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23.5

The Hashemite University	Department of Mathematics	March 26, 2014
Calculus (3)	Second Exam	Time: 1 hour
Name (in Arabic): ________	Student Number: ________	Serial Number: ________
Section number or lecture time:		Instructor name: ________

Please do all your work in this booklet and show all the steps. Calculators and notes are not allowed.

1. (4 points) Find the curvature for the curve $\vec{r}(t) = 2t\,\hat{\imath} + 2\cos(t)\,\hat{\jmath} + 2\sin(t)\,\hat{k}$

$K(t) = \frac{|T'(t)|}{|r'(t)|}$ ⇒ $r'(t) = 2\hat{i} - 2\sin t\,j + 2\cos t\,\hat{k}$

$|r'(t)| = \sqrt{4 + 4(\sin^2 t + \cos^2 t)} = \sqrt{4+4} = 2\sqrt{2}$

$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{2\hat{i} - 2\sin t\,j + 2\cos t\,\hat{k}}{2\sqrt{2}} = \frac{\sqrt{2}}{2}\hat{i} - \frac{\sin t}{\sqrt{2}} j + \frac{\cos t}{\sqrt{2}}\hat{k}$

$T'(t) = -\frac{\cos t}{\sqrt{2}} j - \frac{\sin t}{\sqrt{2}} k$ $\quad |T'(t)| = \sqrt{\frac{\sin^2 t}{2} + \frac{\cos^2 t}{2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

$= \frac{\frac{1}{\sqrt{2}}}{2\sqrt{2}} = \frac{1}{4} = K$

2. (6 points) Find the limit if it exists or show the limit does not exist.

(a) $\lim_{(x,y,z)\to(0,0,0)} \frac{3x^2 + 3y^2 + 3z^2}{\ln(x^2+y^2+z^2)}$ Let $t = x^2 + y^2 + z^2$ then

$\lim_{t\to 0} \frac{3t}{\ln t} \overset{H.R}{=} \lim_{t\to 0} \frac{3}{\frac{1}{t}} = 0$

(b) $\lim_{(x,y)\to(0,0)} \frac{x^4 y}{x^8 + y^2}$ Let $y = x^4$ then → along $y = x^4$

$\lim_{x\to 0} \frac{x^4 x^4}{x^8 + x^8} = \frac{1}{2}$

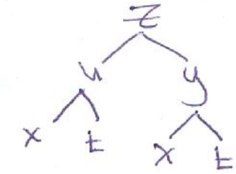
along y - axis $x = 0$

$\lim_{y\to 0} \frac{(0)y}{0 + y^2} = 0$

$\lim_{(x,y)\to(0,0)} \frac{x^4 y}{x^8 + y^2} =$ Does not exist

3. (3 points) If $z = f(3x-3t) + g(4x-4t)$, show that z satisfies the equation $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial t} = 0$

Let $u = 3x-3t$ and $y = 4x-4t$ then $z = F(u) + g(y)$



$\frac{dz}{dx} = \frac{dz}{du}\frac{du}{dx} + \frac{dz}{dy}\frac{dy}{dx}$

$z_u = F'(u)$ $z_y = g'(y)$

$u_x = 3$ $y_x = 4$

$3F'(u) + 4g'(y) = \frac{dz}{dx}$

$\frac{dz}{dt} = \frac{dz}{du}\frac{du}{dt} + \frac{dz}{dy}\frac{dy}{dt}$

$u_t = -3$ $y_t = -4$

$\frac{dz}{dt} = -3F'(u) - 4g'(y)$

$\frac{dz}{dx} + \frac{dz}{dt} = 3F'(u) + 4g'(y) - 3F'(u) - 4F'(y)$

$= 0$

//

4. (6 points) Let $f(x,y,z) = xyz + xze^y$.

(a) Find the directional derivative of f at (1,0,0) in the direction of $\vec{u} = 1\hat{i} + 2\hat{j} + 2\hat{k}$.

$D_uF = \nabla f \cdot u$

$\nabla f = \langle f_x, f_y, F_z \rangle$

$f_x = yz + ze^y \Rightarrow f_x(1,0,0) = 0$

$f_y = xz + xze^y \Rightarrow f_y(1,0,0) = 0$

$f_z = xy + xe^y \Rightarrow f_z(1,0,0) = 1$

$\nabla F = \langle 0, 0, 1 \rangle$

$D_uF = (\hat{k}) \cdot (i + 2j + 2k) =$

$D_uF = i + 2j + 2\hat{k}$

$-\frac{1}{2}$

(b) What is the maximum rate of change for f at the point (1,0,0), in what direction does it occur.

maximum rate of chang $= |\nabla F|$

$\nabla F = \langle 0, 0, 1 \rangle$

$|\nabla f| = \sqrt{0 + 0 + 1} = 1$

in the direction of ∇f $\Rightarrow$ $u = \langle 1, 2, 2 \rangle$

$-\frac{1}{2}$

5. (4 points) Find equations of the tangent plane and normal line to the surface $z - 2x^2 + y^2 = 3$ at the point (1,1,4).

$z = 2x^2 - y^2 + 3$

normal line $= \langle z_x, z_y, -1 \rangle$

$z_x = 4x = 4$

$z_y = -2y = -2$ -1 line

normal Line $= \langle 4, -2, -1 \rangle$

$(z - z_0) = z_x(x - x_0) + z_y(y - y_0)$

$z - 4 = 4(x-1) + (-2)(y-1)$

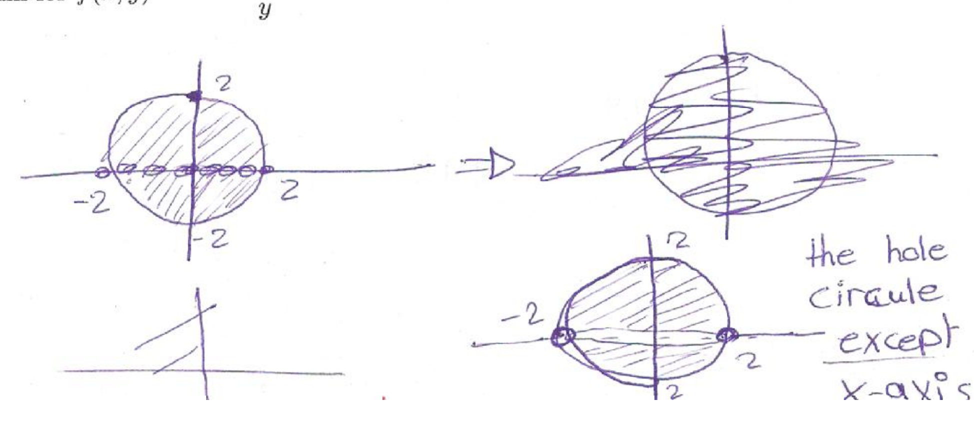
$z = 4x - 2y + 2 \Rightarrow$ tangent plane

6. (2 points) Sketch the domain for $f(x,y) = \frac{\sqrt{4 - y^2 - x^2}}{y}$

$4 - y^2 - x^2 \geq 0$

$4 \geq y^2 + x^2$

$y \neq 0$



the hole circule except x-axis

The Hashemite University | Department of Mathematics | 2-12-2013

Calculus (3) | Second Exam | Time: 1 Hour

الاسم: .... الرقم الجامعي: .... الرقم المتسلسل:(   )

رقم الشعبة(وقت المحاضرة):.. اسم المدرس:

	1	2	3	4	5	6	7	8	9	10	11	12
A		X	X				X	X	X			X
B				X	X							
C	X									X	X	
D						X						

Qusetion1: (**2 Points for each**) choose the best correct answer:

1. If $4z^2 + ln(y-x) = y^2$. Then z_x at (1,2,1) =

A) -8   C) 8

B) 1/8   D) -1/8

2. The unit tangent vector $\vec{T}(t)$ of the vector $\vec{r}(t) =< tan^2 t, cos^2 t, sin^2 t >$at the point $t = \frac{\pi}{4}$ is:

A) $< \frac{4}{\sqrt{18}}, -\frac{1}{\sqrt{18}}, \frac{1}{\sqrt{18}} >$ C) $< \frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}}, -\frac{1}{\sqrt{18}} >$

B) $< -\frac{1}{\sqrt{18}}, \frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}} >$ D) $< \frac{1}{\sqrt{18}}, -\frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}} >$

3. The point on the curve $\vec{r}(t) =< 4cost, 4sint, e^t >, 0 \le t \le \pi$, where the tangent lines is parallel to the plane $x + y = 1$ is:

A) $(2\sqrt{2}, 2\sqrt{2}, e^{\frac{\pi}{4}})$ C) $(\sqrt{3}, 1, e^{\frac{\pi}{3}})$

B) $(\sqrt{3}, 1, e^{\frac{\pi}{6}})$ D) $(\sqrt{2}, \sqrt{2}, e^{\frac{\pi}{4}})$

4. If $\vec{r}'(t) =< 2t, 3t^2, \sqrt{t} >$ and $\vec{r}(0) =< 1,2,0 >$. Then $\vec{r}(t)$ =

A) $< t^2 + 1, t^3, \frac{2}{3}t^{\frac{3}{2}} - \frac{2}{3} >$ C) $< t^2 + 1, t^3, \frac{2}{3}t^{\frac{3}{2}} - \frac{1}{3} >$

B) $< t^2 + 1, t^3 + 2, \frac{2}{3}t^{\frac{3}{2}} >$ D) $< t^2 + 2, t^3 + 1, \frac{2}{3}t^{\frac{3}{2}} >$

5. The curvature of $\vec{r}(t) =< 5\cos t, 5\sin t, 5t >$ at the point (5,0,0) is:

A) 1/4   C) 1/8

B) 1/10   D) 1/6

6. $\lim_{(x,y,z)\to(0,0,0)} \frac{e^{x^2+y^2+z^2}-1}{5x^2+5y^2+5z^2} =$

A) 1/2   C) 1/5

B) 1   D) 0

7. Let $z = f(5x - y, y - 5x)$. Which of the following is true:

A) $z_x - 5z_y = 0$ C) $z_x + 5z_y = 0$

B) $z_x + z_y = 0$ D) $z_x - z_y = 0$

8. The maximum rate of change of $f(x, y) = y\sqrt{x}$ at the point (1,2) is:

A) $\sqrt{8}$ C) $\sqrt{65}$

B) $\frac{\sqrt{65}}{2}$ D) $\sqrt{2}$

9. $\int_0^1 (\frac{2t}{1+t^2}\hat{\imath} + \frac{4}{1+t^2}\hat{k})dt$

A) $(ln2)\hat{\imath} + \pi\hat{k}$ C) $\pi\hat{\imath} + (ln2)\hat{k}$

B) $\pi\hat{\jmath} + (ln2)\hat{k}$ D) $(ln2)\hat{\imath} + \pi\hat{\jmath}$

10. The equation of the tangent plane of z=ln(2y-3x) at the point (1,2,0) is:

A) $z = 2x - y - 1$ C) $z = -3x + 2y - 1$

B) $z = 3x - 2y - 1$ D) $z = x - 2y - 1$

Good Luck

11. $\lim_{(x,y)\to(0,0)} \frac{6xy}{\sqrt{x^2+y^2}} =$

A) 2

B) 0

C) Does not exist

D) 6

12. The range of the function $f(x,y) = \ln(x^4 + y^2 + 1)$

A) $\mathbb{R}^2$

B) $\mathbb{R}$

C) $[0,\infty)$

D) $(-\infty,0]$

Good Luck

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The Hashemite University
Calculus (3)
Name (in Arabic): [illegible]
Section number or lecture time: 10:20 - 11:20

Department of Mathematics
Second Exam

July 4, 2013
Time: 1 hour

Student Number: ______ Serial Number: 35

Instructor name: سمير حمزة الدلايش

Please do all your work in this booklet and show all the steps. Calculators, cell phones and notes are not allowed.

8 5 $7\frac{1}{2}$

1. (5 points) Let, $f(x,y) = xy^2 + e^{2xy} + \sin(2x)$.

(a) Compute $f_{xyx}(0,1) - f_x(1,0)$.

$f_x = y^2 + 2ye^{2xy} + 2\cos 2x$

$f_{xy} = 2y + 2y \cdot 2x e^{2xy} + e^{2xy} \cdot 2$

$f_{xyx} = 4yx(2ye^{2xy}) + e^{2xy}(4y) + 4ye^{2xy}$

when $(0,1) \Rightarrow 4 + 4 = 8 = f_{xyx}$

$f_x(1,0) = 2\cos 2$

$f_{xyx}(0,1) - f_x(1,0) = \boxed{8 - 2\cos 2}$

(b) Find the directional derivative of f at the point $(0,1)$ in the direction of $\vec{u} = 3i + 4j$. $\Rightarrow |\vec{u}| = 5$

$D_{\hat{u}} f(x,y) = \nabla f \cdot \hat{u}$

$\nabla f = \langle y^2 + 2ye^{2xy} + 2\cos 2x \,,\, 2xy + 2xe^{2xy} \rangle$

$\nabla f \big|_{(0,1)} = \langle 1 + 2 + 2 \,,\, 0 \rangle = \langle 5, 0 \rangle$

$D_{\hat{u}} f(x,y) = \langle 5, 0 \rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle = 3 + 0 = ③$

2. (3 points) Evaluate the curvature for $\vec{r}(t) = -\cos t\,\hat{i} + 2t\,\hat{j} + \sin t\,\hat{k}$

$\kappa(t) = \frac{|r' \times r''|}{|r'|^3}$

$r' = \langle \sin t, 2, \cos t \rangle \qquad |r'| = \sqrt{5}$

$r'' = \langle \cos t, 0, -\sin t \rangle \qquad |r''| = 1$

$(r' \cdot r'')^2 = (\sin t \cos t + 0 - \sin t \cos t)^2 = 0$

$|r' \times r''| = \sqrt{(\sqrt{5})^2 \cdot 1 - 0} = \sqrt{5}$

$\kappa(t) = \frac{\sqrt{5}}{(\sqrt{5})^3} = \frac{\sqrt{5}}{5\sqrt{5}} = \left(\frac{1}{5}\right)$

3. (6 points) Find the limit if it exists or show the limit does not exist.

(6)

(a) $\lim_{(x,y)\to(0,0)} \dfrac{x^4+y^4+2x^2y^2}{3\sin(x^2+y^2)}$ $\quad 0/0 \quad$ d.n.e

C_1: $x = t$ $\quad y = 0$

$(x,y)\to(0,0) \to t\to 0$

(0)

$\lim_{t\to 0} \dfrac{t^4}{3\sin t^2} = \lim_{t\to 0} \dfrac{4t^3}{6t\cos t^2} = \lim_{t\to 0} \dfrac{2}{3}\dfrac{t^2}{\cos t^2} = \dfrac{2}{3}\cdot\dfrac{0}{1} = \dfrac{0}{3} = 0$

??

~~C_2: $x = t$ $\quad y = t$~~

~~$\lim_{t\to 0} \dfrac{2t^4+2t^4}{3\sin 2t^2} = \lim_{t\to 0} \dfrac{4t^4}{3\sin 2t^2} = \lim_{t\to 0} \dfrac{16t^3}{12t\cos 2t^2} =$~~

(b) $\lim_{(x,y)\to(0,-2)} \dfrac{x^3-(y+2)^5}{4x^3+(y+2)^5}$ $\quad 0/0 \quad$ d.n.e

C_1: $x = t$ $\quad y = -2$

$\lim_{t\to 0} \dfrac{t^3-0}{4t^3} = \dfrac{1}{4}$ $\quad (x,y)\to(0,-2) \to t\to 0$

$(x,y)\to(0,-2) \to t\to 0$

(3)

C_2: $x = t$ $\quad y = t-2$

$\lim_{t\to 0} \dfrac{t^3-(t-2+2)^5}{4t^4+(t-2+2)^5} = \lim_{t\to 0} \dfrac{t^3-t^5}{4t^4+t^5} = \lim_{t\to 0} \dfrac{3t^2-5t^4}{16t^3+5t^4}$

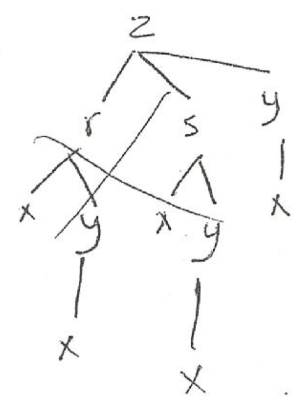
$\lim_{t\to 0} \dfrac{6t-20t^3}{48t^2+20t^3} = \lim_{t\to 0} \dfrac{6-60t}{96t+60t^2} = \dfrac{6}{0}$ d.n.e

4. (3 points) If $z = f(r,s,y)$, $r = g(x,y)$, $s = h(x,y)$ and $y = 2x$. Write a formula for $\dfrac{dz}{dx}$

(2)

$\dfrac{dz}{dx} = \dfrac{dz}{dr}\cdot\dfrac{dr}{dx} + \dfrac{dz}{ds}\cdot\dfrac{ds}{dx} + \dfrac{dz}{dy}\cdot\dfrac{dy}{dx}$

$\dfrac{dz}{dx} = z_r \cdot r_x + z_s \cdot s_x + z_y \cdot 2$



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5. (3 points) If the directional derivative of f at the point (1,2) in the direction of $\hat{u} = <\frac{3}{5}, \frac{4}{5}>$ equals $\frac{6}{5}$ and the directional derivative of f at the point (1,2) in the direction of $\hat{v} = <\frac{\sqrt{3}}{2}, \frac{1}{2}>$ equals $\sqrt{3}$. Find the maximum directional derivative of f at (1,2).

$D_{\hat{u}} f(x,y) = \nabla f(x,y) \cdot \hat{u}$

$\frac{6}{5} = \nabla f \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle \Rightarrow \langle a,b \rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle = \boxed{\frac{3}{5}a + \frac{4}{5}b = \frac{6}{5}}$

$\sqrt{3} = \nabla f \cdot \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle \Rightarrow \langle a,b \rangle \cdot \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle = \boxed{\frac{\sqrt{3}a}{2} + \frac{1}{2}b = \sqrt{3}}$

~~Maximum = |∇f|~~

$3a + 4b = 6$

$-4(\sqrt{3}a + b = 2\sqrt{3})$

$= -4\sqrt{3}a - 4b = -8\sqrt{3}$

$3a + 4b = 6$

$3 - 4\sqrt{3}a = 6 - 8\sqrt{3}$

$a = \frac{6 - 8\sqrt{3}}{3 - 4\sqrt{3}}$

$\Rightarrow b \Rightarrow \frac{18 - 24\sqrt{3}}{15 - 20\sqrt{3}} + \frac{4}{5}b = \frac{6}{5}$

$b = \left(\frac{18 - 24\sqrt{3}}{15 - 20\sqrt{3}} + -\frac{6}{5}\right) \cdot \frac{5}{4}$

0

6. (5 points) Let, $f(x,y) = 3xy^2 - 3x^2 - 6y^2 + 1$

(a) Show that the critical points of f are (0,0), (2,−2) and (2,2).

$f_x = 3y^2 - 6x = 0$

$f_y = 6xy - 12y = 0$

$y(6x - 12) = 0$

$y = 0 \Rightarrow$ $6x = 12 \Rightarrow \boxed{x = 2}$

$y = 0 \Rightarrow (0,0)$

$x = 2 \Rightarrow 3y^2 = 12 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$

(2,2) (2,−2)

Critical pts

(0,0)

(2,2)

(2,−2)

2

(b) Classify the critical points in part (a) as local minimum, local maximum and saddle point.

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$f_{xx} = -6$

$f_{yy} = 6x - 12$

$f_{xy} = 6y$

$D = -36x + 72 - 36y^2$

3

Pt	D	f_{xx}	Conclusion
(0,0)	72	−6	Maximum local pt at (0,0)
(2,2)	−36·4		Saddle pt
(2,−2)	−36·4		Saddle pt

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(8)

" وما توفيقي إلا بالله "

The Hashemite University	Second Exam	December 10, 2012
Department of Mathematics	Calculus (3)	Time: One Hour..

اسم الطالب: [redacted]   الرقم الجامعي: [redacted]   رقم التسلسل: 3

مدرس المادة: وقت المحاضرة:

Question One (13 points): Complete each of the following sentences by filling your answer in the box

(11)

2 points 1. Two curves C_1 and C_2 that pass through the origin and show that the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$$

does not exist are:

C_1: $x = t$ $y = t$
and
C_2: $x = t$ $y = t^2$

2 points 2. If $\frac{\ln(1+z)+4}{xy^2-3z} = 1$, then

$\left.\frac{\partial z}{\partial y}\right|_{(1,2,0)} =$ ~~[crossed out]~~ $-\left(\frac{-4}{4}\right) = +1$

2 points 3. If $z = \frac{x}{y} + y,\ x = \ln(2-t^2),\ y = t^2 - s^2 + 4$, then $\frac{\partial z}{\partial t}$ at the point $P_0(s,t) = (1,-1)$ equals

$\left.\frac{\partial z}{\partial t}\right|_{P_0} = \frac{-22}{16} = \frac{-11}{8}$

Final solution.

2 points 4. If $f(x,y) = e^{xy}$, then

$f_{xyx}(0,4) =$ $f_{xyx} = xy^2e^{xy} + e^{xy}\cdot y + y\cdot e^{xy}$
$= 0(16)e^0 + e^0(4) + 4(e^0) = 0 + 4 + 4 = 8$

2 points 5. As relative maximum, relative minimum, or saddle point, the function $f(x,y) = x^3 + y^3 - 3x - 12y$

has ....Saddle Point......... at the critical point $(1,-2)$

3 points 6. The value of the number k that makes the function

$$f(x,y) = \begin{cases} \frac{x^4+6x^2y^2+9y^4}{x^2+3y^2} + 1 & ,(x,y) \neq (0,0) \\ 3k & ,(x,y) = (0,0) \end{cases}$$

continuous everywhere is

$k = \frac{1}{3}$

3 الفقرة

الجواب الصحيح هو d.

9

Question Two (12 points): Choose the best correct answer and fill it in the following table.

10

	1	2	3	4	5	6
a						
b						
c						
d						

1. $\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{3(x^2+y^2)^2} =$

(a) $\frac{1}{9}$ (b) $\frac{1}{6}$ (c) $-\frac{1}{9}$ (d) $\frac{1}{12}$

2. The directional derivative of $f(x,y,z) = x^3 - x^2y + z^2$ at the point $P(0,-1,-2)$ in the direction of $\vec{a} = 2i - j + 2k$ equals

(a) $-\frac{8}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{4}{3}$

3. The curvature $\kappa(t)$ for the circular helix $x = 3\cos t,\ y = 3\sin t,\ z = t$ is

(a) $\kappa(t) = \frac{5}{26}$ (b) $\kappa(t) = \frac{4}{17}$ (c) $\kappa(t) = \frac{2}{5}$ (d) $\kappa(t) = \frac{3}{10}$

4. The directional derivative of $f(x,y,z)$ at the point $P(x_0,y_0,z_0)$ in the direction of $\vec{a} = -2i - j + 2k$ equals -6 and $\|\nabla f(x_0,y_0,z_0)\| = 6$. Then $\nabla f(x_0,y_0,z_0) =$

(a) $4i + 2j - 4k$ (b) $-4i + 2j + 4k$ (c) $-4i - 2j + 4k$ (d) $-4i + 2j - 4k$

5. The equation of the tangent plane to the surface $x^2 - yz + z^2 = 2$ at the point $P(1,0,-1)$ is

(a) $-y + 3z = 4$ (b) $2x + z = 2$ (c) $2x + y - 2z = 4$ (d) $-2x - z = 2$

6. The parametric equations of the normal line to the surface $x^2 - yz + z^2 = 2$ at the point $P(1,0,-1)$ are

(a) $x = 1 - t,\ y = 0,\ z = -1 - 3t$
(b) $x = 1 + 3t,\ y = 0,\ z = -1 + t$
(c) $x = 1 + 2t,\ y = t,\ z = -1 - 2t$
(d) $x = 1,\ y = -t,\ z = -1 + 3t$

End of Exam

Good Luck

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Excellent

(10)

The Hashemite University	Second Exam	December 10, 2012
Department of Mathematics	Calculus (3)	Time: One Hour....

اسم الطالب: [redacted]   الرقم الجامعي: [redacted]   رقم التسلسل: ٥

مدرس المادة: د. عمر حرز الله   وقت المحاضرة: ١٠ — ١١

Question One (13 points): Complete each of the following sentences by filling your answer in the box

13

2 points

1. Two curves C_1 and C_2 that pass through the origin and show that the limit
$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2}$$
does not exist are:

C_1: $x = t$ $y = t$
and
C_2: $x = t^{1/3}$ $y = t$

2 points

2. If $\frac{\ln(1+z)+4}{xy^2-5z} = 1$, then

$\left.\frac{\partial z}{\partial y}\right|_{(1,2,0)} = +\frac{2}{13}$

2 points

3. If $z = \frac{x}{y} + y,\ x = \ln(2-t^2),\ y = t^2 - s^2 + 1$, then $\frac{\partial z}{\partial t}$ at the point $P_0(s,t) = (1,-1)$ equals

$\left.\frac{\partial z}{\partial t}\right|_{P_0} = 0$

2 points

4. If $f(x,y) = e^{xy}$, then

$f_{xyx}(0,1) = 2$

2 points

5. As relative maximum, relative minimum, or saddle point, the function $f(x,y) = x^3 + y^3 - 3x - 12y$

has ...the relative maximum... at the critical point $(-1,-2)$

3 points

6. The value of the number k that makes the function
$$f(x,y) = \begin{cases} \frac{x^4+6x^2y^2+9y^4}{x^2+3y^2} + 1 & ,(x,y) \neq (0,0) \\ 5k & ,(x,y) = (0,0) \end{cases}$$
continuous everywhere is

$k = \frac{1}{5}$

11

Question Two (12 points): Choose **the best correct** answer and fill it in the following table.

12

	1	2	3	4	5	6
a						
b						
c						
d						

1. $\lim_{(x,y)\to(0,0)} \dfrac{1-\cos(x^2+y^2)}{5(x^2+y^2)^2} =$

(a) $-\frac{1}{15}$ (b) $\frac{1}{15}$ (c) $\frac{1}{20}$ (d) $\frac{1}{10}$

2. The directional derivative of $f(x,y,z) = x^3 - x^2y + z^2$ at the point $P(0,-1,2)$ in the direction of $\vec{a} = 2i - j + 2k$ equals

(a) $-\frac{8}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{4}{3}$

3. The curvature $\kappa(t)$ for the circular helix $x = 5\cos t,\ y = 5\sin t,\ z = t$ is

(a) $\kappa(t) = \frac{5}{26}$ (b) $\kappa(t) = \frac{4}{17}$ (c) $\kappa(t) = \frac{2}{5}$ (d) $\kappa(t) = \frac{3}{10}$

4. The directional derivative of $f(x,y,z)$ at the point $P(x_0,y_0,z_0)$ in the direction of $\vec{a} = 2i - j + 2k$ equals -6 and $\|\nabla f(x_0,y_0,z_0)\| = 6$. Then $\nabla f(x_0,y_0,z_0) =$

(a) $4i + 2j - 4k$ (b) $-4i + 2j + 4k$ (c) $-4i - 2j + 4k$ (d) $-4i + 2j - 4k$

5. The equation of the tangent plane to the surface $x^2 - yz + z^2 = 1$ at the point $P(1,-1,0)$ is

(a) $-y + 3z = 4$ (b) $2x + z = 2$ (c) $2x + y - 2z = 4$ (d) $-2x - z = 2$

6. The parametric equations of the normal line to the surface $x^2 - yz + z^2 = 1$ at the point $P(1,-1,0)$ are

(a) $x = 1 - 3t,\ y = -1,\ z = -t$
(b) $x = 1 + 2t,\ y = -1,\ z = t$
(c) $x = 1 + 2t,\ y = -1 + t,\ z = -2t$
(d) $x = 1,\ y = -1,\ z = 3t$

End of Exam

Good Luck.

The Hashemite University Department of Mathematics Calculus (3)		Second Exam Date:1/5/2012 Time : One Hour

S.Name [redacted] S.Univ.Number: 1031817

Time or Sec. Num.. Serial Num: 12

*Choose the correct answer and fill it in the following table.

QI	1	2	3	4	5	6	7	8	9	10	11	12	13
(a)		✓			✓	✓				✓			
(b)	✓						✓		✓				
(c)			✓	✓							✓	✓	
(d)								✓					✓

Q_1) The arc length parameterization of the line $x = 1 + t,\ y = 3 - 2t,\ z = 4 + 2t$, that has reference point $(1, 3, 4)$ and the same orientation as the given line is:

(a) $x = 1 - \frac{s}{3},\ y = 3 + \frac{2s}{3},\ z = 4 - \frac{2s}{3}$ (b) $x = 1 + \frac{s}{3},\ y = 3 - \frac{2s}{3},\ z = 4 + \frac{2s}{3}$

(c) $x = \frac{s}{3},\ y = \frac{-2s}{3},\ z = \frac{2s}{3}$ (d) $x = 1 + \frac{s}{3},\ y = 3 + \frac{2s}{3},\ z = 4 - \frac{2s}{3}$

Q_2) The unit normal vector $N(t)$ to the graph $C: r(t) = \cos t\, i + \sin t\, j + k$ at $t = \frac{\pi}{4}$ is

(a) $-\frac{\sqrt{2}}{2}(i + j)$ (b) $-\frac{1}{2}(i + \sqrt{3}\, j)$ (c) $\frac{\sqrt{2}}{2}(i + j)$ (d) $\frac{1}{2}(-i + \sqrt{3}\, j)$

Q_3) The curvature $\kappa(t)$ of the function $r(t) = e^t\, i + e^{-t}\, j + tk$ at $t = 0$ is

(a) $\frac{1}{3\sqrt{2}}$ (b) $\frac{\sqrt{2}}{2\sqrt{3}}$ (c) $\frac{\sqrt{2}}{3}$ (d) $\frac{\sqrt{2}}{3\sqrt{3}}$

Q_4) The graph of the function $z = x^2 - 2x + y^2$ represents

(a) Cylinder (b) an upper hemisphere

(c) Circular paraboloid (d) an upper half of a cone

Q_5) Let $f(x, y) = \begin{cases} \frac{\sin(x^2 + y^2)}{x^2 + y^2} & , (x, y) \neq (0, 0) . \\ 1 & ,(x, y) = (0, 0) . \end{cases}$, one of the following statements is true:

(a) $f(x, y)$ is continuous at (0.0).

(b) $\lim_{(x,y)\to(0,0)} f(x, y)$ does not exist.

(c) $f(x, y)$ is not continuous at $(1, 2)$.

(d) $f(x, y)$ is continuous only on the region $R = \{(x, y) : x > 0, y > 0\}$.

Q6) $\lim_{(x,y)\to(1,1)} \dfrac{xy + 2 - 2x - y}{x - 1} =$

(a) D.N.E (b) -1 (c) 0 (d) 1

Q7) The slope of the surface $f(x, y) = x^2 + 3xy + y - 1$ at the point $(4, -5)$ in the x-direction is :

(a) 13 (b) -7 (c) 7 (d) -5

Q8) If $z = f(x, y)$ is a function that is defined implicitly as $yz - \ln z = x + y$, then $\dfrac{\partial z}{\partial x} =$

(a) $\dfrac{z}{xz - 1}$ (b) $\dfrac{z(1 - z)}{yz - 1}$ (c) $\dfrac{z(1 - z)}{xz - 1}$ (d) $\dfrac{z}{yz - 1}$

Q9) Let $w = x^2 + y^2$, $x = r - s$, $y = r + s$, then $\dfrac{\partial w}{\partial s} =$

(a) $2(r - s)$ (b) $4s$ (c) $2r + 2s$ (d) $4r$

Q10) The directional derivative of the function $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $v = 3\,i - 4\,j$ is :

(a) -1 (b) 5 (c) -5 (d) 1

Q11) Let $f(x, y, z) = x + y + z$. For any unit vector $u = a\,i + b\,j + c\,k$, $D_u f(x, y, z) =$

(a) 0 (b) $ax + by + cz$ (c) $a + b + c$ (d) D.N.E

Q12) The equation for the tangent plane to the surface $z = x^{\frac{1}{2}} + y^{\frac{1}{2}}$ at the point $p(4, 9, 5)$ is :

(a) $2x + y - 4z = -6$ (b) $x + 2y - 4z + 6 = 0$
(c) $3x + 2y - 12z = -30$ (d) $2x + 3y - 12z + 30 = 0$

Q13) If $z = x^2 y$, $x = t^2$, $y = t + 7$, then $\dfrac{dz}{dt}\Big|_{t=-1} =$

(a) 33 (b) -60 (c) 144 (d) -23

GOOD LUCK

The Hashemite University
Math. Department

Calculus III, Second Exam
Second semester 2010/2011

Date: Dec 8, 2011
Time: 50 Min

الرقم الجامعي:

اسم الطالب:

الرقم المتسلسل: موعد المحاضرة:

Warning: You MAY NOT use calculators on this exam.

Part I: Choose the best correct answer and fill it in the following table: (2 points each)

	1	2	3	4	5	6	7	8	9	10	11	12	13
a													
b													
c													
d													

1. The maximum value of a directional derivative of $f(x,y) = xy - x^2 + y^2$ at $(2,1)$

(a) 1 (b) 5 (c) 7 (d) -5

2. The curvature κ of the curve $C: \mathbf{r}(t) = 4\cos t\mathbf{i} + 4\sin t\mathbf{j} + 2t\mathbf{k}$ at $t = 0$

(a) $\kappa = \frac{1}{10}$ (b) $\kappa = \frac{1}{5}$ (c) $\kappa = \frac{1}{3}$ (d) $\kappa = \frac{2}{3}$

3. $\lim\limits_{\substack{(x,y)\to(0,0)\\ (\text{along } y=-x)}} \frac{\sin^2(3xy)}{3x^2+y^2} =$

(a) $\frac{-3}{4}$ (b) does not exist (c) 0 (d) $\frac{3}{4}$

4. $\lim\limits_{(x,y)\to(0,0)} e^{1/(x^2+y^2)}$

a) 0 (b) does not exist (c) 1 (d) ∞

5. Let $f(x,y,z) = x^2+y^2+z^2$. Then the directional derivative of f in the direction of $\underline{v} = \underline{i} - \underline{j} + \underline{k}$ at $(1,0,1)$ is

(a) $\frac{4}{\sqrt{3}}$ (b) 4 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

6. Let $f(x,y) = xy + x^2 + y^2 - 6y$. Then f has a

(a) a relative min. at $(4,-2)$ (b) a relative max. at $(-2,4)$
(c) a relative min. at $(-2,4)$ (d) a saddle point at $(4,-2)$

7. Let $f(x,y)$ be a differentiable function on $xy-$plane with $f_x(x,y) = xy^2+2x$, $f_y(x,y) = x^2y-6$, and $f(0,1) = -4$. Then $f(2,2) =$

(a) 0 (b) 2 (c) 26 (d) 11

Consider the function $f(x,y) = x^2+y^2-4x$ on the closed region $R: x^2+y^2 \leq 5,\ y \geq 1$.
Use the function $f(x,y)$ and the region R to answer the questions (8-9).

8. The absolute maximum value of the function $f(x,y)$ on R is

(a) 9 (b) 13 (c) $5+4\sqrt{5}$ (d) $6+4\sqrt{5}$

9. The absolute minimum value of the function $f(x,y)$ on R is

(a) -4 (b) 1 (c) $5-4\sqrt{5}$ (d) -3

10. The unit tangent vector to the graph of the curve $C: \mathbf{r}(t) = 2\cosh t\mathbf{i} + 2\sinh t\mathbf{j}$ at $t=0$ is

(a) $T(0)=\mathbf{i}$ (b) $T(0)=\mathbf{j}$ (c) $T(0)=-\mathbf{i}$ (d) $T(0)=-\mathbf{j}$

11. Consider the surface $S: 2x^2+y^2-z^2=11$. Then the parametric equations for the normal line L_N to S at $(2,2,1)$ are

(a) L_N: $x=2+2t,\ y=1+2t,\ z=1+t$

(b) L_N: $x=8+2t,\ y=4+2t,\ z=-2+t$

(c) L_N: $x=2+4t,\ y=2+2t,\ z=1-t$

(d) L_N: $x=2+t,\ y=1+t,\ z=1-t$

12. Consider the surface $S: z=f(x,y)$ with $f(2,1)=4$ and $\nabla f(2,1) = -4\mathbf{i}+4\mathbf{j}$. Then the equation of the tangent plane P_T to S at $(2,1,4)$ is

(a) $P_T: 4x-4y+z=8$

(b) $P_T: 4x+4y+z=16$

(c) $P_T: -4x+4y+z=8$

(d) $P_T: -4x-4y+z=-8$

13. Suppose $z=x+y,\ x=uv,\ y=v\cos(u)$. Then $\dfrac{\partial z}{\partial v}=$

(a) $v-v\sin(u)$ (b) $u+\cos(u)$ (c) $v-\cos(u)$ (d) $u-v\sin(u)$.

The Hashemite University
Math. Department

Calculus III, Second Exam
Second semester 2010/2011

Date: Dec 6, 2011
Time: 50 Min

16

الرقم الجامعي: ١٠٣٣٦٣٩

اسم الطالب: [redacted]

الرقم المتسلسل:......... موعد المحاضرة: ٩ - ١٠

Warning: You MAY NOT use calculators on this exam.

Part I: Choose the best correct answer and fill it in the following table: (2 points each)

	1	2	3	4	5	6	7	8	9	10	11	12	13
a	✗	✗	✓			✓	✗		✓		✗		✓
b		✓						✗		✓		✗	
c				✗	✓			✓			✓		
d	✓			✓			✓					✓	

12

الإجابة a

1. The maximum value of a directional derivative of $f(x,y) = xy + x^2 + y^2$ at $(1,2)$

(a) $\sqrt{41}$ (b) $2\sqrt{2}$ (c) $\sqrt{7}$ (d) 7

2. The curvature κ of the curve $C : r(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + 4t\mathbf{k}$ at $t = 0$

(a) $\kappa = \frac{1}{10}$ (b) $\kappa = \frac{1}{5}$ (c) $\kappa = \frac{1}{3}$ (d) $\kappa = \frac{2}{3}$

3. $\lim_{\substack{(x,y)\to(0,0) \\ (\text{along } y=x)}} \frac{\sin^2(3xy)}{3x^2+y^2} =$

(a) 0 (b) does not exist (c) 1 (d) $\frac{3}{4}$

4. $\lim_{(x,y)\to(0,0)} e^{-1/(x^2+y^2)}$

(a) 1 (b) does not exist (c) 0 (d) ∞

5. Let $f(x,y,z) = x^2 + y^2 - z$. Then the directional derivative of f in the direction of $v = i - j + k$ at $(1,0,1)$ is

(a) 1 (b) 0 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{5}$

6. Let $f(x,y) = xy + x^2 + y^2 - 6x$. Then f has a

(a) a relative min. at $(4,-2)$
(b) a relative max. at $(4,-2)$
(c) a relative min. at $(-2,4)$
(d) a saddle point at $(4,-2)$

7. Let $f(x,y)$ be a differentiable function on xy−plane with $f_x(x,y) = xy^2 + x^2$, $f_y(x,y) = x^2y - 6$, and $f(0,1) = -4$. Then $f(2,3) =$

(a) $\frac{14}{3}$ (b) $\frac{2}{3}$ (c) 17 (d) 11

Consider the function $f(x,y) = x^2+y^2 - 2x$ on the closed region $R : x^2+y^2 \leq 5,\ y \geq 1$.
Use the function $f(x,y)$ and the region R to answer the questions (8-9)

8. The absolute maximum value of the function $f(x,y)$ on R is

(a) 0 (b) 9 (c) $5+2\sqrt{5}$ (d) $6+2\sqrt{5}$

9. The absolute minimum value of the function $f(x,y)$ on R is

(a) 0 (b) 1 (c) $5-2\sqrt{5}$ (d) -1

10. The unit tangent vector to the graph of the curve $C : r(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j}$ at $t = 0$ is

(a) $T(0) = i$ (b) $T(0) = j$ (c) $T(0) = -i$ (d) $T(0) = -j$

11. Consider the surface $S : x^2+2y^2+z^2 = 7$. Then the parametric equations for the normal line L_N to S at $(2,1,1)$ are

(a) L_N: $x = 2+2t,\ y = 1+2t,\ z = 1+t$
(b) L_N: $x = 4+2t,\ y = 4+t,\ z = 2+t$
(c) L_N: $x = 2+4t,\ y = 1+4t,\ z = 1+t$
(d) L_N: $x = 2+t,\ y = 1+2t,\ z = 1+t$

12. Consider the surface $S : z = f(x,y)$ with $f(2,1) = 4$ and $\nabla f(2,1) = -4i-4j$. Then the equation of the tangent plane P_T to S at $(2,1,4)$ is

(a) $P_T: -4x-4y-z = -8$
(b) $P_T: 4x+4y+z = 16$
(c) $P_T: -4x+4y+z = -16$
(d) $P_T: -4x-4y+z = -8$

13. Suppose $z = x+y,\ x = uv,\ y = v\cos(u)$. Then $\frac{\partial z}{\partial u} =$

(a) $v - v\sin(u)$ (b) $u+\cos(u)$ (c) $v-\cos(u)$ (d) $u - v\sin(u)$.

x_0 y_0 z_0
$(-4, -4, -1)$
a b c
$(2, 1, 4)$

$p(T) = 2(x+4) + (x+4)\ 4(y+1)$
$2x+8+x+4+4y+4$

The Hashemite University
Department of Mathematics
Calculus III

Second Exam
Time: One Hour

Name: [redacted] .... *Serial number:* ....

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
(a)													
(b)													
(c)													
(d)													

***(2 points each)*:** Select the best correct answer and fill your answer in the above table.

1. $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^6}{x^2+y^{12}} =$

a. $\frac{1}{2}$ b. does not exist c. 2 d. 0

2. Let $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$. Then $f_x(0,0)$ is:

a. 1 b. does not exist c. 0 d. none of the previous

3. The function $f(x,y)=x^2-xy+y^2$ has:

a. a relative minimum at $(0,0)$
b. $(0,0)$ is not a critical point of f
c. a relative maximum at $(0,0)$
d. None of the previous

4. $\lim\limits_{(x,y,z)\to(0,0,0)}\left(\dfrac{e^{x^2+y^2+z^2}-x^2-y^2-z^2-1}{x^2+y^2+z^2}\right)$ is :

a. does not exist b. 1 c. -1 d. 0

5. Consider the vector valued function $r(t)=<t,2t,2t>$. An arc length change of parameter that produces an opposite orientation of the given curve and has $t=1$ as the reference point is:

a. $t=-\frac{s}{3}+1$ b. $t=\frac{s}{3}$ c. $t=\frac{s}{3}+1$ d. $t=-\frac{s}{3}$

* Consider the vector valued function $r(t)=\langle x(t),y(t),z(t)\rangle$, and let $T(t),N(t),B(t)$ be the unit tangent vector, the unit normal vector and the binormal vector to the curve C of $r(t)$ at a given value of t. Use this to answer 6,7,8,9, and 10.

6. $r'(t)=$

a. $\langle x'(t),y(t),z(t)\rangle$ b. $\langle x(t),y'(t),z(t)\rangle$

c. $\langle x'(t),y'(t),z'(t)\rangle$ d. $\langle x(t),y(t),z'(t)\rangle$

7. $T(t)=$

a. $r'(t)$ b. $\dfrac{r'(t)}{\|r(t)\|}$ c. $\dfrac{r(t)}{\|r'(t)\|}$ d. $\dfrac{r'(t)}{\|r'(t)\|}$

8. $N(t)=$

a. $\dfrac{T'(t)}{\|T'(t)\|}$ b. $\dfrac{T'(t)}{\|T(t)\|}$ c. $\dfrac{T(t)}{\|T'(t)\|}$ d. $T'(t)$

9. $T(t)\times B(t)=$

a. $N(t)$ b. $-N(t)$ c. $\langle 0,1,0\rangle$ d. $\langle 0,-1,0\rangle$

10. Consider the vector valued function $r(t)=\langle 2\sin t,2\cos t,2t\rangle$. Then the equation of the NB - plane to the curve C of $r(t)$ at $t=0$ is:

a. $y=x$ b. $y=-x$ c. $z=-x$ d. $z=x$

11. The smallest directional derivative of the function $f(x,y)=\cos(x+2y)$ at $(\frac{\pi}{4},\frac{\pi}{8})$ is:

a. $\sqrt{5}$ b. $-\sqrt{5}$ c. 5 d. -5

12. The direction in which the function $f(x,y)=\cos(x+2y)$ increases most rapidly at $(\frac{\pi}{4},\frac{\pi}{8})$ is:

a. $\langle 1,-2\rangle$ b. $\langle 1,2\rangle$ c. $\langle -1,-2\rangle$ d. $\langle -1,2\rangle$

13. The curvature of the curve of the vector valued function $r(t)=\langle 3\cos t,2,3\sin t\rangle$ equals:

a. 2 b. $\dfrac{1}{2}$ c. 3 d. $\dfrac{1}{3}$

The Hashemite University
Department of Mathematics
Calculus III

Second Exam

Time: One Hour

Name: .... *Serial number:* 733746

	1	2	3	4	5	6	7	8	9	10	11	12	13
(a)													
(b)													
(c)													
(d)													

(2 points each): Select the best correct answer and fill your answer in the above table.

Consider the vector valued function $r(t)=\langle x(t), y(t), z(t)\rangle$, and let $T(t), N(t), B(t)$ be the unit tangent vector, the unit normal vector and the binormal vector to the curve C of $r(t)$ at a given value of t. Use this to answer 1,2 ,3, 4, and 5.

1. $r'(t)=$

a. $\langle x'(t), y(t), z(t)\rangle$ b. $\langle x(t), y'(t), z(t)\rangle$

c. $\langle x(t), y(t), z'(t)\rangle$ d. $\langle x'(t), y'(t), z'(t)\rangle$

2. $T(t)=$

a. $\frac{r'(t)}{\|r'(t)\|}$ b. $\frac{r'(t)}{\|r(t)\|}$ c. $\frac{r(t)}{\|r'(t)\|}$ d. $r'(t)$

3. $N(t)=$

a. $\frac{T'(t)}{\|T(t)\|}$ b. $\frac{T'(t)}{\|T'(t)\|}$ c. $\frac{T(t)}{\|T'(t)\|}$ d. $T'(t)$

4. $T(t)\times B(t)=$

a. $N(t)$ b. $\langle 0,1,0\rangle$ c. $-N(t)$ d. $\langle 0,-1,0\rangle$

5. Consider the vector valued function $r(t)=<2\sin t, 2\cos t, 2t>$. Then the equation of the NB – plane to the curve C of $r(t)$ at $t=0$ is:

a. $y=x$ b. $y=-x$ c. $z=x$ d. $z=-x$

6. The largest directional derivative of the function $f(x,y)=\cos(x+2y)$ at $(\frac{\pi}{4},\frac{\pi}{8})$ is:

a. $\sqrt{5}$ b. $-\sqrt{5}$ c. 5 d. -5

7. The direction in which the function $f(x,y)=cos(x+2y)$ decreases most rapidly at $(\frac{\pi}{4},\frac{\pi}{8})$ is:

a. $\langle 1,-2\rangle$ b. $\langle 1,2\rangle$ c. $\langle -1,-2\rangle$ d. $\langle -1,2\rangle$

8. The curvature of the curve of the vector valued function $r(t)=<2cos\,t,3,2sin\,t>$ equals:

a. 2 b. $\frac{1}{2}$ c. $\frac{1}{3}$ d. 3

9. $\lim_{(x,y)\to(0,0)} \frac{xy^6}{x^2+y^{12}} =$

a. $\frac{1}{2}$ b. 0 c. 2 d. does not exist

10. Let $f(x,y)=\begin{cases} \frac{xy}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$. Then $f_x(0,0)$ is:

a. 1 b. 0 c. does not exist d. none of the previous

11. The function $f(x,y)=x^2-xy+y^2$ has:

a. a relative maximum at $(0,0)$
b. $(0,0)$ is not a critical point of f
c. a relative minimum at $(0,0)$
d. None of the previous

12. $\lim_{(x,y,z)\to(0,0,0)} \left(\frac{e^{x^2+y^2+z^2}-x^2-y^2-z^2-1}{x^2+y^2+z^2} \right)$ is :

$\lim_{u\to 0} \frac{e^u-u-1}{u}$

a. 0 b. 1 c. -1 d. does not exist

13. Consider the vector valued function $r(t)=<t,2t,2t>$. An arc length change of parameter that produces an opposite orientation of the given curve and has $t=1$ as the reference point is:

a. $t=\frac{s}{3}+1$ b. $t=\frac{s}{3}$ c. $t=-\frac{s}{3}+1$ d. $t=-\frac{s}{3}$

$\frac{e^u-u-1}{u}$

1. Question *
(2 Points)

Let $\vec{\nabla} f(2,3) = \langle 1, 1 \rangle$.

Find $\lim\limits_{h \to 0} \dfrac{f\left(2 + \frac{h}{\sqrt{2}}, 3 + \frac{h}{\sqrt{2}}\right) - f(2,3)}{h}$.

- $\dfrac{5}{\sqrt{2}}$
- $\dfrac{3}{\sqrt{2}}$
- $\sqrt{7}$
- $\sqrt{3}$
- -2
- $\dfrac{1}{\sqrt{2}}$
- $\sqrt{2}$
- $2\sqrt{2}$

2. Question *

(2 Points)

Let $f(x, y) = 2xe^{-(x^2+y^2)}$. How many critical points $f(x, y)$ has?

- 5
- 9
- 6
- 2
- 3
- 4
- 1
- there is no critical points

3. Question *

(2 Points)

Find f_{xxyxy} if

$$f(x,y) = \frac{y}{x + \ln(x)} - x^3y^2.$$

- $\dfrac{1}{x + \ln(x)}$
- 4
- 6
- -12
- does not exist
- 0
- 2
- 12

4. Question *
(2 Points)

Find $f_y(0,0)$ if

$f(x,y) = \sqrt[3]{8(x^3 - y^3)}$.

- 2
- $-\infty$
- does not exist
- ∞
- -2
- $\sqrt[3]{5}$
- 0
- 4

5. Question *
(2 Points)

Find $\lim_{(x,y)\to(0,0)} \frac{3y^2 \sin(x)}{x^2 + y^2}$.

- 2
- 4
- does not exist
- -1
- 8
- 1
- -2
- 0

7. Question *
(2 Points)

In what direction does
$f(x, y) = xe^{-y} + 3y$
have the minimum rate of
change at the point $(1, 0)$?

- $\langle -1, -2 \rangle$
- $\langle \sqrt{2}, \sqrt{2} \rangle$
- $-\sqrt{5}$
- $\langle 0, 3 \rangle$
- $\langle -1, 3 \rangle$
- $\langle 2, 0 \rangle$
- $\sqrt{5}$
- $\langle 1, 2 \rangle$

8. Question *
(2 Points)

Given the function
$f(x,y) = \sqrt{8 + 8x + 4y - 4x^2 + y^2}$.
The level curves
of $f(x,y)$ are

- rectangles
- lines
- ellipses
- paraboloids
- hyperbolas
- points
- tringles

9. Question *
(2 Points)

Let $f(x, y) = -\sqrt{4 - \frac{x^2}{9} - \frac{y^2}{4}}$.

Find the range of f.

- $[-2, 2]$
- $(-\infty, -2]$
- $[-3, 0]$
- $[-2, 0]$
- $[0, 2]$
- $\mathbb{R}$
- $[2, \infty)$
- $[0, 1]$

10. Question *

(2 Points)

Let $f(x, y) = -x^2 - y^2 + 2x - 2y - 6$.
Which of the following statements best describes the point $(1, -1)$?

- $(1, -1)$ is an absolute min and local min
- $(1, -1)$ is not a critical point
- $(1, -1)$ is a local min
- $(1, -1)$ is a saddle point
- $(1, -1)$ is a local max
- $(1, -1)$ is an absolute max and local max

11. Question *
(2 Points)

If $w = f(x, y)$, where $x = x(t, \theta)$, $y = y(s, t)$, $t = t(\theta)$. Which formula below gives us $\dfrac{\partial w}{\partial \theta}$?

- $\dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial t}\dfrac{dt}{d\theta} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial t}\dfrac{dt}{d\theta} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial \theta}$
- $\dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial t}\dfrac{dt}{d\theta} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial t}\dfrac{dt}{d\theta} + \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial \theta}$
- $\dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial \theta} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial t}$
- $\dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial y}\dfrac{\partial y}{\partial t}$
- $\dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial t}\dfrac{dt}{d\theta} + \dfrac{\partial w}{\partial t}\dfrac{dt}{d\theta}$
- $\dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial t}\dfrac{dt}{d\theta} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial t}\dfrac{dt}{d\theta}$
- $\dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial t}\dfrac{dt}{d\theta} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial \theta}$
- $\dfrac{\partial w}{\partial \theta}\dfrac{\partial \theta}{\partial x} + \dfrac{\partial w}{\partial \theta}\dfrac{\partial \theta}{\partial y}$

12. Question *

(2 Points)

$z = e^x \cos(xy)$. Find the x − intercept of the equation of the tangent plane to the surface at $\left(1, \frac{\pi}{2}, 0\right)$

- $\pi - e$
- $\frac{1}{2}$
- e
- -2
- πe
- $\pi + e$
- $\pi e - 1$
- 2

13. Question *
(2 Points)

Suppose f is a differentiable function,
and define $g(u, v) = f(3u - v, u^2 + v)$.
Find $\frac{\partial g}{\partial v}$ at $(u, v) = (2, -1)$ if
$f(2, -1) = 6$, $g(2, -1) = -7$,
$f_x(2, -1) = 1$, $f_y(2, -1) = 9$
$f(7, 3) = 4$, $g(7, 3) = 2$,
$f_x(7, 3) = -3$, $f_y(7, 3) = 5$

- 8
- −10
- 11
- 3
- 5
- −7
- 6

14. Question *
(2 Points)

Find the absolute minimum value of the function $f(x, y) = x^2 + 3y^2 + 2y$ on the unit disk $x^2 + y^2 \leq 1$.

- 5
- 8
- $\frac{1}{2}$
- -3
- 6
- $-\frac{1}{3}$
- 0
- 4

15. Question *

(2 Points)

Find $\lim_{(x,y)\to(1,1)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

- 16
- 0
- -16
- does not exist
- ∞
- 2
- $-\infty$
- -2

8

Find the equation of the tangent plane to the paraboloid $z=x^2+y^2$ at the point (1,1,2). *

(1 Point)

- ○ $z=x^2+y^2$
- ○ $z=x+y-1$
- ○ $z=x+y+2$
- ◉ $z=2x+2y$
- ○ $z=2x+2y-2$

7

Find the direction in which the maximum rate of change of $f(x,y)=\sin(x\,y)$ at the point (0,1) occurs *

(1 Point)

- $< 1/\sqrt{2}, 1/\sqrt{2} >$
- $< 0, 1/\sqrt{2} >$
- <1,0>
- <0,1>
- $< 1/\sqrt{2}, 1 >$

Let $f(x,y) = \begin{cases} \dfrac{x^3y - xy^3 + x^2 + y^2}{x^2 + y^2}, & if\ (x,y) \neq (0,0) \\ 1, & if\ (x,y) = (0,0) \end{cases}$

Find $f_x(0,0)$

- 0
- -1
- 1
- -2
- D.N.E
- 2

4

At what point is the following function has a local minimum? *

(1 Point)

$f(x, y) = 2x^2 + 2y^2 + 2x^2y + 6$

- ○ (-1,6)
- ◉ (0,0)
- ○ (1,6)
- ○ $(-1, -6)$
- ○ (-1,2)

7

Suppose (1, 1) is a critical point of a function f with continuous second derivatives. what can you say about f

* :if

(1 نقطة)

$f_{xx}(1,1) = 7, f_{yy}(1,1) = 10, f_{yx}(1,1) = 8$

- f has a local minimum at (1,1)
- f has a saddle point at (1,1)
- f has a local maximum at (1,1)
- None of the above

Let $R(s,t) = F(u(s,t), v(s,t))$, where F, u, and v are differentiable, and

$$u(1,0) = 2,\ v(1,0) = 3$$
$$u_s(1,0) = -2,\ v_s(1,0) = 5$$
$$u_t(1,0) = 6,\ v_t(1,0) = 4$$
$$F_u(2,3) = -1,\ F_v(2,3) = 10$$

Find, $R_t(1,0)$.

- 34
- 50
- 32
- 52
- 25

3

Find the critical points of the function. *
(1 Point)

$$f(x, y) = 8 + 76xy + 38x^2 + 240y + \frac{y^4}{4}$$

- ( ) $(-4, 6), (-6, -10), (10, 4)$
- ( ) $(-6, 6), (8, -8), (-8, 8)$
- (•) $(-4, 4), (-6, 6), (10, -10)$
- ( ) $(-4, 4), (-6, 6), (-10, 0)$
- ( ) $(-4, 6), (-8, 6), (8, -6)$

1

Find the equation of the tangent plane to the paraboloid z=x^2+y^2 at the point (1,1,2). *

(1 Point)

- z=2x+2y
- z=x^2+y^2
- z=x+y+2
- z=2x+2y-2
- z=x+y-1

[redacted]

Find all points at which the direction of fastest change of the f(x,y)=x^2+y^2-2x-4y is the vector i+j . *

(1 Point)

- only (0,0)
- (1,1) and (0,0)
- All points on the parabola y=2x^2
- All points on the line y=x+1
- all points on the paine

Find the direction in which the maximum rate of change of f(x,y)=sin(x y) at the point (0,1) occurs *

(1 Point)

- $<0,1>$
- $< 1/\sqrt{2}, 1/\sqrt{2} >$
- $< 0, 1/\sqrt{2} >$
- $<1,0>$
- $< 1/\sqrt{2}, 1 >$

Find the critical points of the function. * (1 Point)

$$f(x,y) = 8 + 76xy + 38x^2 + 240y + \frac{y^4}{4}$$

- $(-4,6)$, $(-8,6)$, $(8,-6)$
- $(-4,4)$, $(-6,6)$, $(10,-10)$
- $(-4,6)$, $(-6,-10)$, $(10,4)$
- $(-6,6)$, $(8,-8)$, $(-8,8)$
- $(-4,4)$, $(-6,6)$, $(-10,0)$

[redacted]

Find the rate of change of f at (-1,1,-1) in the direction of the vector <8,10,-8>, where f(x,y,z)=8x^2-7xy+7xyz *

(1 Point)

$f(x,y,z) = 8x^2 - 7xy + 7xyz$

- -15.099
- 44
- -2.91
- 20
- 14.65856

[redacted]

Suppose (1, 1) is a critical point of a function f with continuous second derivatives. What can you say about f if: *

(1 Point)

$f_{xx}(1,1) = 7, f_{yy}(1,1) = 10, f_{yx}(1,1) = 8$

- ○ f has a saddle point at (1,1)
- ● f has a local minimum at (1,1)
- ○ f has a local maximum at (1,1)
- ○ None of the above

Find the absolute maximum and minimum values of f(x,y)= x^2+y^2-2x on the closed triangular region with vertices (2,0), (0,2) and (0,-2). *

(1 Point)

- Absolute maximum=4, Absolute minimum = -1/2
- Absolute maximum=8, Absolute minimum = -2
- Absolute maximum=0, Absolute minimum =0
- Absolute maximum=0, Absolute minimum = -1/2
- Absolute maximum=4, Absolute minimum = -1
- Absolute maximum=8, Absolute minimum = -1

[redacted]

Find the equation of the tangent plane to the paraboloid z = x ^ 2 + y ^ 2 at the point (1,1,2). *

(1 Point)

- z = 2x + 2y
- z = x ^ 2 + y ^ 2
- z = x + y - 2
- z = 2x + 2y - 2
- z = x + y - 1

2

At what point is the following function has a local minimum? * (1 Point)

$f(x,y) = 2x^2 + 2y^2 + 2x^2y + 6$

- (0,0)
- (-1,6)
- (1,6)
- (-1,2)
- (−1, −6)