

## 7. Question \*

(2 Points)

$z = e^x \cos(xy)$ . Find the  $x$  - intercept  
of the equation of the tangent plane

to the surface at  $\left(1, \frac{\pi}{2}, 0\right)$

$\pi e - 1$

$\frac{1}{2}$

$\pi e$

$-2$

$2$

$\pi - e$

$\pi + e$

$e$

In what direction does  
 $f(x, y) = xe^{-y} + 3y$   
have the minimum rate of  
change at the point  $(1, 0)$ ?

$$(\sqrt{2}, \sqrt{2})$$

$$(1, 2)$$

$$\langle -1, 3 \rangle$$

$$\sqrt{5}$$

$$\sqrt{5}$$

$$(1, 1)$$

$$\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$$

$$(-1, 1)$$

**14. Question \***  
**(2 Points)**

Let  $f(x, y) = -\sqrt{4 - \frac{x^2}{9} - \frac{y^2}{4}}$ .

Find the range of  $f$ .

$\mathbb{R}$

$(-\infty, -2]$

$[-2, 2]$

$[0, 2]$

$[2, \infty)$

$(-2, 0]$

$(-3, 0]$

$[0, 1]$

**15. Question \* [5]**  
**(2 Points)**

Given the function

$$f(x, y) = \sqrt{8 + 8x + 4y - 4x^2 + y^2}.$$

The level curves  
of  $f(x, y)$  are

- ellipses
- hyperbolae
- triangles
- rectangles
- points
- lines
- paraboloids

14. Question \* 

(2 Points)

Let  $f(x, y) = -x^3 - y^2 + 2x - 2y - 6$ .

Which of the following statements  
best describes the point  $(1, -1)$ ?

(1, -1) is a  
local min

(1, -1) is not  
a critical point

(1, -1) is a  
local max

(1, -1) is a  
saddle point

(1, -1) is an  
absolute min  
and local min

(1, -1) is an  
absolute max  
and local max

15. Question \*

**13. Question**   
**(2 Points)**

Find the absolute minimum value of  
the function  $f(x,y) = x^2 + 3y^2 + 2y$ ,  
on the unit disk  $x^2 + y^2 \leq 1$ .

A  $-\frac{1}{3}$

B 0

C 6

D  $-\frac{1}{2}$

E 5

F -3

G 8

H 4

**12. Question \* [5]  
(2 Points)**

Find  $\lim_{(x,y) \rightarrow (1,1)} \frac{xy}{\sqrt{x} - \sqrt{y}}$

2

-8

-16

Does not exist

-2

0

-8

16

**11. Question \* **  
**(2 Points)**

Let  $f(x, y) = -\sqrt{4 - \frac{x^2}{9} - \frac{y^2}}.$

Find the range of  $f.$

[0, 2]

[2,  $\infty$ )

[0, 1]

[-2, 0]

(- $\infty$ , -2]

[-2, 2]

$\mathbb{R}$

[-3, 0]

(2 Points)

$$\text{Let } f(x, y) = \begin{cases} \frac{\cos(y) \sin(x)}{x} & : x \neq 0 \\ k + 1 & : x = 0 \end{cases}$$

Find  $k$  that makes  $f(x, y)$  continuous at the points  $(0, 0)$ .

$k = 4$

$k = 0$

$k = 6$

$k = -3$

$k = 2$

$k = \frac{1}{7}$

$k = \frac{5}{2}$

$k = 7$

**9. Question •**

(2 Points)

Let  $\vec{V}f(2, 3) = (1, 1)$ .

Find  $\lim_{h \rightarrow 0} \frac{f\left(2 + \frac{h}{\sqrt{2}}, 3 + \frac{h}{\sqrt{2}}\right) - f(2, 3)}{h}$ .

A  $\frac{3}{\sqrt{2}}$

B  $\sqrt{7}$

C  $2\sqrt{2}$

D  $\frac{5}{\sqrt{2}}$

E  $\sqrt{3}$

F  $\sqrt{2}$

G  $-2$

H  $\frac{1}{\sqrt{2}}$

**9. Question \* [5]**  
**(2 Points)**

Let  $\vec{\nabla}f(2, 3) = (1, 1)$ .

Find  $\lim_{h \rightarrow 0} \frac{f\left(2 + \frac{h}{\sqrt{2}}, 3 + \frac{h}{\sqrt{2}}\right) - f(2, 3)}{h}$ .

①  $\frac{3}{\sqrt{2}}$

②  $\sqrt{7}$

③  $2\sqrt{2}$

④  $\frac{5}{\sqrt{2}}$

⑤  $\sqrt{3}$

⑥  $\sqrt{2}$

⑦  $-2$

⑧  $\frac{1}{\sqrt{2}}$

**8. Question \* **  
**(2 Points)**

Let  $f(x, y) = 2xe^{-(x^2+y^2)}$ . How many critical points  $f(x, y)$  has?

1 there is no critical points

2 9

3 5

4 1

5 2

6 3

7 4

8 6

## 6. Question \* [5]

(2 Points)

If  $w = f(x, y)$ , where  $x = x(t, \theta)$ ,  
 $y = y(x, t)$ ,  $t = t(\theta)$ . Which formula  
 below gives us  $\frac{\partial w}{\partial \theta}$ ?

A)  $\frac{\partial w}{\partial x} \frac{\partial x}{\partial y} \frac{\partial y}{\partial t}$

B)  $\frac{\partial w}{\partial x} \frac{dx}{dt} \frac{dt}{d\theta} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial \theta}$

C)  $\frac{\partial w}{\partial \theta} \frac{\partial \theta}{\partial x} + \frac{\partial w}{\partial \theta} \frac{\partial \theta}{\partial y}$

D)  $\frac{\partial w}{\partial x} \frac{dx}{dt} \frac{dt}{d\theta} + \frac{\partial w}{\partial y} \frac{\partial y}{dt} \frac{dt}{d\theta}$   
 $+ \frac{\partial w}{\partial y} \frac{\partial y}{\partial \theta}$

E)  $\frac{\partial w}{\partial x} \frac{dx}{dt} \frac{dt}{d\theta} + \frac{\partial w}{\partial y} \frac{\partial y}{dt} \frac{dt}{d\theta}$

F)  $\frac{\partial w}{\partial y} \frac{dy}{dt} \frac{dt}{d\theta} + \frac{\partial w}{\partial t} \frac{dt}{d\theta}$

G)  $\frac{\partial w}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial t}$

H)  $\frac{\partial w}{\partial x} \frac{dx}{dt} \frac{dt}{d\theta} + \frac{\partial w}{\partial y} \frac{\partial y}{dt} \frac{dt}{d\theta}$

**5. Question : [5]  
(2 Points)**

In what direction does

$$f(x, y) = xe^{-y} + 3y$$

have the minimum rate of  
change at the point  $(1, 0)$ ?

(−1, 3)

(−1, −2)

(1, 2)

( $\sqrt{2}, \sqrt{2}$ )

$\sqrt{5}$

$-\sqrt{5}$

(0, 3)

(2, 0)

**4. Question •**   
**(2 Points)**

Suppose  $f$  is a differentiable function,  
and define  $g(u, v) = f(3u - v, u^2 + v)$ .

Find  $\frac{\partial g}{\partial v}$  at  $(u, v) = (2, -1)$  if

$$f(2, -1) = 6, \quad g(2, -1) = -7,$$

$$f_x(2, -1) = 1, \quad f_y(2, -1) = 9,$$

$$f(7, 3) = 4, \quad g(7, 3) = 2,$$

$$f_x(7, 3) = -3, \quad f_y(7, 3) = 5$$

6

-10

-7

3

11

5

8

### 3. Question \*

(2 Points)

Find  $f_x(0, 0)$  if

$$f(x, y) = \sqrt{8(x^3 - y^3)}.$$

$\infty$

0

-2

4

2

$-\infty$

does not exist

$\sqrt{5}$

**2. Question •**   
**(2 Points)**

Find  $f_{x,y}$ , if

$$f(x, y) = \frac{y}{x + \ln(x)} - x^3y^2.$$

2

$\frac{1}{x + \ln(x)}$

12

4

does not exist

6

-12

0

1. Question  (2 Points)

Find  $\lim_{(x,y) \rightarrow (0,0)} \frac{3y^2 \sin(x)}{x^2 + y^2}$

does not exist

-2

2

8

0

-1

4

1