 2. e. $\frac{1}{2}$. f. $-\frac{1}{2}$.

10. Find the area of the region that enclosed by $r = \cos\theta$.

a. 3π. b. 2π. c. π. d. 4π. e. $\frac{\pi}{4}$. f. 9π.

11. $\int_1^e 2\ln x\, dx =$

a. π. b. 2. c. 1. d. $-\pi$. e. -1. f. ~~-2~~.

By using a trigonometric substitution $x = 4\sec\theta$, where $0 \le \theta < \frac{\pi}{2}$, the integral $\int \frac{16}{x^2\sqrt{x^2-16}}\, dx$ can be ...nsformed into one of the following integrals.

a. $\int \cos(\theta)\, d\theta$. b. $\int \csc(\theta)\, d\theta$. c. $\int \sin^2(\theta)\, d\theta$. d. $\int \csc^2(\theta)\, d\theta$. e. $\int \cot^2(\theta)\, d\theta$. f. $\int \cos^2(\theta)\, d\theta$.

13. Integrating $\int \tan^{-1} x\, dx$ by parts in correct way, we obtain an expression of the form $A - \int B\, dx$, then B=

a. $\frac{x^2}{2(1+x^2)}$. b. $-\frac{x}{\sqrt{1-x^2}}$. c. $\frac{x}{\sqrt{1-x^2}}$. d. $\frac{x^3}{2\sqrt{1-x^2}}$. e. $\frac{x}{1+x^2}$. f. $-\frac{x^3}{2\sqrt{1-x^2}}$.

14. Find the area that the parametric curve $x = 6\sin(t)$, $y = 3\cos(t)$, $0 \le t \le 2\pi$ encloses it.

a. 10π. b. 8π. c. 12π. d. 18π. e. 14π. f. 6π.

15. The interval of convergence for the series $\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$ is

a. $[\frac{5}{2}, \frac{7}{2})$. b. $[\frac{5}{2}, \frac{7}{2}]$. c. $[-6, 2)$. d. $(\frac{5}{2}, \frac{7}{2}]$. e. $(-6, 2]$. f. $[-6, 2]$.

16. The interval of convergence for the series $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{4^n . \ln(n)}$ is

a. $(-4, 4)$. b. $(-5, 5)$. c. $[-4, 4]$. d. $[-4, 4)$. e. $(-5, 5]$. f. $(-4, 4]$.

17. Find a power series representation for the function $f(x) = \frac{3}{x+2}$.

a. $\sum_{n=0}^{\infty} \frac{2.x^n}{3^{n+1}}$. b. $\sum_{n=0}^{\infty} \frac{3.x^n}{2^{n+1}}$. c. $\sum_{n=0}^{\infty} \frac{2.(-1)^n.x^n}{3^{n+1}}$. d. $\sum_{n=0}^{\infty} \frac{2.x^n}{3^n}$. e. $\sum_{n=0}^{\infty} \frac{3.(-1)^n.x^n}{2^{n+1}}$. f. $\sum_{n=0}^{\infty} \frac{3.x^n}{2^n}$.

The End .

Q1) Dir or con — نريد معرفة

— أي يجب إجراء أي Test من اي Test درسناهم و كل Test له اسلوب معين

1) ? مش شايف السؤال

2) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1} \rightarrow$ Alt Test $\rightarrow$ لوجود $(-1)^n$

مرتبة الحل $\rightarrow \lim_{n\to\infty} \frac{1}{n+1} = \frac{1}{\infty} = 0$ (a_n)

conv By Alt

3) $\sum_{n=4}^{\infty} \frac{1}{n \sqrt[9]{\ln n}}$

تأكيد $\rightarrow \frac{1}{x \sqrt[p]{\ln x}} \rightarrow p > 1$ conv

4) مش شايف السؤال

يتبع

①

5) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$?

$\Rightarrow$ Ratio Test $\rightarrow$ لوجود Power ، ! of n

$a_{n+1} = \frac{2^{n+1}}{(n+1)!}$

قلوب a_n

$\lim_{n\to\infty} \left|\frac{2^{n+1}}{(n+1)!}\right| \cdot \left|\frac{n!}{2^n}\right|$

$\lim_{n\to\infty} \frac{\cancel{2^n}\cdot 2^1}{(n+1)\cancel{(n)!}} \cdot \frac{\cancel{n!}}{\cancel{2^n}}$

$\lim_{n\to\infty} \frac{2}{n+1} = \frac{2}{\infty} = 0 < 1$

con By Ratio

2

Q2) Sum :

عندما نحصل على ثابت
مرفوع لـ (n) و يجانبه
ثابت آخر ، نضربه
Geometric series

① $\sum_{n=1}^{\infty} \left(\frac{2}{e}\right)^{n}$

عدد نيبيري

$|r| = \left|\frac{2}{2.72}\right| < 1$ conv

شرط بداية العدّاد من ٠

$\sum_{n=0}^{\infty} \left(\frac{2}{e}\right)^{n+1}$

$\sum_{n=0}^{\infty} \left(\frac{2}{e}\right)^{n} \left(\frac{2}{e}\right)$

$Sum = \frac{a}{1-r} = \frac{2/e}{1-2/e}$

$= \frac{\frac{2}{e}}{\frac{e*1}{e*1} - \frac{2}{e}} = \frac{\frac{2}{\cancel{e}}}{\frac{e-2}{\cancel{e}}}$

$= \frac{2}{e-2}$

3

② $$\sum_{n=0}^{\infty} \frac{(-1)^n \cdot (2)^{2n+1}}{e^{2n+1} \cdot (2n+1)!}$$

↳ لدينا صيغة حفظ $\Rightarrow$ $$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)!}$$

$\Rightarrow$ نرتب سؤالنا على الصيغة

$$\sum_{n=0}^{\infty} \frac{(-1)^n \cdot \left(\frac{2}{e}\right)^{2n+1}}{(2n+1)!} = \sin\left(\frac{2}{e}\right)$$

---

③ $$\sum_{n=0}^{\infty} \frac{2^n}{n!} \; ?$$

$\Rightarrow$ صيغة جاهزة. $\Rightarrow$ $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\text{So, } \sum_{n=0}^{\infty} \frac{2^n}{n!} = e^2$$

④

(4) $\sum_{n=0}^{\infty} \frac{(-1)^n \cdot \pi^{2n}}{3^{2n} (2n)!}$

صيغة حفظ $\rightarrow \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n}}{(2n)!}$

So, $\sum_{n=0}^{\infty} \frac{(-1)^n \cdot \left(\frac{\pi}{3}\right)^{2n}}{(2n)!} = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

Q3) (1) $\int_2^4 \frac{4x^2}{x-1}\,dx$?

درجة البسط ≥ درجة المقام

← قسمة طويلة

$\int$ الناتج + $\frac{\text{الباقي}}{\text{المقسوم عليه}}$

$$\begin{array}{r} 4x+4 \\ x-1 \overline{)\, 4x^2 \qquad} \\ \ominus\, 4x^2 \oplus 4x \\ \hline 4x \\ \ominus\, 4x \oplus 4 \\ \hline 4 \end{array}$$

$\int_2^4 4x+4 + \frac{4}{x-1}$

$\frac{4x^2}{2} + 4x + 4\ln|x-1| \Big]_2^4$

$2x^2 + 4x + 4\ln|x-1| \Big]_2^4$

$= [32 + 16 + 4\ln 3] - [8 + 8 + 4\cancel{\ln 1}^{\,0}]$

$= 32 + \cancel{16} + 4\ln 3 - \cancel{16} = 32 + 4\ln 3$

$= 32 + \ln 3^4$

(S) شبع

(2) حسن واضح

(3) $\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3} = \frac{x}{(x-1)(x-2)(x-3)}$

نوحد المقام

$A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) = x$

$x=3$ $x=3$

بدنا C ، لازم نصفر A ، B

when $x=3$

$\hookrightarrow 0 + 0 + C(3-1)(3-2) = 3$

$C(2)(1) = 3$

$\boxed{C = \frac{3}{2}}$

---

(4) ~~$r^2 = \sec\theta\csc\theta$~~

$r^2 = \sec\theta \csc\theta$

$r^2 = \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$ ضرب تبادلي

(6)

$r^2 \cos\theta \sin\theta = 1$

$\underbrace{r\cos\theta} \cdot \underbrace{r\sin\theta} = 1$

$x \cdot y = 1 \rightarrow \boxed{xy = 1}$

5

a) $a_n = 5^n \rightarrow \lim\limits_{n\to\infty} 5^n = 5^\infty = \infty$ Div

b) $a_n = (0.7)^n \rightarrow \lim\limits_{n\to\infty} (0.7)^\infty = 0$ conv

(كسر)$^\infty = 0$

c) $a_n = \dfrac{5n-1}{2n+3}$

درجة البسط = المقام

$\lim\limits_{n\to\infty} \dfrac{5n-1}{2n+3} = \dfrac{5}{2}$

لـه $\dfrac{\text{معامل البسط}}{\text{معامل المقام}}$

conv

d) $a_n = \sqrt[n]{2}$

$\hookrightarrow \lim\limits_{n\to\infty} 2^{\frac{1}{n}} = 2^{\frac{1}{\infty}} = 2^0 = 1$ conv

e) $a_n = (0.8)^n$?

$\lim\limits_{n\to\infty} (0.8)^\infty = 0$

(كسر)$^\infty = 0$ conv

الجواب a

7

6. سيتم ارفاقه
حله لاحقاً

7. Geo → Div شرط

$|r| > 1$

a) $|\cos 1| = 0 < 1$ conv

b) $\left|\frac{1}{\sqrt{2}}\right|^{k} = \frac{1}{\sqrt{2}} < 1$ con $\because$ $\boxed{1^{k} = 1}$

c) $\left|\frac{e}{5}\right| = \left|\frac{2.72}{5}\right| < 1$ conv

d) $|0.2| < 1$ conv

e) $|2.72| > 1$ Div

f) $\left|\frac{1}{\pi}\right| < 1$ conv

الجواب e

8

8) $\lim_{n \to \infty} n e^{-n}$

$\lim_{n \to \infty} \frac{n}{e^n} = \frac{\infty}{\infty}$! L.R $\frac{\text{مشتقة البسط}}{\text{مشتقة المقام}}$

$= \frac{1}{e^n \cdot 1 \cdot \cancel{\ln e}} = \frac{1}{e^n}$

$\lim_{n \to \infty} \frac{1}{e^n} = \frac{1}{\infty} = 0$

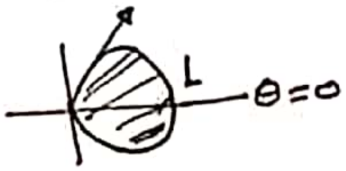
conv

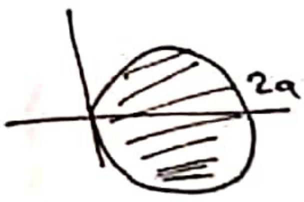
9) $\sum a_n = \lim_{n \to \infty} S_n$ $\boxed{\text{قاعدة}}$

$= \lim_{n \to \infty} \frac{n-1}{2n+1} = \frac{1}{2}$

10) $r = \cos\theta \longrightarrow$ قاعدة

$r = 2a \cos\theta$

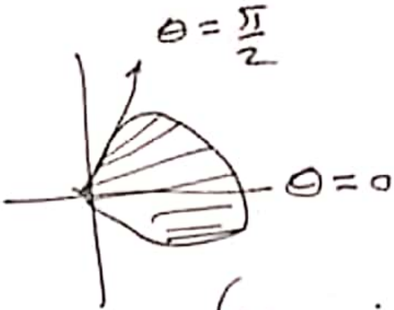




نصفر r لايجاد θ

$0 = \cos\theta \rightarrow \theta = \frac{\pi}{2} \rightarrow$ نهاية الربع الأول

9



تماثل ، نضرب التكامل بـ 2

و نكامل جزء واحد

$$A = \frac{1}{2}\int_{\alpha}^{\beta} (f(\theta))^2 - (g(\theta))^2 \, d\theta$$

$$A = 2 * \frac{1}{2}\int_{0}^{\frac{\pi}{2}} \cos^2\theta - 0 \, d\theta$$

لـ تماثل

$$A = \int_{0}^{\frac{\pi}{2}} \cos^2\theta \, d\theta$$

متطابقة

$$A = \int_{0}^{\frac{\pi}{2}} \frac{1}{2}(1 + \cos 2\theta) d\theta$$

$$A = \frac{1}{2}\int_{0}^{\frac{\pi}{2}} 1 + \cos 2\theta \, d\theta$$

$$\frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_{0}^{\frac{\pi}{2}}$$

١٠

و يكمل --

(11) $\int_1^e 2 \ln x \, dx =$

Sol:

$2 \int_1^e \ln x \, dx$ By Parts

$= \left(\cancel{x} \ln x \cdot x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} dx \right)$

$\left(x \ln x - \int dx \right)$

$x \ln x - x \Big]_1^e$

$2 \left[\left[e \cancel{\ln e}^{\,1} - e \right] - \left[1 \cancel{\ln 1}^{\,0} - 1 \right] \right]$

$2 \left[\cancel{e} - \cancel{e} + 1 \right] = 2$

[redacted]

(12) $x = 4\sec\theta$ $\displaystyle\int \frac{16}{x^2\sqrt{x^2-16}}$

بده شكل التكامل بـ θ باستخدام

Trig sub

$\Rightarrow x = 4\sec\theta \longrightarrow dx = 4\sec\theta\tan\theta\, d\theta$

$\boxed{x^2 = 16\sec^2\theta}$

$x^2-16 = 16\sec^2\theta - 16$

$= 16(\sec^2\theta - 1)$

$= 16\tan^2\theta$

$\sqrt{x^2-16} = 4\tan\theta$

$$\int \frac{\cancel{16}}{\cancel{16}\sec^{\cancel{2}}\theta \cdot \cancel{4\tan\theta}}\ \cancel{4\sec\theta}\cancel{\tan\theta}\, d\theta$$

$$\int \frac{1}{\sec\theta}\, d\theta = \int \cos\theta\, d\theta$$

(12)

(13)

بدي تحت B بعد

ما نوصل لشكل التكامل

$$A - \int B\,dx$$

$$\hookrightarrow \int \tan^{-1}x\,dx$$

$$\Rightarrow \tan^{-1}x \cdot x - \underbrace{\int x \cdot \frac{1}{1+x^2}\,dx}_{B}$$

$$B = \frac{x}{1+x^2}$$

---

(14) ~~$x = 6\sin t$~~ ~~$0 < t < 2\pi$~~

~~$y = 3\cos t$~~

~~لايجاد التكامل سنعوض احد الحدود في~~

~~الاقترانات البارامترية عن هو الاقتران~~

~~الذي استخدمناه في بداية التكامل~~

~~(سهل طريقة سريعة)~~

~~التعويض الاكبر عشان الاقتران الاصغر~~

التوجل

~~B~~

~~نتبع~~

13

CamScanner الممسوحة ضوئيا بـ

(15) $\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$ is?

لإيجاد Interval of convergence نضطر إما Ratio أو Root

دودوقًا طبقوا الـ Ratio

$a_{n+1} = \frac{2^{n+1} \cdot (x-3)^{n+1}}{\sqrt{n+4}}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$\lim_{n\to\infty} \left|\frac{2^{n+1} \cdot (x-3)^{n+1}}{\sqrt{n+4}}\right| \cdot \left|\frac{\sqrt{n+3}}{2^n (x-3)^n}\right|$

$\lim_{n\to\infty} \left|\frac{\cancel{2^n} \cdot 2^1 \cancel{(x-3)^n} (x-3)}{\sqrt{n+4}}\right| \left|\frac{\sqrt{n+3}}{\cancel{2^n} \cdot \cancel{(x-3)^n}}\right|$

النهاية تخص (n)

$\lim_{n\to\infty} 2 \cancel{\frac{\sqrt{n+3}}{\sqrt{n+4}}} |x-3|$

درجة البسط = المقام

الجواب: معامل أعلى درجة في البسط / معامل المقام

$\Rightarrow 2(1)|x-3|$

يتبع (14)

$2|x-3| < 1 \rightarrow$ شرط ratio في conv دوما

$$\frac{2}{2}|x-3| < \frac{1}{2}$$

$$|x-3| < \frac{1}{2}$$

$$-\frac{1}{2} < x-3 < \frac{1}{2}$$

$$-\frac{1}{2} + \frac{3 * 2}{1 * 2} < x < \frac{1}{2} + \frac{3 * 2}{1 * 2}$$

$$\frac{5}{2} < x < \frac{7}{2}$$

الآن نعوض قيمة كل (X) بسؤالنا و نختبر السيريز على أي Test تعلمناه

اذا conv ← فترة مغلقة

Div ← فترة مفتوحة

15

CamScanner الممسوحة ضوئيا بـ

$X = \frac{5}{2}$;

$$\sum_{n=1}^{\infty} \frac{2^n \left(\frac{5}{2} - \frac{3}{1}\right)^n}{\sqrt{n+3}}$$

$$\sum_{n=1}^{\infty} \frac{2^n \left(\frac{-1}{2}\right)^n}{\sqrt{n+3}} = \sum_{n=1}^{\infty} \frac{2^n (-1)^n \cdot \left(\frac{1}{2}\right)^n}{\sqrt{n+1}}$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{\left(\cancel{2} * \frac{1}{\cancel{2}}\right)^n (-1)^n}{\sqrt{n+1}}$$

$$\sum_{n=1}^{\infty} \frac{\cancel{(1)^n}^{\,1} (-1)^n}{\sqrt{n+1}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$$

So, $\lim_{n \to \infty} \frac{1}{\sqrt{n+1}} = \frac{1}{\infty} = 0$

conv By Alt

16

$x = \frac{7}{2}$,

$$\sum_{n=1}^{\infty} \frac{2^n \left(\frac{7}{2} - 3\right)^n}{\sqrt{n+3}}$$

$$\sum_{n=1}^{\infty} \frac{2^n \left(\frac{1}{2}\right)^n}{\sqrt{n+3}} \to \sum_{n=1}^{\infty} \frac{2^n \cdot \left(\frac{1}{2}\right)^n}{\sqrt{n+3}}$$

$$\sum_{n=1}^{\infty} \frac{\left(2 * \frac{1}{2}\right)^n}{\sqrt{n+3}} \to \sum_{n=1}^{\infty} \frac{(1)^n}{\sqrt{n+3}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+3}}$$

so, Apply L.C.T

$$b_n = \frac{1}{(n)^{\frac{1}{2}}} = \frac{1}{n^{\frac{1}{2}}}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{\sqrt{n+3}} \cdot \sqrt{n}$$

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+3}} = 1$$

17

$\frac{\text{معامل البسط}}{\text{معامل المقام}}$

$$0 < 1 < \infty$$

$\sum b_n = \frac{1}{n^{1/2}}$, $p < 1$ Div By P-series

$\sum a_n$ Div By L.C.T

يتبع

So, Interval $\left[\frac{5}{2}, \frac{7}{2}\right]$

---

(16) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{4^n \ln n}$ is :

نفس السابق

sol

$$a_{n+1} = \frac{(-1)^{n+1} \cdot x^{n+1}}{4^{n+1} \ln(n+1)}$$

$$\lim_{n\to\infty} \left| \frac{\cancel{(-1)^{n+1}} \cdot x^{n+1}}{4^{n+1} \cdot \ln(n+1)} \right| \cdot \left| \frac{4^n \cdot \ln n}{\cancel{(-1)^n}\, x^n} \right|$$

مطلق

$$\lim_{n\to\infty} \left| \frac{\cancel{x^n} \cdot x}{\cancel{4^n} \cdot 4 \cdot \ln(n+1)} \right| \cdot \left| \frac{\cancel{4^n} \cdot \ln n}{\cancel{x^n}} \right|$$

$$\lim_{n\to\infty} \frac{1}{4} \cdot |x| \cdot \frac{\ln n}{\ln(n+1)}$$

$$\lim_{n\to\infty} \frac{1}{4} \frac{\ln\infty}{\ln\infty} |x|$$

$$\frac{1}{4}\left(\frac{\infty}{\infty} LR\right)|x|$$

يتبع

(18)

CamScanner الممسوحة ضوئيا بـ

$$\frac{1}{4} \cdot \frac{\frac{1}{n}}{\frac{1}{n+1}} \cdot |x|$$

$$\frac{1}{4} \quad \frac{n\cancel{+1}}{\cancel{n}} |x|$$

$$\frac{1}{4} |x| < 1$$

$$|x| < 4$$

$$-4 < x < 4$$

$$x=4 ; \sum_{n=1}^{\infty} \frac{(-1)^n \cancel{(4)^n}}{\cancel{4^n} \ln n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$$

$$\lim_{n\to\infty} \frac{1}{\ln n} = \frac{1}{\ln \infty} = \frac{1}{\infty} = 0$$

Conv By Alt

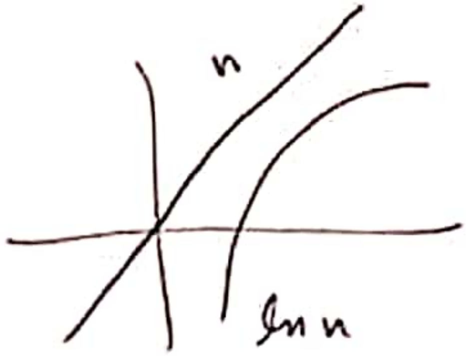
$$x=-4 ; \sum_{n=1}^{\infty} \frac{(-1)^n (-4)^n}{4^n \ln n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n (-1)^n \cdot 4^n}{4^n \ln n} = \sum_{n=1}^{\infty} \frac{\cancel{((-1)^2)^n}^{\,1} \cdot \cancel{4^n}}{\cancel{4^n} \ln n}$$

$$= \sum_{n=1}^{\infty} \frac{1}{\ln n}$$

يتبع

[illegible]



$\ln n < n$

$b_n \quad \dfrac{1}{\ln n} > \dfrac{1}{n} \quad a_n$

Comparison Test

$\sum a_n = \sum \frac{1}{n}$ $p \le 1$ Div

So, $\sum b_n$ Div

Int $\cancel{[4, -4)}$ $(-4, 4]$

---

17) $f(x) = \dfrac{3}{x+2}$?

$$\Rightarrow \frac{3}{x+2} = 3 \cdot \frac{1}{x+2} = 3 \cdot \frac{1}{2+x}$$

$$= \frac{3}{2\left(1+\frac{x}{2}\right)} = \frac{3}{2} \cdot \frac{1}{1+\frac{x}{2}} = \frac{3}{2} \sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{2}\right)^n$$

$$= \sum_{n=0}^{\infty} \frac{3(-1)^n x^n}{2^{n+1}}$$

20

Thm : If $\lim\limits_{n\to\infty} a_n = L$, and f is cont at L, then

$$\lim_{n\to\infty} f(a_n) = f\left(\lim_{n\to\infty} a_n\right) = f(L)$$

Ex: $a_0 = 1$, $a_{n+1} = \frac{1}{2}\left(a_n + \frac{2}{a_n}\right)$.

Show that $\lim\limits_{n\to\infty} a_n = \sqrt{2}$?

Solution:

$$a_1 = \frac{1}{2}\left(a_0 + \frac{2}{a_0}\right) = \frac{1}{2}(1+2) = \frac{3}{2}$$

$$a_2 = \frac{1}{2}\left(a_1 + \frac{2}{a_1}\right) = \frac{1}{2}\left(\frac{3}{2} + \frac{2}{\frac{3}{2}}\right) = \frac{8}{12} = \frac{4}{6}$$

نلاحظ أن المسلسلة متزايدة لأن الحد التالي أكبر من الحد السابق وهكذا.

$\Rightarrow$ let $\lim\limits_{n\to\infty} a_n = L$, $\lim\limits_{n\to\infty} a_{n+1} = L$

So, $$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \frac{1}{2}\left(a_n + \frac{2}{a_n}\right) = \lim_{n\to\infty} \frac{a_n}{2} + \lim_{n\to\infty} \frac{1}{a_n}$$

$$L = \frac{L}{2} + \frac{1}{L} \Rightarrow L - \frac{L}{2} = \frac{1}{L} \Rightarrow \frac{L}{2} = \frac{1}{L}$$

$L^2 = 2 \Rightarrow L = \pm\sqrt{2}$, So $L = \sqrt{2}$ (since is inc)