

1.

$$(a) \quad T = 4(7.5 - 2.1)10^{-3} = 21.6 \times 10^{-3}, \quad \omega = \frac{2\pi 10^3}{21.6} = 290.9t \text{ rad/s}$$

$$\therefore f(t) = 8.5 \sin(290.9t + \Phi) \quad \therefore 0 = 8.5 \sin(290.9 \times 2.1 \times 10^{-3} + \Phi)$$

$$\therefore \Phi = -0.6109^{\text{rad}} + 2\pi = 5.672^{\text{rad}} \text{ or } 325.0^\circ$$

$$\therefore f(t) = \boxed{8.5 \sin(290.9t + 325.0^\circ)}$$

$$(b) \quad 8.5 \sin(290.9t + 325.0^\circ) = 8.5$$

$$\cos(290.9t + 235^\circ) = \boxed{8.5 \cos(290.9t - 125^\circ)}$$

$$(c) \quad 8.5 \cos(-125^\circ) \cos \omega t + 8.5 \sin 125^\circ$$

$$\sin \omega t = \boxed{-4.875^+ \cos 290.9t + 6.963 \sin 290.9t}$$

2.

(a) $-10 \cos \omega t + 4 \sin \omega t + A \cos(\omega t + \Phi)$, $A > 0$, $-180^\circ < \Phi \leq 180^\circ$

$A = \sqrt{116} = 10.770$, $A \cos \Phi = -10$, $A \sin \Phi = -4 \therefore \tan \Phi = 0.4$, 3^d quad

$\therefore \Phi = 21.80^\circ = 201.8^\circ$, too large $\therefore \Phi = 201.8^\circ - 360^\circ = -158.20^\circ$

(b) $200 \cos(5t + 130^\circ) = F \cos 5t + G \sin 5t \therefore F = 200 \cos 130^\circ = -128.6$

$G = -200 \sin 130^\circ = -153.2$

(c) $i(t) = 5 \cos 10t - 3 \sin 10t = 0$, $0 \leq t \leq 1$ s

$\therefore \frac{\sin 10t}{\cos 10t} = \frac{5}{3}$, $10t = 1.0304$,

$t = 0.10304$ s; also, $10t = 1.0304 + \pi$, $t = 0.4172$ s; $10t = 1.0304 + 2\pi$, $t = 0.7314$ s

(d) $0 < t < 10$ ms, $10 \cos 100\pi t \geq 12 \sin 100\pi t$; let $10 \cos 100\pi t = 12 \sin 100\pi t$

$\therefore \tan 100\pi t = \frac{10}{12}$, $100\pi t = 0.6947 \therefore t = 2.211$ ms $\therefore 0 < t < 2.211$ ms

3.

- (a) Note that $A \cos x + B \sin x = \sqrt{A^2 + B^2} \cos \left(x + \tan^{-1} \left(\frac{-B}{A} \right) \right)$. For $f(t)$, the angle is in the second quadrant; most calculators will return -30.96° , which is off by 180° .

$$f(t) = -50 \cos \omega t - 30 \sin \omega t = 58.31 \cos (\omega t + 149.04^\circ)$$

$$g(t) = 55 \cos \omega t - 15 \sin \omega t = 57.01 \cos (\omega t + 15.255^\circ)$$

$$\therefore \text{ampl. of } f(t) = 58.31, \text{ ampl. of } g(t) = 57.01$$

- (b) $f(t)$ leads $g(t)$ by $149.04^\circ - 15.255^\circ = 133.8^\circ$

4. $i(t) = A \cos(\omega t - \theta)$, and

$$L(di/dt) + Ri = V_m \cos \omega t$$

$$\therefore L[-\omega A \sin(\omega t - \theta)] + RA \cos(\omega t - \theta) = V_m \cos \omega t$$

$$-\omega LA \sin \omega t \cos \theta + \omega LA \cos \omega t \sin \theta + RA \cos \omega t \cos \theta + RA \sin \omega t \sin \theta$$

$$= V_m \cos \omega t$$

$$\therefore \omega LA \cos \theta = RA \sin \theta$$

$$\text{and } \omega LA \sin \theta + RA \cos \theta = V_m$$

$$\text{Thus, } \tan \theta = \frac{\omega L}{R} \quad *$$

$$\text{and } \omega LA \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} + RA \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = V_m$$

$$\text{so that } \left(\frac{\omega^2 L^2}{\sqrt{R^2 + \omega^2 L^2}} + \frac{R^2}{\sqrt{R^2 + \omega^2 L^2}} \right) A = V_m$$

$$\text{Thus, } \left(\sqrt{R^2 + \omega^2 L^2} \right) A = (V_m) \text{ and therefore we may write } A = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \quad *$$

5. $f = 13.56 \text{ MHz}$ so $\omega = 2\pi f = 85.20 \text{ Mrad/s}$.

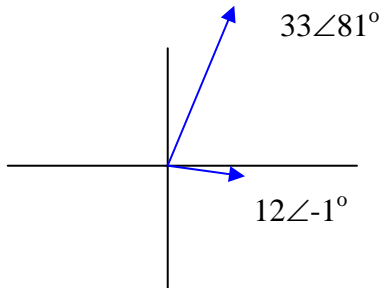
Delivering 300 W (peak) to a $5\text{-}\Omega$ load implies that $\frac{V_m^2}{5} = 300$ so $V_m = 38.73 \text{ V}$.

Finally, $(85.2 \times 10^6)(21.15 \times 10^{-3}) + \phi = n\pi$, $n = 1, 3, 5, \dots$

Since $(85.2 \times 10^6)(21.15 \times 10^{-3}) = 1801980$, which is 573588π , we find that

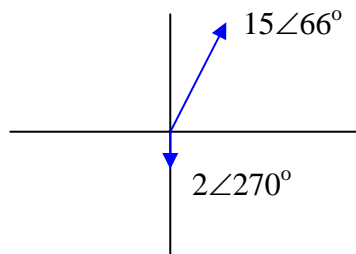
$$\phi = 573589\pi - (85.2 \times 10^6)(21.15 \times 10^{-3}) = 573589\pi - 573588\pi = \pi$$

6. (a) $-33 \sin(8t - 9^\circ) \rightarrow -33 \angle (-9-90)^\circ = 33 \angle 81^\circ$
 $12 \cos(8t - 1^\circ) \rightarrow 12 \angle -1^\circ$



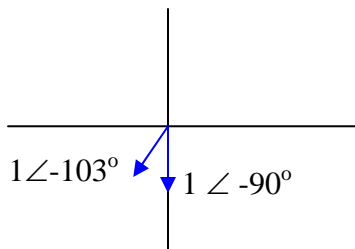
$-33 \sin(8t - 9^\circ)$ leads $12 \cos(8t - 1^\circ)$ by $81 - (-1) = 82^\circ$.

- (b) $15 \cos(1000t + 66^\circ) \rightarrow 15 \angle 66^\circ$
 $-2 \cos(1000t + 450^\circ) \rightarrow -2 \angle 450^\circ = -2 \angle 90^\circ = 2 \angle 270^\circ$



$15 \cos(1000t + 66^\circ)$ leads $-2 \cos(1000t + 450^\circ)$ by $66 - -90 = 156^\circ$.

- (c) $\sin(t - 13^\circ) \rightarrow 1 \angle -103^\circ$
 $\cos(t - 90^\circ) \rightarrow 1 \angle -90^\circ$

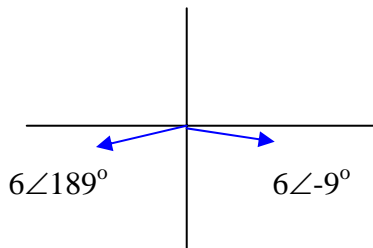


$\cos(t - 90^\circ)$ leads $\sin(t - 13^\circ)$ by $66 - -90 = 156^\circ$.

- (d) $\sin t \rightarrow 1 \angle -90^\circ$
 $\cos(t - 90^\circ) \rightarrow 1 \angle -90^\circ$

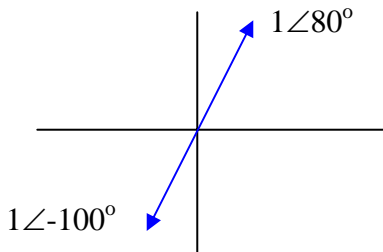
These two waveforms are *in phase*. Neither leads the other.

7. (a) $6 \cos(2\pi 60t - 9^\circ) \rightarrow 6 \angle -9^\circ$
 $-6 \cos(2\pi 60t + 9^\circ) \rightarrow 6 \angle 189^\circ$



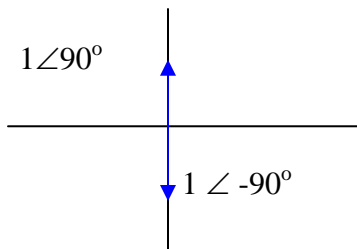
$-6 \cos(2\pi 60t + 9^\circ)$ lags $6 \cos(2\pi 60t - 9^\circ)$ by $360 - 9 - 189 = 162^\circ$.

(b) $\cos(t - 100^\circ) \rightarrow 1 \angle -100^\circ$
 $-\cos(t - 100^\circ) \rightarrow -1 \angle -100^\circ = 1 \angle 80^\circ$



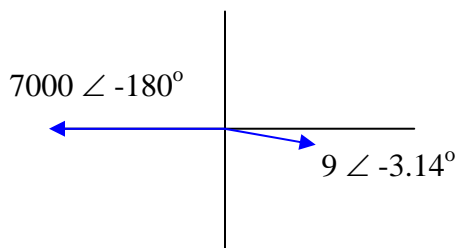
$-\cos(t - 100^\circ)$ lags $\cos(t - 100^\circ)$ by 180° .

(c) $-\sin t \rightarrow -1 \angle -90^\circ = 1 \angle 90^\circ$
 $\sin t \rightarrow 1 \angle -90^\circ$



$-\sin t$ lags $\sin t$ by 180° .

(d) $7000 \cos(t - \pi) \rightarrow 7000 \angle -\pi = 7000 \angle -180^\circ$
 $9 \cos(t - 3.14^\circ) \rightarrow 9 \angle -3.14^\circ$



$7000 \cos(t - \pi)$ lags $9 \cos(t - 3.14^\circ)$ by $180 - 3.14 = 176.9^\circ$.

$$8. \quad v(t) = V_1 \cos \omega t - V_2 \sin \omega t \quad [1]$$

We assume this can be written as a single cosine such that

$$v(t) = V_m \cos (\omega t + \phi) = V_m \cos \omega t \cos \phi - V_m \sin \omega t \sin \phi \quad [2]$$

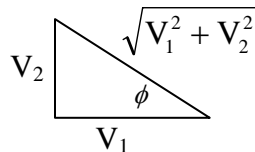
Equating terms on the right hand sides of Eqs. [1] and [2],

$$V_1 \cos \omega t - V_2 \sin \omega t = (V_m \cos \phi) \cos \omega t - (V_m \sin \phi) \sin \omega t$$

yields

$$V_1 = V_m \cos \phi \quad \text{and} \quad V_2 = V_m \sin \phi$$

Dividing, we find that $\frac{V_2}{V_1} = \frac{V_m \sin \phi}{V_m \cos \phi} = \tan \phi$ and $\phi = \tan^{-1}(V_2/V_1)$



Next, we see from the above sketch that we may write $V_m = V_1 / \cos \phi$ or

$$V_m = \frac{V_1}{V_1 / \sqrt{V_1^2 + V_2^2}} = \sqrt{V_1^2 + V_2^2}$$

Thus, we can write $v(t) = V_m \cos (\omega t + \phi) = \sqrt{V_1^2 + V_2^2} \cos [\omega t + \tan^{-1}(V_2/V_1)]$.

9. (a) In the range $0 \leq t \leq 0.5$, $v(t) = t/0.5$ V.
Thus, $v(0.4) = 0.4/0.5 = 0.8$ V.
- (b) Remembering to set the calculator to radians, 0.7709 V.
- (c) 0.8141 V.
- (d) 0.8046 V.

$$\begin{aligned}
 10. \quad (a) \quad V_{\text{rms}} &= \left[\frac{V_m^2}{T} \int_0^T \cos^2 \omega t \, dt \right]^{1/2} \\
 &= \left[\frac{V_m^2}{T} \int_0^T \cos^2 \frac{2\pi t}{T} \, dt \right]^{1/2} \\
 &= \left[\frac{V_m^2}{2T} \int_0^T \left(1 + \cos \frac{4\pi t}{T} \right) dt \right]^{1/2} \\
 &= \left[\frac{V_m^2}{2T} \int_0^T dt + \frac{V_m^2}{2T} \int_0^T \cos \frac{4\pi t}{T} dt \right]^{1/2} \\
 &= \left[\frac{V_m^2}{2T} T + \frac{V_m^2}{8\pi} \cos u \Big|_0^{4\pi} \right]^{1/2} \\
 &= \frac{V_m}{\sqrt{2}} *
 \end{aligned}$$

$$(b) \quad V_m = 110\sqrt{2} = 155.6 \text{ V}, \quad 115\sqrt{2} = 162.6 \text{ V}, \quad 120\sqrt{2} = 169.7 \text{ V}$$

11. We begin by defining a clockwise current i . Then, KVL yields

$$-2 \times 10^{-3} \cos 5t + 10i + v_C = 0.$$

Since $i = i_C = C \frac{dv_C}{dt}$, we may rewrite our KVL equation as

$$30 \frac{dv_C}{dt} + v_C = 2 \times 10^{-3} \cos 5t \quad [1]$$

We anticipate a response of the form $v_C(t) = A \cos(5t + \theta)$. Since $\frac{dv_C}{dt} = -5A \sin(5t + \theta)$,

we now may write Eq. [1] as $-150A \sin(5t + \theta) + A \cos(5t + \theta) = 2 \times 10^{-3} \cos 5t$. Using a common trigonometric identity, we may combine the two terms on the left hand side into a single cosine function:

$$\sqrt{(150A)^2 + A^2} \cos\left(5t + \theta + \tan^{-1} \frac{150A}{A}\right) = 2 \times 10^{-3} \cos 5t$$

Equating terms, we find that $A = 13.33 \mu\text{V}$ and $\theta = -\tan^{-1} 150 = -89.62^\circ$. Thus,

$$v_C(t) = 13.33 \cos(5t - 89.62^\circ) \mu\text{V}.$$

12. KVL yields

$$-6\cos 400t + 100i + v_L = 0.$$

Since $v_L = L \frac{di}{dt} = 2 \frac{di}{dt}$, we may rewrite our KVL equation as

$$2 \frac{di}{dt} + 100i = 6 \cos 400t \quad [1]$$

We anticipate a response of the form $i(t) = A \cos(400t + \theta)$. Since

$$\frac{di}{dt} = -400A \sin(400t + \theta),$$

we now may write Eq. [1] as

$$-800A \sin(400t + \theta) + 100A \cos(400t + \theta) = 6 \cos 400t.$$

Using a common trigonometric identity, we may combine the two terms on the left hand side into a single cosine function:

$$\sqrt{(800A)^2 + (100A)^2} \cos\left(400t + \theta + \tan^{-1} \frac{800A}{100A}\right) = 6 \cos 400t$$

Equating terms, we find that $A = 7.442$ mA and $\theta = -\tan^{-1} 8 = -82.88^\circ$. Thus,

$$i(t) = 7.442 \cos(400t - 82.88^\circ) \text{ mA, so } v_L = L \frac{di}{dt} = 2 \frac{di}{dt} = \boxed{5.954 \cos(400t + 7.12^\circ)}$$

13. $20\cos 500t \text{ V} \rightarrow 20\angle 0^\circ \text{ V}$. $20 \text{ mH} \rightarrow j10 \Omega$.

Performing a quick source transformation, we replace the voltage source/20- Ω resistor series combination with a $1\angle 0^\circ$ A current source in parallel with a 20- Ω resistor.

$20 \parallel 60\text{k} = 19.99 \Omega$. By current division, then,

$$\mathbf{I}_L = \frac{19.99}{19.99 + 5 + j10} = 0.7427\angle -21.81^\circ \text{ A. Thus, } i_L(t) = 742.7 \cos(500t - 21.81^\circ) \text{ mA.}$$

14.

$$\text{At } x-x: R_{th} = 80 \parallel 20 = 16\Omega$$

$$v_{oc} = -0.4(15 \parallel 85) \frac{80}{85} \cos 500t$$

$$\therefore v_{oc} = 4.8 \cos 500t \text{ V}$$

$$\begin{aligned} \text{(a)} \quad i_L &= \frac{4.8}{\sqrt{16^2 + 10^2}} \cos\left(500t - \tan^{-1} \frac{10}{15}\right) \\ &= \boxed{0.2544 \cos(500t - 32.01^\circ) \text{ A}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad v_L &= Li_L' = 0.02 \times 0.02544(-500) \\ &\sin(500t - 32.01^\circ) = -2.544 \sin(500t - 32.01^\circ) \text{ V} \\ \therefore v_L &= 2.544 \cos(500t + 57.99^\circ) \text{ V}, i_x \\ &= \boxed{31.80 \cos(500t + 57.99^\circ) \text{ mA}} \end{aligned}$$

15.

$$(a) \quad i = \frac{100}{\sqrt{500^2 + 800^2}} \cos\left(10^5 t - \frac{800}{500}\right) = 0.10600 \cos(10^5 t - 57.99^\circ) \text{ A}$$

$$p_R = 0 \text{ when } i = 0 \therefore 10^5 t - \frac{57.99^\circ}{180} \pi = \frac{\pi}{2}, t = \boxed{25.83 \mu\text{s}}$$

$$(b) \quad \pm v_L = Li' = 8 \times 10^{-3} \times 0.10600 (-10^5) \sin(10^5 t - 57.99^\circ)$$

$$\therefore v_L = -84.80 \sin(10^5 t - 57.99^\circ)$$

$$\therefore p_L = v_L i = -8.989 \sin(10^5 t - 57.99^\circ)$$

$$\cos(10^5 t - 57.99^\circ) = -4.494 \sin(2 \times 10^5 t - 115.989^\circ)$$

$$\therefore p_L = 0 \text{ when } 2 \times 10^5 t - 115.989^\circ = 0^\circ, 180^\circ,$$

$$\therefore t = \boxed{10.121 \text{ or } 25.83 \mu\text{s}}$$

$$(c) \quad p_s = v_s i_L = 10.600 \cos 10^5 t \cos(10^5 t - 57.99^\circ)$$

$$\therefore p_s = 0 \text{ when } 10^5 t = \frac{\pi}{2}, t = \boxed{15.708 \mu\text{s}} \text{ and also } t = \boxed{25.83 \mu\text{s}}$$

16. $v_s = 3 \cos 10^5 t$ V, $i_s = 0.1 \cos 10^5 t$ A

v_s in series with $30\Omega \rightarrow 0.1 \cos 10^5 t$ A $\parallel 30\Omega$

Add, getting $0.2 \cos 10^5 t$ A $\parallel 30\Omega$

change to $6 \cos 10^5 t$ V in series with 30Ω ; $30\Omega + 20\Omega = 50\Omega$

$$\therefore i_L = \frac{6}{\sqrt{50^2 + 10^2}} \cos\left(10^5 t - \tan^{-1} \frac{10}{50}\right) = 0.11767 \cos(10^5 t - 11.310^\circ) \text{ A}$$

At $t = 10\mu\text{s}$, $10^5 t = 1 \therefore i_L = 0.1167 \cos(1^{\text{rad}} - 11.310^\circ) = 81.76\text{mA}$

$\therefore v_L = 0.11767 \times 10 \cos(1^{\text{rad}} - 11.30^\circ + 90^\circ) = -0.8462\text{V}$

17. $\cos 500t$ V \rightarrow $1\angle 0^\circ$ V. 0.3 mH \rightarrow $j0.15$ Ω .

Performing a quick source transformation, we replace the voltage source-resistor series combination with a $0.01\angle 0^\circ$ A current source in parallel with a $100\text{-}\Omega$ resistor. Current division then leads to

$$(0.01 + 0.2\mathbf{I}_L) \frac{100}{100 + j0.15} = \mathbf{I}_L$$

$$1 + 20\mathbf{I}_L = (100 + j0.15) \mathbf{I}_L$$

Solving, we find that $\mathbf{I}_L = 0.0125\angle -0.1074^\circ$ A,

so that $i_L(t) = \boxed{12.5\cos(500t - 0.1074^\circ)}$ mA.

18. $v_{s1} = V_{s2} = 120 \cos 120 \pi t \text{ V}$
 $\frac{120}{60} = 2 \text{ A}, \frac{120}{12} = 1 \text{ A}, 2 + 1 = 3 \text{ A}, 60 \parallel 120 = 40 \Omega$
 $3 \times 40 = 120 \text{ V}, \omega L = 12 \pi = 37.70 \Omega$
 $\therefore i_L = \frac{120}{\sqrt{40^2 + 37.70^2}} \cos \left(120 \pi t - \tan^{-1} \frac{37.70}{40} \right)$
 $= 2.183 \cos (120 \pi t - 43.30^\circ) \text{ A}$
- (a) $\therefore \omega_L = \frac{1}{2} \times 0.1 \times 2.183^2 \cos^2 (120 \pi t - 43.30^\circ)$
 $= 0.2383 \cos^2 (120 \pi t - 43.30^\circ) \text{ J}$
- (b) $\omega_{L,av} = \frac{1}{2} \times 0.2383 = 0.11916 \text{ J}$

19. $v_{s1} = 120 \cos 400t$ V, $v_{s2} = 180 \cos 200t$ V

Performing two quick source transformations,

$$\frac{120}{60} = 2 \text{ A}, \frac{180}{120} = 1.5 \text{ A}, \text{ and noting that } 60 \parallel 120 = 40 \Omega,$$

results in two current sources (with different frequencies) in parallel, and also in parallel with a 40Ω resistor and the 100 mH inductor.

Next we employ superposition. Open-circuiting the 200 rad/s source first, we perform a source transformation to obtain a voltage source having magnitude $2 \times 40 = 80$ V. Applying Eqn. 10.4,

$$i_L' = \frac{80}{\sqrt{40^2 + 400^2(0.1)^2}} \cos(400t - \tan^{-1} \frac{400(0.1)}{40})$$

Next, we open-circuit the 400 rad/s current source, and perform a source transformation to obtain a voltage source with magnitude $1.5 \times 40 = 60$ V. Its contribution to the inductor current is

$$i_L'' = \frac{60}{\sqrt{40^2 + 200^2(0.1)^2}} \cos(200t - \tan^{-1} \frac{200(0.1)}{40}) \text{ A}$$

so that $i_L = 1.414 \cos(400t - 45^\circ) + 1.342 \cos(200t - 26.57^\circ) \text{ A}$

20.

$$R_i = \infty, R_o = 0, A = \infty, \text{ ideal}, R_1 C_1 = \frac{L}{R}$$

$$i_{upper} = -\frac{V_m \cos \omega t}{R}, i_{lower} = \frac{v_{out}}{R_1}$$

$$\therefore i_{c1} = i_{upper} + i_{lower} = \frac{i}{R_1} (v_{out} - V_m \cos \omega t) = -C_1 v'_{out}$$

$$\therefore V_m \cos \omega t = v_{out} + R_1 C_1 v'_{out} = v_{out} + \frac{L}{R} v'_{out}$$

$$\text{For RL circuit, } V_m \cos \omega t = v_r + L \frac{d}{dt} \left(\frac{v_R}{R} \right)$$

$$\therefore V_m \cos \omega t = v_R + \frac{L}{R} v'_R$$

By comparison, $v_R = v_{out}$ *

21.

(a) $V_m \cos \omega t = Ri + \frac{1}{C} \int idt$ (ignore I.C)

$$\therefore -\omega V_m \sin \omega t = Ri' + \frac{1}{C} i$$

(b) Assume $i = A \cos(\omega t + \Phi)$

$$\therefore -\omega V_m \sin \omega t = -R\omega A \sin(\omega t + \Phi) + \frac{A}{C} \cos(\omega t + \Phi)$$

$$\therefore -\omega V_m \sin \omega t = -R\omega A \cos \Phi \sin \omega t - R\omega A \sin \Phi \cos \omega t + \frac{A}{C} \cos \omega t \cos \Phi - \frac{A}{C} \sin \omega t \sin \Phi$$

Equating terms on the left and right side,

[1] $R\omega A \sin \Phi = \frac{A}{C} \cos \Phi \therefore \tan \Phi = \frac{1}{\omega CR}$ so $\Phi = \tan^{-1}(1/\omega CR)$, and

[2] $-\omega V_m = -R\omega A \frac{\omega CR}{\sqrt{1+\omega^2 C^2 R^2}} - \frac{A}{C} \frac{1}{\sqrt{1+\omega^2 C^2 R^2}}$

$$\therefore \omega V_m = \frac{A}{C} \left[\frac{R^2 \omega^2 C^2 + 1}{\sqrt{1+\omega^2 C^2 R^2}} \right] = \frac{A}{C} \sqrt{1+\omega^2 C^2 R^2} \therefore A = \frac{\omega C V_m}{\sqrt{1+\omega^2 C^2 R^2}}$$

$$\therefore i = \frac{\omega C V_m}{\sqrt{1+\omega^2 C^2 R^2}} \cos \left(\omega t + \tan^{-1} \frac{1}{\omega CR} \right)$$

22. (a) $7 \angle -90^\circ = -j7$
- (b) $3 + j + 7 \angle -17^\circ = 3 + j + 6.694 - j 2.047 = 9.694 - j 1.047$
- (c) $14e^{j15^\circ} = 14 \angle 15^\circ = 14 \cos 15^\circ + j 14 \sin 15^\circ = 13.52 + j 3.623$
- (d) $1 \angle 0^\circ = 1$
- (e) $-2(1 + j9) = -2 - j18 = 18.11 \angle -96.34^\circ$
- (f) $3 = 3 \angle 0^\circ$

23. (a) $3 + 15 \angle -23^\circ = 3 + 13.81 - j 5.861 = 16.81 - j 5.861$
- (b) $(j 12)(17 \angle 180^\circ) = (12 \angle 90^\circ)(17 \angle 180^\circ) = 204 \angle 270^\circ = -j 204$
- (c) $5 - 16(9 - j 5) / (33 \angle -9^\circ) = 5 - (164 \angle -29.05^\circ) / (33 \angle -9^\circ)$
 $= 5 - 4.992 \angle -20.05^\circ = 5 - 4.689 - j 1.712 = 0.3109 + j 1.712$

24. (a) $5 \angle 9^\circ - 9 \angle -17^\circ = 4.938 + j 0.7822 - 8.607 + j 2.631 = -3.668 + j 3.414$
 $= 5.011 \angle 137.1^\circ$
- (b) $(8 - j 15)(4 + j 16) - j = 272 + j 68 - j = 272 + j 67 = 280.1 \angle 13.84^\circ$
- (c) $(14 - j 9) / (2 - j 8) + 5 \angle -30^\circ = (16.64 \angle -32.74^\circ) / (8.246 \angle -75.96^\circ) + 4.330 - j 2.5$
 $= 1.471 + j 1.382 + 4.330 - j 2.5 = 5.801 - j 1.118 = 5.908 \angle -10.91^\circ$
- (d) $17 \angle -33^\circ + 6 \angle -21^\circ + j 3 = 14.26 - j 9.259 + 5.601 - j 2.150 + j 3$
 $= 19.86 - j 8.409 = 21.57 \angle -22.95^\circ$

25. (a) $e^{j14^\circ} + 9 \angle 3^\circ - (8 - j6)/j^2 = 1 \angle 14^\circ + 9 \angle 3^\circ - (8 - j6)/(-1)$
 $= 0.9703 + j0.2419 + 8.988 + j0.4710 + 8 - j6 = 17.96 - j5.287 = 18.72 \angle -16.40^\circ$

(b) $(5 \angle 30^\circ)/(2 \angle -15^\circ) + 2e^{j5^\circ}/(2 - j2)$
 $= 2.5 \angle 45^\circ + (2 \angle 5^\circ)/(2.828 \angle -45^\circ) = 1.768 + j1.768 + 0.7072 \angle 50^\circ$
 $= 1.768 + j1.768 + 0.4546 + j0.5418$
 $= 2.224 + j2.310 = 3.207 \angle 46.09^\circ$

26.

(a) $5\angle -110^\circ = -1.7101 - j4.698$

(b) $6e^{j160^\circ} = -5.638 + j2.052$

(c) $(3 + j6)(2\angle 50^\circ) = -5.336 + j12.310$

(d) $-100 - j40 = 107.70\angle -158.20^\circ$

(e) $2\angle 50^\circ + 3\angle -120^\circ = 1.0873\angle -101.37^\circ$

27.

$$(a) \quad 40\angle -50^\circ - 18\angle 25^\circ = 39.39\angle -76.20^\circ$$

$$(b) \quad 3 + \frac{2}{j} + \frac{2 - j5}{1 + j2} = 4.050\angle -69.78^\circ$$

$$(c) \quad (2.1\angle 25^\circ)^3 = 9.261\angle 75^\circ = 2.397 + j8.945^+$$

$$(d) \quad 0.7e^{j0.3} = 0.7\angle 0.3^{\text{rad}} = 0.6687 + j0.2069$$

28.

$$i_c = 20e^{j(40t+30^\circ)} \text{ A} \therefore v_c = 100 \int 20e^{j(40t+30^\circ)} dt$$

$$v_c = -j50e^{j(40t+30^\circ)}, i_R = -j10e^{j(40t+30^\circ)} \text{ A}$$

$$\therefore i_L = (20 - j10)e^{j(40t+30^\circ)}, v_L = j40 \times 0.08(20 - j10)e^{j(40t+30^\circ)}$$

$$\therefore v_L = (32 + j64)e^{j(40t+30^\circ)} \text{ V} \therefore v_s = (32 + j64 - j50)e^{j(40t+30^\circ)}$$

$$\therefore v_s = \boxed{34.93e^{j(40t-53.63^\circ)} \text{ V}}$$

29.

$$i_L = 20e^{j(10t+25^\circ)} \text{ A}$$

$$v_L = 0.2 \frac{d}{dt} [20e^{j(10t+25^\circ)}] = j40e^{j(10t+25^\circ)}$$

$$v_R = 80e^{j(10t+25^\circ)}$$

$$v_s = (80 + j40)e^{j(10t+25^\circ)}, i_c = 0.08(80 + j40)j10e^{j(10t+25^\circ)}$$

$$\therefore i_c = (-32 + j64)e^{j(10t+25^\circ)} \quad \therefore i_s = (-12 + j64)e^{j(10t+25^\circ)}$$

$$\therefore i_s = \boxed{65.12e^{j(10t+125.62^\circ)} \text{ A}}$$

30. $80 \cos(500t - 20^\circ) \text{ V} \rightarrow 5 \cos(500t + 12^\circ) \text{ A}$

(a) $v_s = 40 \cos(500t + 10^\circ) \therefore i_{out} = 2.5 \cos(500t + 42^\circ) \text{ A}$

(b) $v_s = 40 \sin(500t + 10^\circ) = 40 \cos(500t - 80^\circ)$
 $\therefore i_{out} = 2.5 \cos(500t - 48^\circ) \text{ A}$

(c) $v_s = 40e^{j(500t+10^\circ)} = 40 \cos(500t + 10^\circ)$
 $+ j40 \sin(500t + 10^\circ) \therefore i_{out} = 2.5e^{j(500t+42^\circ)} \text{ A}$

(d) $v_s = (50 + j20)e^{j500t} = 53.85^+ e^{j21.80^\circ + j500t}$
 $\therefore i_{out} = 3.366e^{j(500t+53.80^\circ)} \text{ A}$

31.

(a) $12 \sin(400t + 110^\circ) \text{ A} \rightarrow 12 \angle 20^\circ \text{ A}$

(b) $-7 \sin 800t - 3 \cos 800t \rightarrow j7 - 3$
 $= -3 + j7 = 7.616 \angle 113.20^\circ \text{ A}$

(c) $4 \cos(200t - 30^\circ) - 5 \cos(200t + 20^\circ)$
 $\rightarrow 4 \angle -30^\circ - 5 \angle 20^\circ = 3.910 \angle -108.40^\circ \text{ A}$

(d) $\omega = 600, t = 5 \text{ ms} : 70 \angle 30^\circ \text{ V}$
 $\rightarrow 70 \cos(600 \times 5 \times 10^{-3} \text{ rad} + 30^\circ) = -64.95 \text{ V}$

(e) $\omega = 600, t = 5 \text{ ms} : 60 + j40 \text{ V} = 72.11 \angle 146.3^\circ$
 $\rightarrow 72.11 \cos(3 \text{ rad} + 146.31^\circ) = 53.75 \text{ V}$

32. $\omega = 4000$, $t = 1\text{ms}$

(a) $I_x = 5\angle -80^\circ \text{ A}$

$$\therefore i_x = 5 \cos(4^{\text{rad}} - 80^\circ) = -4.294 \text{ A}$$

(b) $I_x = -4 + j1.5 = 4.272\angle 159.44^\circ \text{ A}$

$$\therefore i_x = 4.272 \cos(4^{\text{rad}} + 159.44^\circ) = 3.750^- \text{ A}$$

(c) $v_x(t) = 50 \sin(250t - 40^\circ)$

$$= 50 \cos(250t - 130^\circ) \rightarrow V_x = 50\angle -130^\circ \text{ V}$$

(d) $v_x = 20 \cos 108t - 30 \sin 108t$

$$\rightarrow 20 + j30 = 36.06\angle 56.31^\circ \text{ V}$$

(e) $v_x = 33 \cos(80t - 50^\circ) + 41 \cos(80t - 75^\circ)$

$$\rightarrow 33\angle -50^\circ + 41\angle -75^\circ = 72.27\angle -63.87^\circ \text{ V}$$

33. $V_1 = 10\angle 90^\circ$ mV, $\omega = 500$; $V_2 = 8\angle 90^\circ$ mV,
 $\omega = 1200$, M by -5 , $t = 0.5$ ms

$$\begin{aligned}v_{out} &= (-5) [10 \cos(500 \times 0.5 \times 10^{-3} \text{rad} + 90^\circ) \\ &\quad + 8 \cos(1.2 \times 0.5 + 90^\circ)] \\ &= 50 \sin 0.25^{\text{rad}} + 40 \sin 0.6^{\text{rad}} = \boxed{34.96 \text{mV}}\end{aligned}$$

34. Begin with the inductor:
 $(2.5 \angle 40^\circ) (j500) (20 \times 10^{-3}) = 25 \angle 130^\circ$ V across the inductor and the 25- Ω resistor.
The current through the 25- Ω resistor is then $(25 \angle 130^\circ) / 25 = 1 \angle 130^\circ$ A.

The current through the unknown element is therefore $2.5 \angle 40^\circ + 1 \angle 130^\circ = 2.693 \angle 61.80^\circ$ A; this is the same current through the 10- Ω resistor as well. Armed with this information, KVL provides that

$$\mathbf{V}_s = 10(2.693 \angle 61.8^\circ) + (25 \angle -30^\circ) + (25 \angle 130^\circ) = 35.47 \angle 58.93^\circ$$

and so $v_s(t) = 35.47 \cos(500t + 58.93^\circ)$ V.

35. $\omega = 5000$ rad/s.

- (a) The inductor voltage = $48\angle 30^\circ = j\omega L \mathbf{I}_L = j(5000)(1.2 \times 10^{-3}) \mathbf{I}_L$
So $\mathbf{I}_L = 8\angle -60^\circ$ and the total current flowing through the capacitor is
 $10\angle 0^\circ - \mathbf{I}_L = 9.165\angle 49.11^\circ$ A and the voltage \mathbf{V}_1 across the capacitor is

$$\mathbf{V}_1 = (1/j\omega C)(9.165\angle 49.11^\circ) = -j2 (9.165\angle 49.11^\circ) = 18.33\angle -40.89^\circ \text{ V.}$$

Thus, $v_1(t) = 18.33 \cos(5000t - 40.89^\circ) \text{ V.}$

- (b) $\mathbf{V}_2 = \mathbf{V}_1 + 5(9.165\angle 49.11^\circ) + 60\angle 120^\circ = 75.88\angle 79.48^\circ \text{ V}$

$\therefore v_2(t) = 75.88 \cos(5000t + 79.48^\circ) \text{ V}$

- (c) $\mathbf{V}_3 = \mathbf{V}_2 - 48\angle 30^\circ = 75.88\angle 79.48^\circ - 48\angle 30^\circ = 57.70\angle 118.7^\circ \text{ V}$

$\therefore v_3(t) = 57.70 \cos(5000t + 118.70^\circ) \text{ V}$

36. $\mathbf{V}_R = 1\angle 0^\circ \text{ V}$, $\mathbf{V}_{\text{series}} = (1 + j\omega - j/\omega)(1\angle 0^\circ)$

$$V_R = 1 \quad \text{and} \quad V_{\text{series}} = \sqrt{1 + (\omega - 1/\omega)^2}$$

We desire the frequency ω at which $V_{\text{series}} = 2V_R$ or $V_{\text{series}} = 2$

Thus, we need to solve the equation $1 + (\omega - 1/\omega)^2 = 4$

$$\text{or } \omega^2 - \sqrt{3}\omega - 1 = 0$$

Solving, we find that $\omega = 2.189 \text{ rad/s}$.

37. With an operating frequency of $\omega = 400$ rad/s, the impedance of the 10-mH inductor is $j\omega L = j4 \Omega$, and the impedance of the 1-mF capacitor is $-j/\omega C = -j2.5 \Omega$.
- $$\therefore V_c = 2\angle 40^\circ (-j2.5) = 5\angle -50^\circ \text{ A}$$
- $$\therefore I_L = 3 - 2\angle 40^\circ = 1.9513\angle -41.211^\circ \text{ A}$$
- $$\therefore V_L = 4 \times 1.9513\angle 90^\circ - 4.211^\circ = 7.805^+ \angle 48.79^\circ \text{ V}$$
- $$\therefore V_x = V_L - V_c = 7.805^+ \angle 48.79^\circ - 5\angle -50^\circ$$
- $$\therefore V_x = 9.892\angle 78.76^\circ \text{ V}, \quad v_x = 9.892 \cos(400t + 78.76^\circ) \text{ V}$$

38.

$$\text{If } I_{s1} = 2\angle 20^\circ \text{ A, } I_{s2} = 3\angle -30^\circ \text{ A} \rightarrow V_{out} = 80\angle 10^\circ \text{ V}$$

$$I_{s1} = I_{s2} = 4\angle 40^\circ \text{ A} \rightarrow V_{out} = 90 - j30 \text{ V}$$

$$\text{Now let } I_{s1} = 2.5\angle -60^\circ \text{ A and } I_{s2} = 2.5\angle 60^\circ \text{ A}$$

$$\text{Let } V_{out} = AI_{s1} + BI_{s2} \therefore 80\angle 10^\circ = A(2\angle 20^\circ) + B(3\angle -30^\circ)$$

$$\text{and } 90 - j30 = (A + B)(4\angle 40^\circ) \therefore A + B = \frac{90 - j30}{4\angle 40^\circ} = 12.415^+ - j20.21$$

$$\therefore \frac{80\angle 10^\circ}{2\angle 20^\circ} = A + B \frac{3\angle -30^\circ}{2\angle 20^\circ} \therefore A = 40\angle -10^\circ - B(1.5\angle -50^\circ)$$

$$\therefore 12.415^+ - j20.21 - B = 40\angle -10^\circ - B(1.5\angle -50^\circ)$$

$$\therefore 12.415^+ - j20.21 - 40\angle -10^\circ = B(1 - 1.5\angle -50^\circ)$$

$$= B(1.1496\angle +88.21^\circ)$$

$$\therefore B = \frac{30.06\angle -153.82^\circ}{1.1496\angle +88.21^\circ} = 26.148\angle 117.97^\circ$$

$$\therefore A = 12.415^+ - j20.21 - 10.800 + j23.81$$

$$= 49.842\angle -60.32^\circ$$

$$V_{out} = (49.842\angle -60.32^\circ)(2.5\angle -60^\circ)$$

$$+ (26.15\angle 117.97^\circ)(2.5\angle 60^\circ)$$

$$= 165.90\angle -140.63^\circ \text{ V}$$

39. We begin by noting that the series connection of capacitors can be replaced by a single equivalent capacitance of value $C = \frac{1}{1 + \frac{1}{2} + \frac{1}{3}} = 545.5 \mu\text{F}$. Noting $\omega = 2\pi f$,

(a) $\omega = 2\pi \text{ rad/s}$, therefore $\mathbf{Z}_C = -j/\omega C = \frac{-j10^6}{2\pi(545.5)} = -j291.8 \Omega$.

(b) $\omega = 200\pi \text{ rad/s}$, therefore $\mathbf{Z}_C = -j/\omega C = \frac{-j10^6}{200\pi(545.5)} = -j2.918 \Omega$.

(c) $\omega = 2000\pi \text{ rad/s}$, therefore $\mathbf{Z}_C = -j/\omega C = \frac{-j10^6}{2000\pi(545.5)} = -j291.8 \text{ m}\Omega$.

(d) $\omega = 2 \times 10^9 \pi \text{ rad/s}$, therefore $\mathbf{Z}_C = -j/\omega C = \frac{-j10^6}{2 \times 10^9 \pi(545.5)} = -j291.8 \text{ n}\Omega$.

40. We begin by noting that the parallel connection of inductors can be replaced by a single equivalent inductance of value $L = \frac{1}{1+5} = \frac{5}{6}$ nH. In terms of impedance, then, we have

$$\mathbf{Z} = \frac{5 \left(j\omega \frac{5}{6} \times 10^{-9} \right)}{5 + j\omega \frac{5}{6} \times 10^{-9}}$$

Noting $\omega = 2\pi f$,

(a) $\omega = 2\pi$ rad/s, therefore $\mathbf{Z} = j5.236 \times 10^{-9} \Omega$ (the real part is essentially zero).

(b) $\omega = 2 \times 10^3 \pi$ rad/s, therefore $\mathbf{Z} = 5.483 \times 10^{-12} + j5.236 \times 10^{-6} \Omega$.

(c) $\omega = 2 \times 10^6 \pi$ rad/s, therefore $\mathbf{Z} = 5.483 \times 10^{-6} + j5.236 \times 10^{-6} \Omega$.

(d) $\omega = 2 \times 10^9 \pi$ rad/s, therefore $\mathbf{Z} = 2.615 + j2.497 \Omega$.

(e) $\omega = 2 \times 10^{12} \pi$ rad/s, therefore $\mathbf{Z} = 5 + j4.775 \times 10^{-3} \Omega$.

41.

(a) $\omega = 800: 2\mu\text{F} \rightarrow -j625, 0.6\text{H} \rightarrow j480$

$$\begin{aligned}\therefore Z_{in} &= \frac{300(-j625)}{300 - j625} + \frac{600(j480)}{600 + j480} \\ &= 478.0 + j175.65\Omega\end{aligned}$$

(b) $\omega = 1600: Z_{in} = \frac{300(-j312.5)}{300 - j312.5}$

$$+ \frac{600(j960)}{600 + j960} = 587.6 + j119.79\Omega$$

42.

At $\omega = 100$ rad/s, $2 \text{ mF} \rightarrow -j5 \Omega$; $0.1 \text{ H} \rightarrow j10 \Omega$.

$$\begin{aligned} \text{(a)} \quad (10 + j10) \parallel (-j5) &= \frac{50 - j50}{10 + j5} = \frac{10 - j10}{2 + j1} \frac{2 - j1}{2 - j1} \\ &= 2 - j6 \Omega \therefore Z_{in} = 20 + 2 - j6 = \boxed{22 - j6 \Omega} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \text{SC}_{a,b}: 20 \parallel 10 &= 6.667, (6.667 - j5) \parallel j10 \\ &= \frac{50 + j66.67}{6.667 + j5} = \frac{150 + j200}{20 + j15} = \frac{30 + j40}{4 + j3} \times \frac{4 - j3}{4 - j3} \\ &= Z_{in} \therefore Z_{in} (1.2 + j1.6)(4 - j3) = \boxed{9.6 + j2.8 \Omega} \end{aligned}$$

43.

$$\omega = 800: 2\mu\text{F} \rightarrow -j625, 0.6\text{H} \rightarrow j480$$

$$\therefore Z_{in} = \frac{300(-j625)}{300 - j625} + \frac{600(j480)}{600 + j480}$$

$$= 478.0 + j175.65\Omega$$

$$\therefore I = \frac{120}{478.0 + j175.65} \times \frac{-j625}{300 - j625}$$

$$\text{or } I = 0.2124 \angle -45.82^\circ \text{ A}$$

Thus, $i(t) = 212.4 \cos(800t - 45.82^\circ) \text{ mA}$.

44.

(a) $3\Omega + 2\text{mH}: V = (3\angle -20^\circ)(3 + j4) = 15\angle 33.13^\circ \text{ V}$

(b) $3\Omega + 125\mu\text{F}: V = (3\angle -20^\circ)(3 - j4) = 15\angle -73.3^\circ \text{ V}$

(c) $3\Omega 2\text{mH } 125\mu\text{F}: V = (3\angle -20^\circ) 3 = 9\angle -20^\circ \text{ V}$

(d) same: $\omega = 4000 \therefore V = (3\angle -20^\circ)(3 + j8 - j2)$
 $\therefore V = (3\angle -20^\circ)(3 + j6) = 20.12\angle 43.43^\circ \text{ V}$

45.

(a) $C = 20\mu\text{F}$, $\omega = 100$

$$\mathbf{Z}_{in} = \frac{1}{\frac{1}{200} + \frac{1}{j1000} + j1000 \times 20 \times 10^{-6}} = \frac{1}{0.005 - j0.01 + j0.002}$$

$$\therefore \mathbf{Z}_{in} = \frac{1}{0.005 + j0.001} = 196.12 \angle -11.310^\circ \Omega$$

(b) $\omega = 100 \text{ rad/s} \therefore \mathbf{Z}_{in} = \frac{1}{0.005 - j0.001 + j100C}$

$$|\mathbf{Z}_{in}| = 125 = \frac{1}{\sqrt{0.005^2 + (100C - 0.001)^2}}$$

$$\text{or } 64 \times 10^{-6} = 0.005^2 + (100C - 0.001)^2$$

$$\text{so } 6.245 \times 10^{-3} = \sqrt{39 \times 10^{-6}} = 100C - 0.001$$

$$\text{or } C = 72.45 \mu\text{F}$$

(c) $C = 20\mu\text{F} \therefore \mathbf{Z}_{in} = \frac{1}{0.0005 - j0.1/\omega + j2 \times 10^{-5} \omega} = 100 \angle = \frac{1}{0.01 \angle}$

$$\therefore 0.005^2 + \left(2 \times 10^{-5} \omega - \frac{0.1}{\omega}\right)^2 = 0.0001, \left(2 \times 10^{-5} - \frac{0.1}{\omega}\right)^2 = 7.5 \times 10^{-5}$$

$$\therefore 2 \times 10^{-5} - \frac{0.01}{\omega} \mp 866.0 \times 10^{-5} = 0 \therefore 2 \times 10^{-5} \omega^2 \mp 866.0 \times 10^{-5} \omega - 0.1 = 0$$

$$\text{use - sign: } \omega = \frac{866.0 \times 10^{-5} \pm \sqrt{7.5 \times 10^{-5} + 8 \times 10^{-6}}}{4 \times 10^{-5}} = 444.3 \text{ and } < 0$$

$$\text{use + sign: } \omega = \frac{-866.0 \times 10^{-5} \pm \sqrt{7.5 \times 10^{-5} + 8 \times 10^{-6}}}{4 \times 10^{-5}} = 11.254 \text{ and } < 0$$

$$\therefore \omega = 11.254 \text{ and } 444.3 \text{ rad/s}$$

46.

$$(a) \quad \left| \frac{1}{\frac{1}{jx} + \frac{1}{30}} \right| = 25 = \frac{1}{0.04} \therefore \frac{1}{900} + \frac{1}{x^2} = 0.0016$$

$$\therefore X = 45.23 \Omega = 0.002\omega, \omega = 2261 \text{ rad/s}$$

$$(b) \quad \angle Y_{in} = -25^\circ = \angle \text{ of } \left(\frac{1}{30} - j \frac{1}{x} \right) = \tan^{-1} \frac{-30}{x}$$

$$\therefore x = 64.34 = 0.02\omega, \omega = 3217 \text{ rad/s}$$

$$(c) \quad Z_{in} = \frac{30(j0.02\omega)}{30 + j0.02\omega} \times \frac{30 - j0.092\omega}{30 - j0.02\omega} = \frac{0.012\omega^2 + j18\omega}{900 + 0.0004\omega^2}$$

$$\therefore 0.012\omega^2 = 25(900 + 0.0004\omega^2)$$

$$\therefore 0.012\omega^2 = 0.01\omega^2 + 22,500, \omega = 3354 \text{ rad/s}$$

$$(d) \quad 18\omega = 10(900 + 0.0004\omega^2), 0.004\omega^2 - 18\omega + 9000 = 0,$$

$$\omega^2 - 4500\omega + 2.25 \times 10^6 = 0$$

$$\omega = \frac{4500 \pm \sqrt{20.25 \times 10^6 - 9 \times 10^6}}{2} = \frac{4500 \pm 3354}{2} = 572.9, 3927 \text{ rad/s}$$

47. With an operating frequency of $\omega = 400$ rad/s, the impedance of the 10-mH inductor is $j\omega L = j4 \Omega$, and the impedance of the 1-mF capacitor is $-j/\omega C = -j2.5 \Omega$.

$$\therefore V_c = 2\angle 40^\circ (-j2.5) = 5\angle -50^\circ \text{ A}$$

$$\therefore I_L = 3 - 2\angle 40^\circ = 1.9513\angle -41.211^\circ \text{ A}$$

$$I_L = \frac{2\angle 40^\circ (R_2 - j2.5)}{R_1 + j4}$$

$$\therefore R_1 + j4 = \frac{2\angle 40^\circ (R_2 - j2.5)}{1.9513\angle -41.21^\circ}$$

$$= 1.0250\angle 81.21^\circ (R_2 - j2.5)$$

$$= R_2 (1.0250\angle 81.21^\circ) + 2.562\angle -8.789^\circ$$

$$= 0.15662R_2 + j1.0130R_2 + 2.532 - j0.3915$$

$$\therefore R_1 = 2.532 + 0.15662R_2, \quad 4 = 1.0130R_2 - 0.395$$

$$\therefore R_2 = 4.335^+ \Omega, \quad R_1 = 3.211\Omega$$

48. $\omega = 1200 \text{ rad/s.}$

(a) $\omega = 1200$

$$\mathbf{Z}_{in} = \frac{-j \times (200 + j80)}{200 + j(80 - x)} = \frac{(80x - j200x)[200 + j(x - 80)]}{40,000 + 6400 - 160x + x^2}$$

$$X_{in} = 0 \therefore -40,000x + 80x^2 - 6400x = 0$$

$$\therefore 46,400 = 80x, x = 580 \Omega = \frac{1}{1200C} \therefore C = 1.437 \mu\text{F}$$

(b) $\mathbf{Z}_{in} = \frac{80X - j200X}{200 + j(80 - X)} \quad |\mathbf{Z}_{in}| = 100$

$$\therefore \frac{6400X^2 + 40,000X^2}{40,000 + 6400 - 160X + X^2} = 10,000$$

$$\therefore 0.64X^2 + 4X^2 = X^2 - 160X + 46,400$$

$$\therefore 3.64X^2 + 160X - 46,400 = 0,$$

$$X = \frac{-160 \pm \sqrt{25,600 + 675,600}}{7.28} = \frac{-160 \pm 837.4}{7.28}$$

$$\therefore X = 93.05^+ (> 0) = \frac{1}{1200C} \therefore C = 8.956 \mu\text{F}$$

49. At $\omega = 4$ rad/s, the 1/8-F capacitor has an impedance of $-j/\omega C = -j2 \Omega$, and the 4-H inductor has an impedance of $j\omega L = j16 \Omega$.
- (a) Terminals ab open circuited: $\mathbf{Z}_{in} = 8 + j16 \parallel (2 - j2) = 10.56 - j1.92 \Omega$
- (b) Terminals ab short-circuited: $\mathbf{Z}_{in} = 8 + j16 \parallel 2 = 9.969 + j0.2462 \Omega$

50. $f = 1 \text{ MHz}$, $\omega = 2\pi f = 6.283 \text{ Mrad/s}$

$2 \mu\text{F}$	$\rightarrow -j0.07958 \Omega$	$= \mathbf{Z}_1$
$3.2 \mu\text{H}$	$\rightarrow j20.11 \Omega$	$= \mathbf{Z}_2$
$1 \mu\text{F}$	$\rightarrow -j0.1592 \Omega$	$= \mathbf{Z}_3$
$1 \mu\text{H}$	$\rightarrow j6.283 \Omega$	$= \mathbf{Z}_4$
$20 \mu\text{H}$	$\rightarrow j125.7 \Omega$	$= \mathbf{Z}_5$
200 pF	$\rightarrow -j795.8 \Omega$	$= \mathbf{Z}_6$

The three impedances at the upper right, \mathbf{Z}_3 , $700 \text{ k}\Omega$, and \mathbf{Z}_6 reduce to $-j0.01592 \Omega$

Then we form \mathbf{Z}_2 in series with \mathbf{Z}_{eq} : $\mathbf{Z}_2 + \mathbf{Z}_{\text{eq}} = j20.09 \Omega$.

Next we see $10^6 \parallel (\mathbf{Z}_2 + \mathbf{Z}_{\text{eq}}) = j20.09 \Omega$.

Finally, $\mathbf{Z}_{\text{in}} = \mathbf{Z}_1 + \mathbf{Z}_4 + j20.09 = j26.29 \Omega$.

51. As in any true design problem, there is more than one possible solution. Model answers follow:

(a) Using at least 1 inductor, $\omega = 1$ rad/s. $\mathbf{Z} = 1 + j4 \Omega$.

Construct this using a single 1Ω resistor in series with a 4 H inductor.

(b) Force $j\omega L = j/C$, so that $C = 1/L$. Then we construct the network using

a single 5Ω resistor, a 2 H inductor, and a 0.5 F capacitor, all in series (any values for these last two will suffice, provided they satisfy the $C = 1/L$ requirement).

(c) $\mathbf{Z} = 7 \angle 80^\circ \Omega$. $R = \text{Re}\{\mathbf{Z}\} = 7 \cos 80^\circ = 1.216 \Omega$, and $X = \text{Im}\{\mathbf{Z}\} = 7 \sin 80^\circ = 6.894 \Omega$.

We can obtain this impedance at 100 rad/s by placing a resistor of value 1.216Ω in series with an inductor having a value of $L = 6.894/\omega = 68.94$ mH.

(d) A single resistor having value $R = 5 \Omega$ is the simplest solution.

52. As in any true design problem, there is more than one possible solution. Model answers follow:

(a) $1 + j4 \text{ k}\Omega$ at $\omega = 230 \text{ rad/s}$ may be constructed using a $1 \text{ k}\Omega$ resistor in series with an inductor L and a capacitor C such that $j230L - j/(230C) = 4000$. Selecting arbitrarily $C = 1 \text{ F}$ yields a required inductance value of $L = 17.39 \text{ H}$.

Thus, one design is a $1 \text{ k}\Omega$ resistor in series with 17.39 H in series with 1 F .

(b) To obtain a purely real impedance, the reactance of the inductor must cancel the reactance of the capacitor. In a series string, this is obtained by meeting the criterion $\omega L = 1/\omega C$, or $L = 1/\omega^2 C = 1/100C$.

Select a $5 \text{ M}\Omega$ resistor in series with 1 F in series with 100 mH .

(c) If $\mathbf{Z} = 80\angle -22^\circ \Omega$ is constructed using a series combination of a single resistor R and single capacitor C , $R = \text{Re}\{\mathbf{Z}\} = 80\cos(-22^\circ) = 74.17 \Omega$. $X = -1/\omega C = \text{Im}\{\mathbf{Z}\} = 80\sin(-22^\circ) = -29.97 \Omega$. Thus, $C = 667.3 \mu\text{F}$.

(d) The simplest solution, independent of frequency, is a single 300Ω resistor.

53. Note that we may replace the three capacitors in parallel with a single capacitor having value $10^{-3} + 2 \times 10^{-3} + 4 \times 10^{-3} = 7 \text{ mF}$.

(a) $\omega = 4\pi \text{ rad/s}$. $\mathbf{Y} = j4\pi\text{C} = j87.96 \text{ mS}$

(b) $\omega = 400\pi \text{ rad/s}$. $\mathbf{Y} = j400\pi\text{C} = j8.796 \text{ S}$

(c) $\omega = 4\pi \times 10^3 \text{ rad/s}$. $\mathbf{Y} = j4\pi \times 10^3\text{C} = j879.6 \text{ S}$

(d) $\omega = 4\pi \times 10^{11} \text{ rad/s}$. $\mathbf{Y} = j4\pi \times 10^{11}\text{C} = j8.796 \times 10^9 \text{ S}$

54. (a) Susceptance is 0

(b) $B = \omega C = 100 \text{ S}$

(c) $Z = 1 + j100 \Omega$, so $Y = \frac{1}{1 + j100} = \frac{1 - j100}{1 + 100^2} = G + jB$, where $B = -9.999 \text{ mS}$.

55.

$$2\text{H} \rightarrow j2, 1\text{F} \rightarrow -j1 \text{ Let } \mathbf{I}_e = 1\angle 0^\circ \text{ A}$$

$$\therefore \mathbf{V}_L = j2\text{V} \therefore \mathbf{I}_c = \mathbf{I}_{in} + 0.5\mathbf{V}_L = 1 + j1$$

$$\therefore \mathbf{V}_{in} = j2 + (1 + j1)(-j1) = 1 + j1$$

$$\therefore \mathbf{V}_{in} = \frac{1\angle 0^\circ}{\mathbf{V}_{in}} = \frac{1}{1 + j1} \frac{1 - j1}{1 - j1} = 0.5 - j0.5$$

$$\text{Now } 0.5 \text{ S} \rightarrow \boxed{2\Omega} - j0.5 \text{ S} = \frac{1}{j2} \rightarrow \boxed{2 \text{ H}}$$

56.

$$(a) \quad \omega = 500, Z_{inRLC} = 5 + j10 - j1 = 5 + j9$$

$$\therefore Y_{inRLC} = \frac{1}{5 + j9} = \frac{5 - j9}{106} \therefore Y_c = \frac{9}{106} = 500C$$

$$\therefore C = \frac{9}{53,000} = \boxed{169.8 \mu\text{F}}$$

$$(b) \quad R_{in,ab} = \frac{106}{5} = \boxed{21.2 \Omega}$$

$$(c) \quad \omega = 1000 \text{ rad/s} \therefore$$

$$\mathbf{Z}_S = 5 + j2 - j5 = 5 - j3 = 5.831 \angle -30.96^\circ \Omega$$

$$\text{and } \mathbf{Z}_C = -j58.89 \Omega.$$

Thus,

$$\mathbf{Y}_{in,ab} = \frac{1}{\mathbf{Z}_S} + \frac{1}{\mathbf{Z}_C} = \boxed{0.1808 \angle 35.58^\circ \text{ S}}$$
$$= \boxed{147.1 + j105.2 \text{ mS}}$$

57.

$$(a) \quad R_{in} = 550\Omega : Z_{in} = 500 + \frac{j0.1\omega}{100 + j0.001\omega}$$

$$\therefore Z_{in} = \frac{50,000 + j0.6\omega}{100 + j0.001\omega} \times \frac{100 - j0.001\omega}{100 - j0.001\omega}$$

$$\therefore Z_{in} = \frac{5 \times 10^6 + 0.0006\omega^2 + j(60\omega - 50\omega)}{10^4 + 10^{-6}\omega^2}$$

$$\therefore R_{in} = \frac{5 \times 10^6 + 0.006\omega^2}{10^4 + 10^{-6}\omega^2} = 550 \therefore 5.5 \times 10^6$$

$$+ 5.5 \times 10^{-4}\omega^2 = 5 \times 10^6 \times 10^{-4}\omega^2$$

$$\therefore 0.5 \times 10^{-4}\omega^2 = 0.5 \times 10^6, \omega^2 = 10^{10}, \omega = 10^5 \text{ rad/s}$$

$$(b) \quad X_{in} = 50\Omega = \frac{10\omega}{10^4 + 10^{-6}\omega^2} = 0.5 \times 10^6 + 0.5 \times 10^{-4}\omega^2 - 10\omega$$

$$= 0, \omega^2 - 2 \times 10^5\omega + 10^{10} = 0$$

$$\therefore \omega = \frac{2 \times 10^5 \pm \sqrt{4 \times 10^{10} - 4 \times 10^{10}}}{2} = 10^5 \therefore \omega = 10^5 \text{ rad/s}$$

$$(c) \quad G_{in} = 1.8 \times 10^{-3} : Y_{in} = \frac{100 + j0.001\omega}{50,000 + j0.6\omega} \times \frac{50,000 - j0.6\omega}{50,000 - j0.6\omega}$$

$$= \frac{5 \times 10^6 + 6 \times 10^{-4}\omega^2 + j(50\omega - 6\omega)}{25 \times 10^8 + 0.36\omega^2}$$

$$\therefore 1.8 \times 10^3 = \frac{5 \times 10^6 + 6 \times 10^{-4}\omega^2}{25 \times 10^8 + 0.36\omega^2}$$

$$\therefore 5 \times 10^6 + 6 \times 10^{-4}\omega^2 = 4.5 \times 10^6 + 648 \times 10^{-6}\omega^2$$

$$\therefore 0.5 \times 10^6 = 48 \times 10^{-6}\omega^2 \therefore \omega = 102.06 \text{ krad/s}$$

$$(d) \quad B_{in} = 1.5 \times 10^{-4} = \frac{-10\omega}{25 \times 10^8 + 0.36\omega^2}$$

$$\therefore 10\omega = 37.5 \times 10^4 + 54 \times 10^{-6}\omega^2$$

$$\therefore 54 \times 10^{-6}\omega^2 - 10\omega + 37.5 \times 10^4 = 0,$$

$$\omega = 10 \pm \frac{\sqrt{100 - 81}}{108 \times 10^{-6}} = 52.23 \text{ and } 133.95 \text{ krad/s}$$

58.

$$(a) \quad V_1 = \frac{I_1}{Y_1} = \frac{0.1 \angle 30^\circ}{(3 + j4)10^{-3}} = 20 \angle -23.13^\circ \therefore |V_1| = \boxed{20 \text{ V}}$$

$$(b) \quad V_2 = V_1 \therefore |V_2| = \boxed{20 \text{ V}}$$

$$(c) \quad I_2 = Y_2 V_2 = (5 + j2)10^{-3} \times 20 \angle -23.13^\circ = 0.10770 \angle -1.3286^\circ \text{ A} \\ \therefore I_3 = I_1 + I_2 = 0.1 \angle 30^\circ + 0.10770 \angle -1.3286^\circ = 0.2 \angle 13.740^\circ \text{ A} \\ \therefore V_3 = \frac{I_3}{Y_3} = \frac{0.2 \angle 13.740^\circ}{(2 - j4)10^{-3}} = 44.72 \angle 77.18^\circ \text{ V} \therefore |V_3| = \boxed{44.72 \text{ V}}$$

$$(d) \quad V_{in} = V_1 + V_3 + 20 \angle -23.13^\circ + 44.72 \angle 77.18^\circ = 45.60 \angle 51.62^\circ \\ \therefore |V_{in}| = \boxed{45.60 \text{ V}}$$

59.

$$(a) \quad 50\mu\text{F} \rightarrow -j20\Omega \therefore Y_{in} = 0.1 + j0.05$$

$$Y_{in} = \frac{1}{R_1 - j\frac{1000}{C}} \therefore R_1 - j\frac{1000}{C} = \frac{1}{0.1 + j0.05} = 8 - j4$$

$$\therefore R_1 = 8\Omega \text{ and } C_1 = \frac{1}{4\omega} = \boxed{250\mu\text{F}}$$

$$(b) \quad \omega = 2000: 50\mu\text{F} \rightarrow -j10\Omega \therefore Y_{in} = 0.1 + j0.1 = \frac{1}{R_1 - j\frac{500}{C_1}}$$

$$\therefore R_1 - j\frac{500}{C_1} = 5 - j5 \therefore R_1 = 5\Omega, C_1 = \boxed{100\mu\text{F}}$$

60.

$$(a) \quad Z_{in} = 1 + \frac{10}{j\omega} = \frac{10 + j\omega}{j\omega}$$

$$\therefore Y_{in} = \frac{j\omega}{10 + j\omega} \times \frac{10 - j\omega}{10 - j\omega}$$

$$\therefore Y_{in} = \frac{\omega^2 + j10\omega}{\omega^2 + 100}$$

$$G_{in} = \frac{\omega^2}{\omega^2 + 100}, \quad B_{in} = \frac{10\omega}{\omega^2 + 100}$$

ω	G_{in}	B_{in}
0	0	0
1	0.0099	0.0099
2	0.0385	0.1923
5	0.2	0.4
10	0.5	0.5
20	0.8	0.4
∞	1	0

61. As in any true design problem, there is more than one possible solution. Model answers follow:

(a) $\mathbf{Y} = 1 - j4 \text{ S}$ at $\omega = 1 \text{ rad/s}$.

Construct this using a 1 S conductance in parallel with an inductance L such that $1/\omega L = 4$, or $L = 250 \text{ mH}$.

(b) $\mathbf{Y} = 200 \text{ mS}$ (purely real at $\omega = 1 \text{ rad/s}$). This can be constructed using a 200 mS conductance ($R = 5 \Omega$), in parallel with an inductor L and capacitor C such that $\omega C - 1/\omega L = 0$. Arbitrarily selecting $L = 1 \text{ H}$, we find that $C = 1 \text{ F}$.

One solution therefore is a 5 Ω resistor in parallel with a 1 F capacitor in parallel with a 1 H inductor.

(c) $\mathbf{Y} = 7\angle 80^\circ \mu\text{S} = G + jB$ at $\omega = 100 \text{ rad/s}$. $G = \text{Re}\{\mathbf{Y}\} = 7\cos 80^\circ = 1.216 \text{ S}$ (an 822.7 m Ω resistor). $B = \text{Im}\{\mathbf{Y}\} = 7\sin 80^\circ = 6.894 \text{ S}$. We may realize this susceptance by placing a capacitor C in parallel with the resistor such that $j\omega C = j6.894$, or $C = 68.94 \text{ mF}$.

One solution therefore is an 822.7 m Ω resistor in parallel with a 68.94 mF.

(d) The simplest solution is a single conductance $G = 200 \text{ mS}$ (a 5 Ω resistor).

62. As in any true design problem, there is more than one possible solution. Model answers follow:

(a) $\mathbf{Y} = 1 - j4$ pS at $\omega = 30$ rad/s.

Construct this using a 1 pS conductance (a 1 T Ω resistor) in parallel with an inductor L such that $-j4 \times 10^{-12} = -j/\omega L$, or $L = 8.333$ GH.

(b) We may realise a purely real admittance of 5 μ S by placing a 5 μ S conductance (a 200 k Ω resistor) in parallel with a capacitor C and inductance L such that $\omega C - 1/\omega L = 0$. Arbitrarily selecting a value of $L = 2$ H, we find a value of $C = 1.594$ μ F.

One possible solution, then, is a 200 k Ω resistor in parallel with a 2 H inductor and a 1.594 μ F capacitor.

(c) $\mathbf{Y} = 4 \angle -10^\circ$ nS = $G + jB$ at $\omega = 50$ rad/s. $G = \text{Re}\{\mathbf{Y}\} = 4 \times 10^{-9} \cos(-10^\circ) = 3.939$ nS (an 253.9 M Ω resistor). $B = \text{Im}\{\mathbf{Y}\} = 4 \times 10^{-9} \sin(-10^\circ) = -6.946 \times 10^{-10}$ S. We may realize this susceptance by placing an inductor L in parallel with the resistor such that $-j/\omega L = -6.946 \times 10^{-10}$, or $L = 28.78$ μ H.

One possible solution, then, is a 253.9 M Ω resistor in parallel with a 28.78 μ H inductor.

(d) The simplest possible solution is a 60 nS resistor (a 16.67 M Ω resistor).

63.

$$-j5 = \frac{v_1}{3} + \frac{V_1 - V_2}{-j5} + \frac{v_1 - V_2}{j3}, \quad -j75 = 5V_1 + j3V_1 - j3V_2 - j5V_1 + j5V_2$$

$$\therefore (5 - j2)V_1 + j2V_2 = -j75 \quad (1)$$

$$\frac{v_2 - V_1}{j3} + \frac{V_2 - V_1}{-j5} + \frac{V_2}{6} = 10$$

$$-j10V_2 + j10V_1 + j6V_2 - j6V_1 + 5V_2 = 300 \therefore j4V_1 + (5 - j4)V_2 = 300 \quad (2)$$

$$\therefore V_2 = \frac{\begin{vmatrix} 5 - j2 & -j75 \\ j4 & 300 \end{vmatrix}}{\begin{vmatrix} 5 - j2 & j2 \\ j4 & 5 - j4 \end{vmatrix}} = \frac{1500 - j600 - 300}{17 - j30 + 8} = \frac{1200 - j600}{25 - j30} = 34.36 \angle 23.63^\circ \text{ V}$$

64.

$$j3I_B - j5(I_B - I_D) = 0 \therefore -2I_B + j5I_D = 0$$

$$3(I_D + j5) - j5(I_D - I_B) + 6(I_D + 10) = 0$$

$$\therefore j5I_B + (9 - j5)I_D = -60 - j15$$

$$I_B = \frac{\begin{vmatrix} 0 & j5 \\ -60 - j15 & 9 - j5 \end{vmatrix}}{\begin{vmatrix} -j2 & j5 \\ j5 & 9 - j5 \end{vmatrix}} = \frac{-75 + j300}{15 - j18}$$

$$= 13.198 \angle 154.23^\circ \text{ A}$$

65.

$$v_{s1} = 20 \cos 1000t \text{ V}, v_{s2} = 20 \sin 1000t \text{ V}$$

$$\therefore V_{s1} = 20 \angle 0^\circ \text{ V}, V_{s2} = -j20 \text{ V}$$

$$0.01 \text{ H} \rightarrow j10 \Omega, 0.1 \text{ mF} \rightarrow -j10 \Omega$$

$$\therefore \frac{v_x - 20}{j10} + \frac{v_x}{25} + \frac{v_x + j20}{-j10} = 0, 0.04v_x + j2 - 2 = 0,$$

$$V_x = 25(2 - j2) = 70.71 \angle -45^\circ \text{ V}$$

$$\therefore v_x(t) = 70.71 \cos(1000t - 45^\circ) \text{ V}$$

66.

(a) Assume $V_3 = 1V \therefore V_2 = 1 - j0.5V, I_2 = 1 - j0.5 \text{ mA}$
 $\therefore V_1 = 1 - j0.5 + (2 - j0.5)(-j0.5) = 0.75 - j1.5V$
 $\therefore I_1 = 0.75 - j1.5 \text{ mA}, \therefore I_{in} = 0.75 - j1.5 + 2 - j0.5 = 2.75 - j2 \text{ mA}$
 $\therefore V_{in} = 0.75 - j1.5 - j1.5 + (2.75 - j2)(-j0.5)$
 $= -0.25 - j2.875 \text{ V} \therefore V_3 = \frac{100}{-j0.25 - j2.875} = 34.65^\circ \angle 94.97^\circ \text{ V}$

(b) $-j0.5 \rightarrow -jx$ Assume $V_3 = 1V \therefore I_3 = 1A,$
 $V_2 = 1 - jX, I_2 = 1 - jX, \rightarrow I_{12} = 2 - jX$
 $\therefore V_1 = 1 - jX + (2 - jX)(-jX) = 1 - X^2 - j3X, I_1 = 1 - X^2 - j3X, I_{in} = 3 - X^2 - j4X$
 $\therefore V_{in} = 1 - X^2 - j3X - 4X^2 + jX^3 - j3X = 1 - 5X^2 + j(X^3 - 6X) \therefore X^3 - 6X = 0$
 $\therefore X^2 = 6, X = \sqrt{6}, Z_c = -j2.449 \text{ k}\Omega$

67. Define three clockwise mesh currents i_1 , i_2 , i_3 with i_1 in the left mesh, i_2 in the top right mesh, and i_3 in the bottom right mesh.

$$\text{Mesh 1: } -10\angle 0^\circ + (1 + 1 - j0.25)\mathbf{I}_1 - \mathbf{I}_2 - (-j0.25)\mathbf{I}_3 = 0$$

$$\text{Mesh 2: } -\mathbf{I}_1 + (1 + 1 + j4)\mathbf{I}_2 - \mathbf{I}_3 = 0$$

$$\text{Mesh 3: } (-j0.25 + 1 + 1)\mathbf{I}_3 - \mathbf{I}_2 - (-j0.25\mathbf{I}_1) = 0$$

$$\mathbf{I}_x = \frac{\begin{vmatrix} 2 - j0.25 & -1 & 10 \\ -1 & 2 + j4 & 0 \\ j0.25 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} 2 - j0.25 & -1 & j0.25 \\ -1 & 2 + j4 & -1 \\ j0.25 & -1 & 2 - j0.25 \end{vmatrix}}$$

$$\begin{aligned} \therefore \mathbf{I}_x &= \frac{10(1+1-j0.5)}{j0.25(2-j0.5) + (-2+j0.25+j0.25) + (2-j0.25)(4+1-j0.5+j8-1)} \\ &= \frac{20-j5}{8+j15} \therefore \mathbf{I}_x = 1.217\angle -75.96^\circ \text{ A, } \boxed{i_x(t) = 1.2127 \cos(100t - 75.96^\circ) \text{ A}} \end{aligned}$$

68.

$$V_1 - 10 - j0.25V_1 + j0.25V_x + V_1 - V_2 = 0$$

$$\therefore (2 - j0.25)V_1 - V_2 + j0.25V_x = 10$$

$$V_2 - V_1 + V_2 - V_x + j4V_2 = 0$$

$$-V_1 + (2 + j4)V_2 - V_x = 0$$

$$-j0.25V_x + j0.25V_1 + V_x + V_x - V_2$$

$$\therefore j0.25V_1 - V_2 + (2 - j0.25)V_x = 0$$

$$V_x = \frac{\begin{vmatrix} 2 - j0.25 & -1 & 10 \\ -1 & 2 + j4 & 0 \\ j0.25 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} -j0.25 & -1 & j0.25 \\ -1 & 2 + j4 & -1 \\ j0.25 & -1 & 2 - j0.25 \end{vmatrix}}$$

$$= \frac{10(1 + 1 - j0.5)}{j0.25(2 - j0.5) + (-2 + j0.25 + j0.25) + (2 - j0.25)(4 + 1 - j0.5 + j8 - 1)}$$

$$= \frac{20 - j5}{8 + j15} = 1.2127 \angle -75.96^\circ \text{ V}$$

$$\therefore v_x = 1.2127 \cos(100t - 75.96^\circ) \text{ V}$$

69.

(a) $R_1 = \infty, R_o = 0, A = -V_o / V_i \gg 0$

$$I = \frac{V_1 + AV_i}{R_f} = j\omega C_1 (V_s - V_i)$$

$$\therefore V_i(1 + A + j\omega C_1 R_f) = j\omega C_1 R_f V_s$$

$$V_o = -AV_i \therefore -\frac{V_o}{A}(1 + A + j\omega C_1 R_f) = j\omega C_1 R_f V_s$$

$$\therefore \frac{V_o}{V_s} = -\frac{j\omega C_1 R_f A}{1 + A + j\omega C_1 R_f} \text{ As } A \rightarrow \infty, \frac{V_o}{V_s} \rightarrow -j\omega C_1 R_f$$

(b) $R_f \parallel C_f = \frac{1}{j\omega C_f + \frac{1}{R_f}} = \frac{R_f}{1 + j\omega C_f R_f}$

$$I = \frac{(V_1 + AV_i)}{R_f}(1 + j\omega C_f R_f) = (V_s - V_i) j\omega C_1, V_o = -AV_i$$

$$\therefore V_i(1 + A)(1 + j\omega C_f R_f) = V_s j\omega C_1 R_f - j\omega C_1 R_f V_i,$$

$$V_i [(1 + A)(1 + j\omega C_f R_f) + j\omega C_1 R_f] = j\omega C_1 R_f V_s$$

$$\therefore -\frac{V_o}{A} [(1 + A)(1 + j\omega C_f R_f) + j\omega C_1 R_f] = j\omega C_1 R_f V_s$$

$$\therefore \frac{V_o}{V_s} = \frac{-j\omega C_1 R_f A}{(1 + A)(1 + j\omega C_f R_f) + j\omega C_1 R_f} \text{ As } A \rightarrow \infty, \frac{V_o}{V_s} \rightarrow \frac{-j\omega C_1 R_f}{1 + j\omega C_f R_f}$$

70. Define the nodal voltage $v_1(t)$ at the junction between the two dependent sources. The voltage source may be replaced by a $3\angle-3^\circ$ V source, the $600\text{-}\mu\text{F}$ capacitor by a $-j/0.6\ \Omega$ impedance, the $500\text{-}\mu\text{F}$ capacitor by a $-j2\ \Omega$ impedance, and the inductor by a $j2\ \Omega$ impedance.

$$5\mathbf{V}_2 + 3\mathbf{V}_2 = \frac{\mathbf{V}_1 - 3\angle-3^\circ}{100 - j/0.6} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j2} \quad [1]$$

$$-5\mathbf{V}_2 = \frac{(\mathbf{V}_2 - \mathbf{V}_1)}{-j2} + \frac{\mathbf{V}_2}{j2} \quad [2]$$

Solving, we find that $\mathbf{V}_2 = 9.81 \angle -13.36^\circ$ mV.
Converting back to the time domain,

$$v_2(t) = 9.81 \cos(10^3 t - 13.36^\circ) \text{ mV}$$

71. Define three clockwise mesh currents: $i_1(t)$ in the left-most mesh, $i_2(t)$ in the bottom right mesh, and $i_3(t)$ in the top right mesh. The 15- μF capacitor is replaced with a $-j/0.15 \Omega$ impedance, the inductor is replaced by a $j20 \Omega$ impedance, the 74 μF capacitor is replaced by a $-j1.351 \Omega$ impedance, the current source is replaced by a $2\angle 0^\circ$ mA source, and the voltage source is replaced with a $5\angle 0^\circ$ V source.

Around the 1, 2 supermesh: $(1 + j20) \mathbf{I}_1 + (13 - j1.351) \mathbf{I}_2 - 5 \mathbf{I}_3 = 0$
and

$$-\mathbf{I}_1 + \mathbf{I}_2 = 2 \times 10^{-3}$$

Mesh 3: $5\angle 0^\circ - 5 \mathbf{I}_2 + (5 - j6.667) \mathbf{I}_3 = 0$

Solving, we find that $\mathbf{I}_1 = 148.0\angle 179.6^\circ$ mA. Converting to the time domain,

$$i_1(t) = 148.0 \cos(10^4 t + 179.6^\circ) \mu\text{A}$$

$$\begin{aligned} \text{Thus, } P_{1\Omega} &= [i_1(1 \text{ ms})]^2 \cdot 1 \\ &= (16.15 \times 10^{-3})(1) \text{ W} = \boxed{16.15 \text{ mW}} \end{aligned}$$

72. We define an additional clockwise mesh current $i_4(t)$ flowing in the upper right-hand mesh. The inductor is replaced by a $j0.004 \Omega$ impedance, the $750 \mu\text{F}$ capacitor is replaced by a $-j/0.0015 \Omega$ impedance, and the $1000 \mu\text{F}$ capacitor is replaced by a $-j/2 \Omega$ impedance. We replace the left voltage source with a $6 \angle -13^\circ \text{ V}$ source, and the right voltage source with a $6 \angle 0^\circ \text{ V}$ source.

$$(1 - j/0.0015) \mathbf{I}_1 + j/0.0015 \mathbf{I}_2 - \mathbf{I}_3 = 6 \angle -13^\circ \quad [1]$$

$$(0.005 + j/0.0015) \mathbf{I}_1 + (j0.004 - j/0.0015) \mathbf{I}_2 - j0.004 \mathbf{I}_4 = 0 \quad [2]$$

$$-\mathbf{I}_1 + (1 - j500) \mathbf{I}_3 + j500 \mathbf{I}_4 = -6 \angle 0^\circ \quad [3]$$

$$-j0.004 \mathbf{I}_2 + j500 \mathbf{I}_3 + (j0.004 - j500) \mathbf{I}_4 = 0 \quad [4]$$

Solving, we find that

$$\mathbf{I}_1 = 2.002 \angle -6.613^\circ \text{ mA}, \mathbf{I}_2 = 2.038 \angle -6.500^\circ \text{ mA}, \text{ and } \mathbf{I}_3 = 5.998 \angle 179.8^\circ \text{ A}.$$

Converting to the time domain,

$$\begin{aligned} i_1(t) &= 1.44 \cos(2t - 6.613^\circ) \text{ mA} \\ i_2(t) &= 2.038 \cos(2t - 6.500^\circ) \text{ mA} \\ i_3(t) &= 5.998 \cos(2t + 179.8^\circ) \text{ A} \end{aligned}$$

73. We replace the voltage source with a $115\sqrt{2} \angle 0^\circ$ V source, the capacitor with a $-j/2\pi C_1 \Omega$ impedance, and the inductor with a $j0.03142 \Omega$ impedance.

Define \mathbf{Z} such that $\mathbf{Z}^{-1} = 2\pi C_1 - j/0.03142 + 1/20$

By voltage division, we can write that $6.014 \angle 85.76^\circ = 115\sqrt{2} \frac{\mathbf{Z}}{\mathbf{Z} + 20}$

Thus, $\mathbf{Z} = 0.7411 \angle 87.88^\circ \Omega$. This allows us to solve for C_1 :

$$2\pi C_1 - 1/0.03142 = -1.348 \text{ so that } C_1 = 4.85 \text{ F.}$$

74. Defining a clockwise mesh current $i_1(t)$, we replace the voltage source with a $115\sqrt{2} \angle 0^\circ$ V source, the inductor with a $j2\pi L$ Ω impedance, and the capacitor with a $-j1.592$ Ω impedance.

$$\text{Ohm's law then yields } \mathbf{I}_1 = \frac{115\sqrt{2}}{20 + j(2\pi L - 1.592)} = 8.132 \angle 0^\circ$$

Thus, $20 = \sqrt{20^2 + (2\pi L - 1.592)^2}$ and we find that $L = 253.4$ mH.

75. (a) By nodal analysis:

$$0 = (\mathbf{V}_\pi - 1)/R_S + \mathbf{V}_\pi/R_B + \mathbf{V}_\pi/r_\pi + j\omega C_\pi \mathbf{V}_\pi + (\mathbf{V}_\pi - \mathbf{V}_{\text{out}})j\omega C_\mu \quad [1]$$

$$-g_m \mathbf{V}_\pi = (\mathbf{V}_{\text{out}} - \mathbf{V}_\pi)j\omega C_\mu + \mathbf{V}_{\text{out}}/R_C + \mathbf{V}_{\text{out}}/R_L \quad [2]$$

Simplify and collect terms:

$$\left[\left(\frac{1}{R_S} + \frac{1}{R_B} + \frac{1}{r_\pi} \right) + j\omega(C_\pi + C_\mu) \right] \mathbf{V}_\pi - j\omega C_\mu \mathbf{V}_{\text{out}} = \frac{1}{R_S} \quad [1]$$

$$(-g_m + j\omega C_\mu) \mathbf{V}_\pi - (j\omega C_\mu + 1/R_C + 1/R_L) \mathbf{V}_{\text{out}} = 0 \quad [2]$$

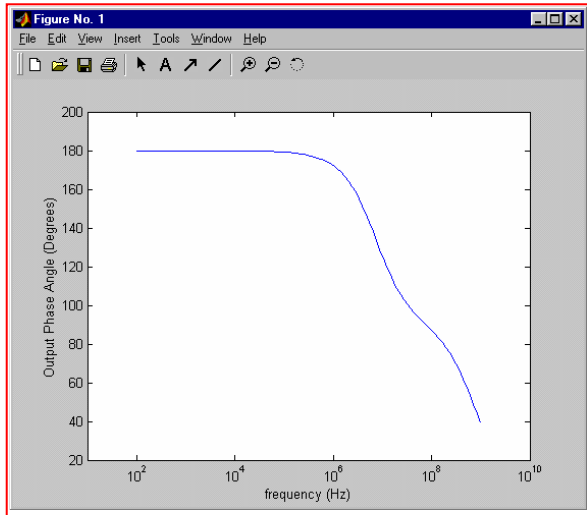
Define $\frac{1}{R_S'} = \frac{1}{R_S} + \frac{1}{R_B} + \frac{1}{r_\pi}$ and $R_L' = R_C \parallel R_L$

Then $\Delta = \frac{-1}{R_S' R_L'} + \omega^2(2C_\mu^2 + C_\mu C_\pi) - j\omega \left(g_m C_\mu + \frac{C_\mu + C_\pi}{R_L'} + \frac{C_\mu}{R_S'} \right)$

And $\mathbf{V}_{\text{out}} = \frac{g_m R_S - j\omega C_\mu / R_S}{\frac{-1}{R_S' R_L'} + \omega^2(2C_\mu^2 + C_\mu C_\pi) - j\omega \left(g_m C_\mu + \frac{C_\mu + C_\pi}{R_L'} + \frac{C_\mu}{R_S'} \right)}$

Therefore, $\text{ang}(\mathbf{V}_{\text{out}}) = \tan^{-1} \left(\frac{-j\omega C_\mu}{g_m R_S^2} \right) - \tan^{-1} \left(\frac{-\omega \left(g_m C_\mu + \frac{C_\mu + C_\pi}{R_L'} + \frac{C_\mu}{R_S'} \right)}{\frac{-1}{R_S' R_L'} + \omega^2(2C_\mu^2 + C_\mu C_\pi)} \right)$

(b)



(c) The output is $\sim 180^\circ$ out of phase with the input for $f < 10^5$ Hz; only for $f = 0$ is it exactly 180° out of phase with the input.

76.

$$\text{OC: } -\frac{V_x}{20} + \frac{100 - V_x}{-j10} - 0.02V_x = 0$$

$$j10 = (0.05 + j0.1 + 0.02) V_x, V_x = \frac{j10}{0.07 + j0.1}$$

$$\therefore V_x = 67.11 + j46.98$$

$$\therefore V_{ab,oc} = 100 - V_x = 32.89 - j46.98 = 57.35 \angle -55.01^\circ \text{ V}$$

$$\text{SC: } V_x = 100 \therefore \downarrow I_{SC} = 0.02 \times 100 + \frac{100}{20} = 7 \text{ A}$$

$$\therefore Z_{th} = \frac{57.35 \angle -55.01^\circ}{7} = 4.698 - j6.711 \Omega$$

77.

Let $\mathbf{I}_{in} = 1\angle 0$. Then $\mathbf{V}_L = j2\omega\mathbf{I}_{in} = j2\omega \therefore 0.5\mathbf{V}_L = j\omega$

$$\therefore \mathbf{V}_{in} = (1 + j\omega)\frac{1}{j\omega} + j2\omega$$

$$= 1 + \frac{1}{j\omega} + j2\omega$$

$$\therefore \mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{1} = 1 + \frac{1}{j\omega} + j2\omega \quad \text{so } \mathbf{Y}_{in} = \frac{\omega}{\omega + j(2\omega^2 - 1)}$$

At $\omega = 1$, $\mathbf{Z}_{in} = 1 - j1 + j2 = 1 + j$

$$\therefore \mathbf{Y}_m = \frac{1}{1 + j1} = 0.5 - j0.5$$

$$\mathbf{R} = 1/0.5 = 2 \Omega \quad \text{and} \quad \mathbf{L} = 1/0.5 = 2 \text{ H.}$$

78.

$$(a) \quad V_s : \frac{(1-j1)1}{2-j1} \times \frac{2+j1}{2+j1} = \frac{3-j1}{5} \therefore V_1 = \frac{-15}{j2+0.6-j0.2} \times 0.6-j0.2$$

$$\therefore V_1 = 5 \angle 90^\circ \therefore v_1(t) = 5 \cos(1000t + 90^\circ) \text{ V}$$

(b) I_s :

$$j2 \parallel 1 = \frac{j2}{1+j2} \frac{1-j2}{1-j2} = 0.8 + j0.4 \therefore V_1$$

$$= j25 \frac{0.8 + j0.4}{1-j1+0.8+j0.4} = \frac{-10 + j20}{1.8-j0.6} = 11.785^\circ \angle 135^\circ \text{ V}$$

$$\text{so } v_1(t) = 11.79 \cos(1000t + 135^\circ) \text{ V.}$$

79.

$$\text{OC: } V_L = 0 \therefore V_{ab,oc} = 1 \angle 0^\circ \text{ V}$$

$$\text{SC: } \downarrow I_N \therefore V_L = j2I_N \therefore 1 \angle 0^\circ = -j1[0.25(j2I_N) + I_N] + j2I_N$$

$$\therefore 1 = (0.5 - j + j2)I_N = (0.5 + j1)I_N$$

$$\therefore I_N = \frac{1}{0.5 + j1} = 0.4 - j0.8 \therefore Y_N = \frac{I_N}{1 \angle 0^\circ} = 0.4 - j0.8$$

$$\therefore R_N = \frac{1}{0.4} = 2.5 \Omega, \frac{1}{j\omega L_N} = \frac{1}{jL_N} = -j0.8, L_N = \frac{1}{0.8} = 1.25 \text{ H}$$

$$I_N = 0.4 - j0.8 = 0.8944 \angle -63.43^\circ \text{ A}$$

80. To solve this problem, we employ superposition in order to separate sources having different frequencies. First considering the sources operating at $\omega = 200$ rad/s, we open-circuit the 100 rad/s current source. This leads to $\mathbf{V}'_L = (j)(2\angle 0) = j2$ V. Therefore, $v'_L(t) = 2\cos(200t + 90^\circ)$ V. For the 100 rad/s source, we find

$$\mathbf{V}''_L = \frac{j}{2}(1\angle 0), v''_L = 0.5\cos(100t + 90^\circ) \text{ V}$$

$$\therefore v_L(t) = 2\cos(200t + 90^\circ) + 0.5\cos(100t + 90^\circ) \text{ V}$$

81.

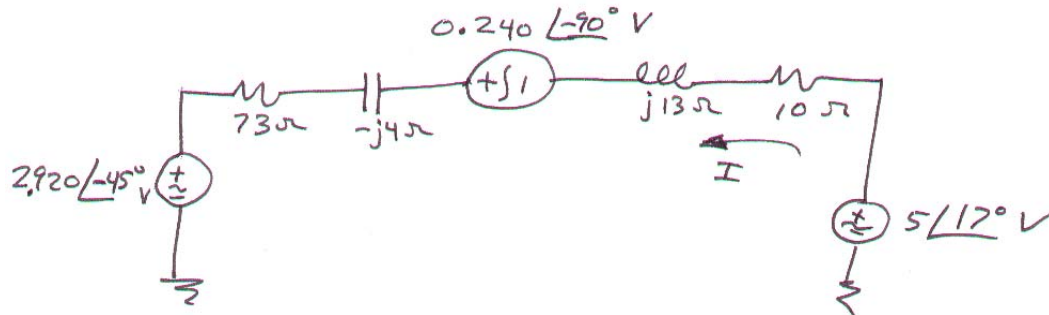
Use superposition. Left: $V_{ab} = 100 \frac{j100}{j100 - j300}$

$= -50 \angle 0^\circ \text{ V}$ Right: $V_{ab} = j100 \frac{-j300}{-j300 + j100} = j150 \text{ V}$

$\therefore V_{th} = -50 + j150 = 158.11 \angle 108.43^\circ \text{ V}$

$Z_{th} = j100 \parallel -j300 = \frac{30,000}{-j200} = j150 \Omega$

82. This problem is easily solved if we first perform two source transformations to yield a circuit containing only voltage sources and impedances:

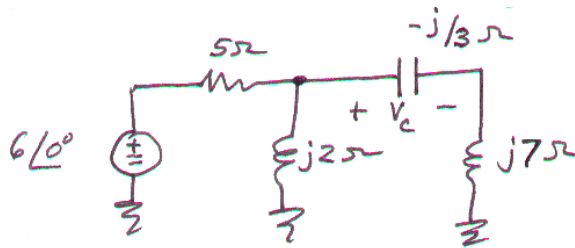


$$\begin{aligned} \text{Then } \mathbf{I} &= \frac{5\angle 17^\circ + 0.240\angle -90^\circ - 2.920\angle -45^\circ}{73 + 10 + j13 - j4} \\ &= (4.264\angle 50.42^\circ) / (83.49\angle 6.189^\circ) = 51.07\angle 44.23^\circ \text{ mA} \end{aligned}$$

Converting back to the time domain, we find that

$$i(t) = 51.07 \cos(10^3 t + 43.23^\circ) \text{ mA}$$

83.



(a) There are a number of possible approaches: Thévenizing everything to the left of the capacitor is one of them.

$$\mathbf{V}_{\text{TH}} = 6(j2)/(5 + j2) = 2.228 \angle 68.2^\circ \text{ V}$$

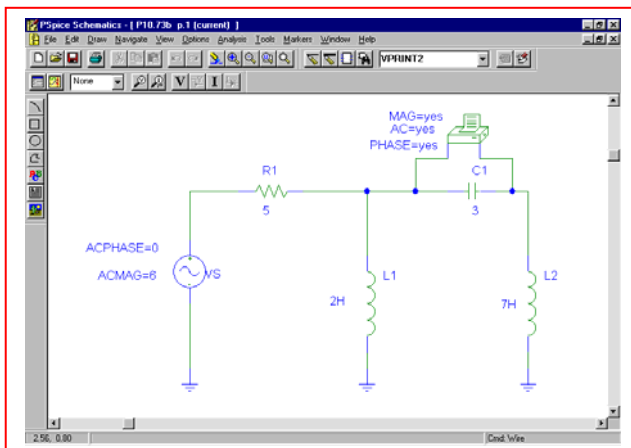
$$\mathbf{Z}_{\text{TH}} = 5 \parallel j2 = j10/(5 + j2) = 1.857 \angle 68.2^\circ \Omega$$

Then, by simple voltage division, we find that

$$\begin{aligned} \mathbf{V}_C &= (2.228 \angle 68.2^\circ) \frac{-j/3}{1.857 \angle 68.2^\circ - j/3 + j7} \\ &= 88.21 \angle -107.1^\circ \text{ mV} \end{aligned}$$

Converting back to the time domain, $v_C(t) = 88.21 \cos(t - 107.1^\circ) \text{ mV}$.

(b) PSpice verification.



Running an ac sweep at the frequency $f = 1/2\pi = 0.1592 \text{ Hz}$, we obtain a phasor magnitude of 88.23 mV, and a phasor angle of -107.1° , in agreement with our calculated result (the slight disagreement is a combination of round-off error in the hand calculations and the rounding due to expressing 1 rad/s in Hz).

```
* C:\My Documents\HKD-6th Edition\SOLUTIONS MANUAL\Chapter 10\P10.73b.sch
**** AC ANALYSIS TEMPERATURE = 27.000 DEG C
*****
FREQ      VH({#N_0002,#N_0003})VP({#N_0002,#N_0003})
1.592E-01  8.823E-02  -1.071E+02
JOB CONCLUDED
TOTAL JOB TIME .06
```

84. (a) Performing nodal analysis on the circuit,

$$\text{Node 1: } 1 = \mathbf{V}_1/5 + \mathbf{V}_1/(-j10) + (\mathbf{V}_1 - \mathbf{V}_2)/(-j5) + (\mathbf{V}_1 - \mathbf{V}_2)/j10 \quad [1]$$

$$\text{Node 2: } j0.5 = \mathbf{V}_2/10 + (\mathbf{V}_2 - \mathbf{V}_1)/(-j5) + (\mathbf{V}_2 - \mathbf{V}_1)/j10 \quad [2]$$

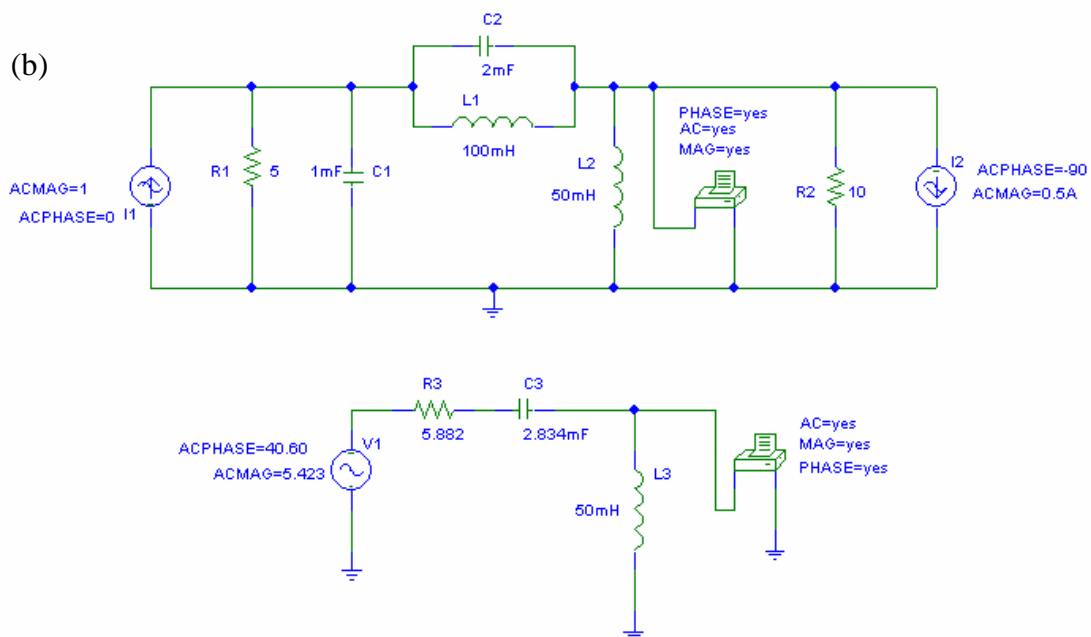
Simplifying and collecting terms,

$$(0.2 + j0.2) \mathbf{V}_1 - j0.1 \mathbf{V}_2 = 1 \quad [1]$$

$$-j \mathbf{V}_1 + (1 + j) \mathbf{V}_2 = j5 \quad [2]$$

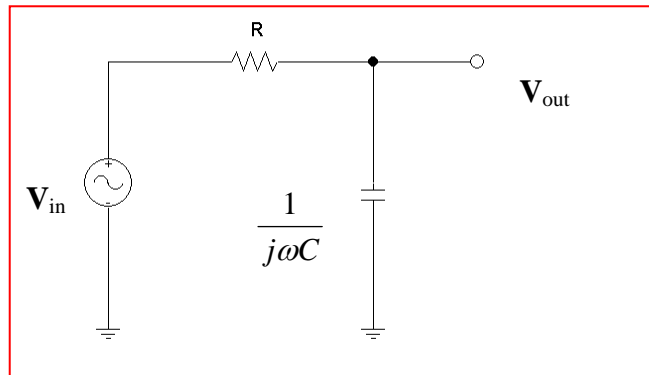
Solving, we find that $\mathbf{V}_2 = \mathbf{V}_{\text{TH}} = 5.423 \angle 40.60^\circ \text{ V}$

$$\mathbf{Z}_{\text{TH}} = 10 \parallel [(j10 \parallel -j5) + (5 \parallel -j10)] = 10 \parallel (-j10 + 4 - j2) = 5.882 - j3.529 \Omega.$$



FREQ	VM (\$N_0002, 0)	VP (\$N_0002, 0)
1.592E+01	4.474E+00	1.165E+02
FREQ	VM (\$N_0005, 0)	VP (\$N_0005, 0)
1.592E+01	4.473E+00	1.165E+02

85. Consider the circuit below:



Using voltage division, we may write:

$$\mathbf{V}_{\text{out}} = \mathbf{V}_{\text{in}} \frac{1/j\omega C}{R + 1/j\omega C}, \quad \text{or} \quad \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{1}{1 + j\omega RC}$$

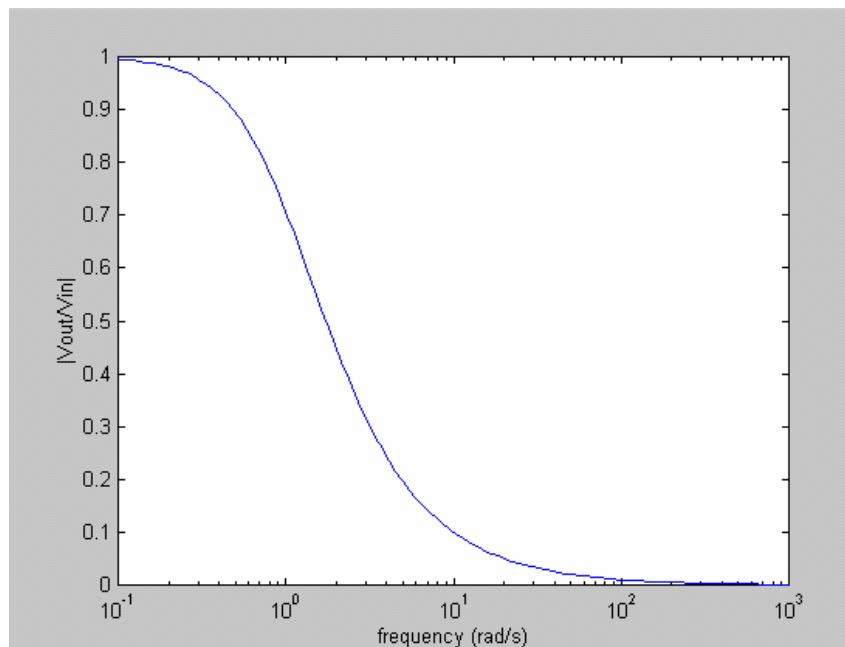
The magnitude of this ratio (consider, for example, an input with unity magnitude and zero phase) is

$$\left| \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} \right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

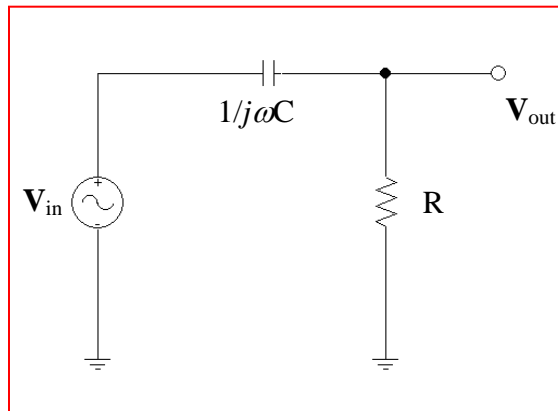
As $\omega \rightarrow 0$, this magnitude $\rightarrow 1$, its maximum value.

As $\omega \rightarrow \infty$, this magnitude $\rightarrow 0$; the capacitor is acting as a short circuit to the ac signal.

Thus, low frequency signals are transferred from the input to the output relatively unaffected by this circuit, but high frequency signals are attenuated, or “filtered out.” This is readily apparent if we plot the magnitude as a function of frequency (assuming $R = 1 \, \Omega$ and $C = 1 \, \text{F}$ for convenience):



86. Consider the circuit below:



Using voltage division, we may write:

$$\mathbf{V}_{\text{out}} = \mathbf{V}_{\text{in}} \frac{R}{R + 1/j\omega C}, \quad \text{or} \quad \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{j\omega RC}{1 + j\omega RC}$$

The magnitude of this ratio (consider, for example, an input with unity magnitude and zero phase) is

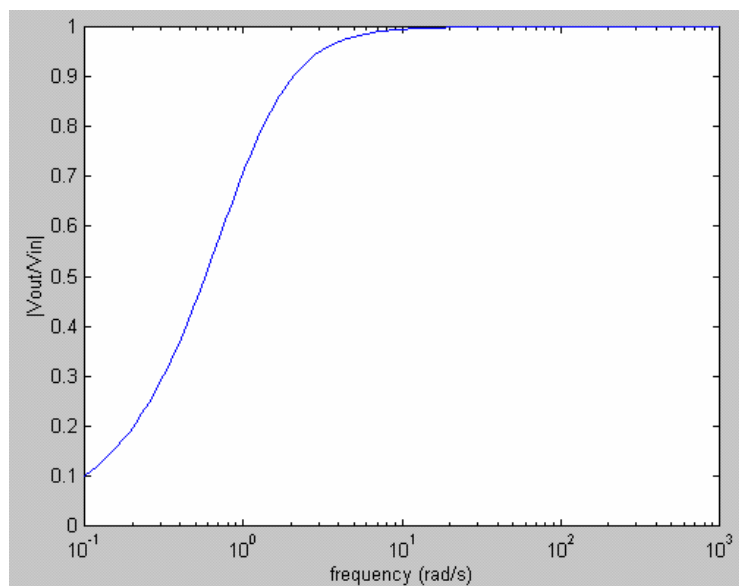
$$\left| \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} \right| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

As $\omega \rightarrow \infty$, this magnitude $\rightarrow 1$, its maximum value.

As $\omega \rightarrow 0$, this magnitude $\rightarrow 0$; the capacitor is acting as an open circuit to the ac signal.

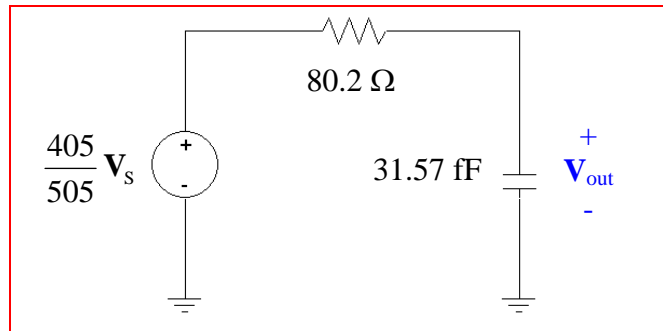
Thus, high frequency signals are transferred from the input to the output relatively unaffected by this circuit, but low frequency signals are attenuated, or “filtered out.”

This is readily apparent if we plot the magnitude as a function of frequency (assuming $R = 1 \Omega$ and $C = 1 \text{ F}$ for convenience):



87. (a) Removing the capacitor temporarily, we easily find the Thévenin equivalent:

$$\mathbf{V}_{th} = (405/505) \mathbf{V}_S \quad \text{and} \quad \mathbf{R}_{th} = 100 \parallel (330 + 75) = 80.2 \, \Omega$$

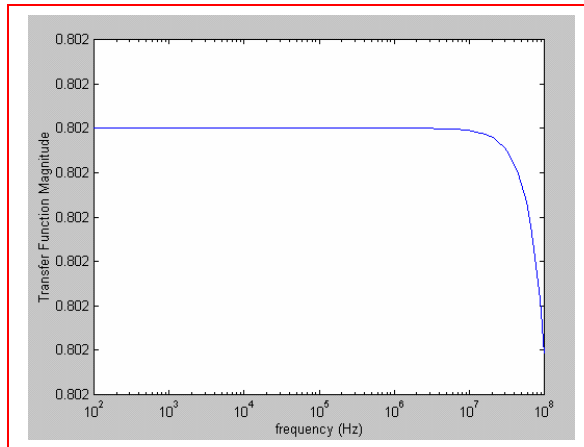


$$(b) \quad \mathbf{V}_{out} = \frac{405}{505} \mathbf{V}_S \frac{1/j\omega C}{80.2 + 1/j\omega C} \quad \text{so} \quad \frac{\mathbf{V}_{out}}{\mathbf{V}_S} = \left(\frac{405}{505} \right) \frac{1}{1 + j2.532 \times 10^{-12} \omega}$$

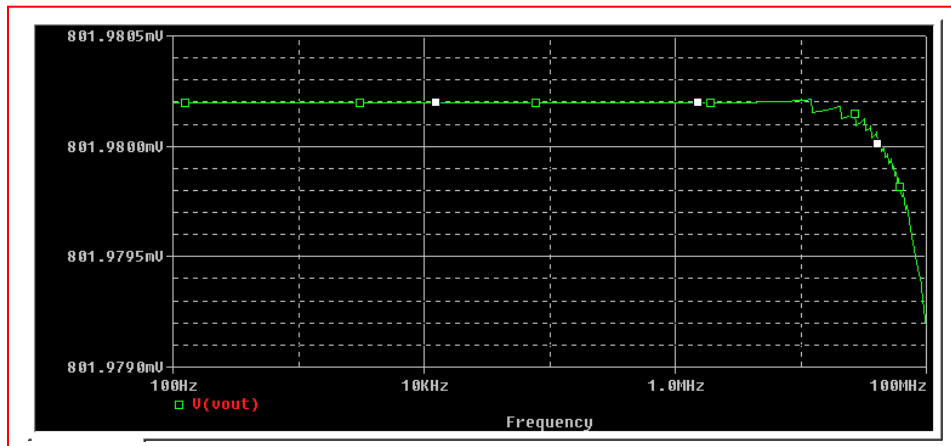
and hence

$$\left| \frac{\mathbf{V}_{out}}{\mathbf{V}_S} \right| = \frac{0.802}{\sqrt{1 + 6.411 \times 10^{-24} \omega^2}}$$

(c)



Both the MATLAB plot of the frequency response and the PSpice simulation show essentially the same behavior; at a frequency of approximately 20 MHz, there is a sharp roll-off in the transfer function magnitude.



88. From the derivation, we see that

$$\frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{-g_m(\mathbf{R}_C \parallel \mathbf{R}_L) + j\omega(\mathbf{R}_C \parallel \mathbf{R}_L)C_\mu}{1 + j\omega(\mathbf{R}_C \parallel \mathbf{R}_L)C_\mu}$$

so that

$$\left| \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} \right| = \left[\frac{g_m^2 \left(\frac{\mathbf{R}_C \mathbf{R}_L}{\mathbf{R}_C + \mathbf{R}_L} \right)^2 + \omega^2 \left(\frac{\mathbf{R}_C \mathbf{R}_L}{\mathbf{R}_C + \mathbf{R}_L} \right)^2 C_\mu^2}{1 + \omega^2 \left(\frac{\mathbf{R}_C \mathbf{R}_L}{\mathbf{R}_C + \mathbf{R}_L} \right)^2 C_\mu^2} \right]^{1/2}$$

This function has a maximum value of $g_m(\mathbf{R}_C \parallel \mathbf{R}_L)$ at $\omega = 0$. Thus, the capacitors reduce the gain at high frequencies; this is the frequency regime at which they begin to act as short circuits. Therefore, the maximum gain is obtained at frequencies at which the capacitors may be treated as open circuits. If we do this, we may analyze the circuit of Fig. 10.25*b* without the capacitors, which leads to

$$\left. \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_S} \right|_{\text{low frequency}} = -g_m \left(\frac{\mathbf{R}_C \mathbf{R}_L}{\mathbf{R}_C + \mathbf{R}_L} \right) \frac{(r_\pi \parallel \mathbf{R}_B)}{\mathbf{R}_S + r_\pi \parallel \mathbf{R}_B} = -g_m \left(\frac{\mathbf{R}_C \mathbf{R}_L}{\mathbf{R}_C + \mathbf{R}_L} \right) \frac{r_\pi \mathbf{R}_B}{\mathbf{R}_S(r_\pi + \mathbf{R}_B) + r_\pi \mathbf{R}_B}$$

The resistor network comprised of r_π , \mathbf{R}_S , and \mathbf{R}_B acts as a voltage divider, leading to a reduction in the gain of the amplifier. In the situation where $r_\pi \parallel \mathbf{R}_B \gg \mathbf{R}_S$, then it has minimal effect and the gain will equal its “maximum” value of $-g_m(\mathbf{R}_C \parallel \mathbf{R}_L)$.

(b) If we set $\mathbf{R}_S = 100 \Omega$, $\mathbf{R}_L = 8 \Omega$, $\mathbf{R}_C |_{\text{max}} = 10 \text{ k}\Omega$ and $r_\pi g_m = 300$, then we find that

$$\frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_S} = -g_m (7.994) \frac{r_\pi \parallel \mathbf{R}_B}{100 + r_\pi \parallel \mathbf{R}_B}$$

We seek to maximize this term within the stated constraints. This requires a large value of g_m , but also a large value of $r_\pi \parallel \mathbf{R}_B$. This parallel combination will be less than the smaller of the two terms, so even if we allow $\mathbf{R}_B \rightarrow \infty$, we are left with

$$\frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_S} \approx -(7.994) \frac{g_m r_\pi}{100 + r_\pi} = \frac{-2398}{100 + r_\pi}$$

Considering this simpler expression, it is clear that if we select r_π to be small, (*i.e.* $r_\pi \ll 100$), then g_m will be large and the gain will have a maximum value of approximately -23.98 .

(c) Referring to our original expression in which the gain $\mathbf{V}_{\text{out}}/\mathbf{V}_{\text{in}}$ was computed, we see that the critical frequency $\omega_c = [(\mathbf{R}_C \parallel \mathbf{R}_L) C_\mu]^{-1}$. Our selection of maximum \mathbf{R}_C , $\mathbf{R}_B \rightarrow \infty$, and $r_\pi \ll 100$ has not affected this frequency.

89. Considering the $\omega = 2 \times 10^4$ rad/s source first, we make the following replacements:

$$100 \cos(2 \times 10^4 t + 3^\circ) \text{ V} \rightarrow 100 \angle 3^\circ \text{ V}$$

$$33 \mu\text{F} \rightarrow -j1.515 \Omega \quad 112 \mu\text{H} \rightarrow j2.24 \Omega \quad 92 \mu\text{F} \rightarrow -j0.5435 \Omega$$

Then

$$(\mathbf{V}_1' - 100 \angle 3^\circ) / 47 \times 10^3 + \mathbf{V}_1' / (-j1.515) + (\mathbf{V}_1' - \mathbf{V}_2') / (56 \times 10^3 + j4.48) = 0 \quad [1]$$

$$(\mathbf{V}_2' - \mathbf{V}_1') / (56 \times 10^3 + j4.48) + \mathbf{V}_2' / (-j0.5435) = 0 \quad [2]$$

Solving, we find that

$$\mathbf{V}_1' = 3.223 \angle -87^\circ \text{ mV} \quad \text{and} \quad \mathbf{V}_2' = 31.28 \angle -177^\circ \text{ nV}$$

Thus, $v_1'(t) = 3.223 \cos(2 \times 10^4 t - 87^\circ) \text{ mV}$ and $v_2'(t) = 31.28 \cos(2 \times 10^4 t - 177^\circ) \text{ nV}$

Considering the effects of the $\omega = 2 \times 10^5$ rad/s source next,

$$100 \cos(2 \times 10^5 t - 3^\circ) \text{ V} \rightarrow 100 \angle -3^\circ \text{ V}$$

$$33 \mu\text{F} \rightarrow -j0.1515 \Omega \quad 112 \mu\text{H} \rightarrow j22.4 \Omega \quad 92 \mu\text{F} \rightarrow -j0.05435 \Omega$$

Then

$$\mathbf{V}_1'' / -j0.1515 + (\mathbf{V}_1'' - \mathbf{V}_2'') / (56 \times 10^3 + j44.8) = 0 \quad [3]$$

$$(\mathbf{V}_2'' - \mathbf{V}_1'') / (56 \times 10^3 + j44.8) + (\mathbf{V}_2'' - 100 \angle 3^\circ) / 47 \times 10^3 + \mathbf{V}_2'' / (-j0.05435) = 0 \quad [4]$$

Solving, we find that

$$\mathbf{V}_1'' = 312.8 \angle 177^\circ \text{ pV} \quad \text{and} \quad \mathbf{V}_2'' = 115.7 \angle -93^\circ \mu\text{V}$$

Thus,

$$v_1''(t) = 312.8 \cos(2 \times 10^5 t + 177^\circ) \text{ pV} \quad \text{and} \quad v_2''(t) = 115.7 \cos(2 \times 10^5 t - 93^\circ) \mu\text{V}$$

Adding, we find

$$v_1(t) = 3.223 \times 10^{-3} \cos(2 \times 10^4 t - 87^\circ) + 312.8 \times 10^{-12} \cos(2 \times 10^5 t + 177^\circ) \text{ V} \quad \text{and}$$

$$v_2(t) = 31.28 \times 10^{-9} \cos(2 \times 10^4 t - 177^\circ) + 115.7 \times 10^{-12} \cos(2 \times 10^5 t - 93^\circ) \text{ V}$$

90. For the source operating at $\omega = 4$ rad/s,

$7 \cos 4t \rightarrow 7 \angle 0^\circ$ V, $1 \text{ H} \rightarrow j4 \Omega$, $500 \text{ mF} \rightarrow -j0.5 \Omega$, $3 \text{ H} \rightarrow j12 \Omega$, and $2 \text{ F} \rightarrow -j/8 \Omega$.

Then by mesh analysis, (define 4 clockwise mesh currents $\mathbf{I}_1, \mathbf{I}_2, \mathbf{I}_3, \mathbf{I}_4$ in the top left, top right, bottom left and bottom right meshes, respectively):

$$\begin{aligned} (9.5 + j4) \mathbf{I}_1 - j4 \mathbf{I}_2 - 7 \mathbf{I}_3 & - 4 \mathbf{I}_4 & = 0 & [1] \\ -j4 \mathbf{I}_1 + (3 + j3.5) \mathbf{I}_2 & - 3 \mathbf{I}_4 & = -7 & [2] \\ -7 \mathbf{I}_1 + & (12 - j/8) \mathbf{I}_3 + j/8 \mathbf{I}_4 & = 0 & [3] \\ & -3 \mathbf{I}_2 + j/8 \mathbf{I}_3 + (4 + j11.875) \mathbf{I}_4 & = 0 & [4] \end{aligned}$$

Solving, we find that $\mathbf{I}_3 = 365.3 \angle -166.1^\circ$ mA and $\mathbf{I}_4 = 330.97 \angle 72.66^\circ$ mA.

For the source operating at $\omega = 2$ rad/s,

$5.5 \cos 2t \rightarrow 5.5 \angle 0^\circ$ V, $1 \text{ H} \rightarrow j2 \Omega$, $500 \text{ mF} \rightarrow -j \Omega$, $3 \text{ H} \rightarrow j6 \Omega$, and $2 \text{ F} \rightarrow -j/4 \Omega$.

Then by mesh analysis, (define 4 clockwise mesh currents $\mathbf{I}_A, \mathbf{I}_B, \mathbf{I}_C, \mathbf{I}_D$ in the top left, top right, bottom left and bottom right meshes, respectively):

$$\begin{aligned} (9.5 + j2) \mathbf{I}_A - j2 \mathbf{I}_B - 7 \mathbf{I}_C & - 4 \mathbf{I}_D & = 0 & [1] \\ -j2 \mathbf{I}_A + (3 + j) \mathbf{I}_B & - 3 \mathbf{I}_D & = -7 & [2] \\ -7 \mathbf{I}_A + & (12 - j/4) \mathbf{I}_C + j/4 \mathbf{I}_D & = 0 & [3] \\ & -3 \mathbf{I}_2 + j/4 \mathbf{I}_C + (4 + j5.75) \mathbf{I}_D & = 0 & [4] \end{aligned}$$

Solving, we find that $\mathbf{I}_C = 783.8 \angle -4.427^\circ$ mA and $\mathbf{I}_D = 134 \angle -25.93^\circ$ mA.

$\mathbf{V}_1' = -j0.25 (\mathbf{I}_3 - \mathbf{I}_4) = 0.1517 \angle 131.7^\circ$ V and $\mathbf{V}_1'' = -j0.25 (\mathbf{I}_C - \mathbf{I}_D) = 0.1652 \angle -90.17^\circ$ V

$\mathbf{V}_2' = (1 + j6) \mathbf{I}_4 = 2.013 \angle 155.2^\circ$ V and $\mathbf{V}_2'' = (1 + j6) \mathbf{I}_D = 0.8151 \angle 54.61^\circ$ V

Converting back to the time domain,

$$\begin{aligned} v_1(t) &= 0.1517 \cos(4t + 131.7^\circ) + 0.1652 \cos(2t - 90.17^\circ) \text{ V} \\ v_2(t) &= 2.013 \cos(4t + 155.2^\circ) + 0.8151 \cos(2t + 54.61^\circ) \text{ V} \end{aligned}$$

91.

$$(a) \quad I_L = \frac{100}{j2.5 + \frac{-2}{2-j1}} = \frac{100(2-j1)}{2.5+j3} = 57.26 \angle -76.76^\circ (2.29in)$$

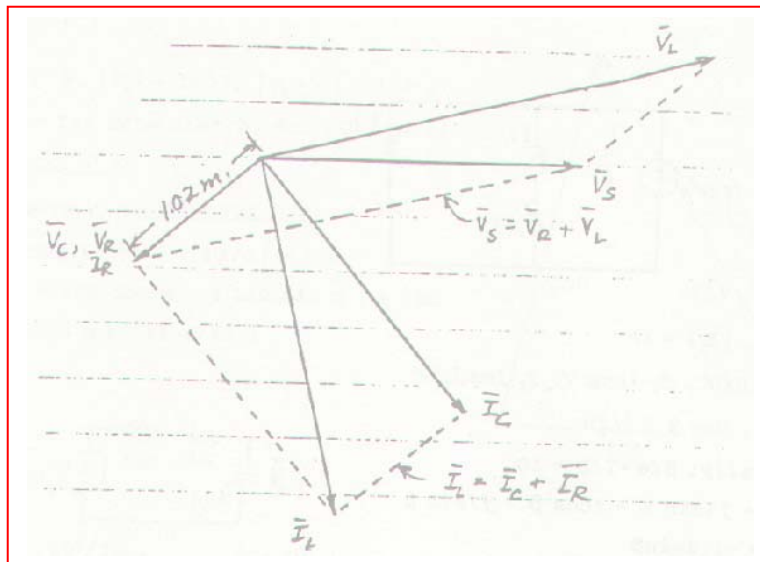
$$I_R = (57.26 \angle -76.76^\circ) \frac{-j1}{2-j1} = 25.61 \angle -140.19^\circ (1.02in)$$

$$I_C = (57.26 \angle -76.76^\circ) \frac{2}{2-j1} = 51.21 \angle -50.19^\circ (2.05in)$$

$$V_L = 2.5 \times 57.26 \angle 90^\circ - 76.76^\circ = 143.15 \angle 13.24^\circ (2.86in)$$

$$V_R = 2 \times 25.61 \angle -140.19^\circ = 51.22 \angle -140.19^\circ (1.02in)$$

$$V_C = 51.21 \angle -140.19^\circ (1.02in)$$



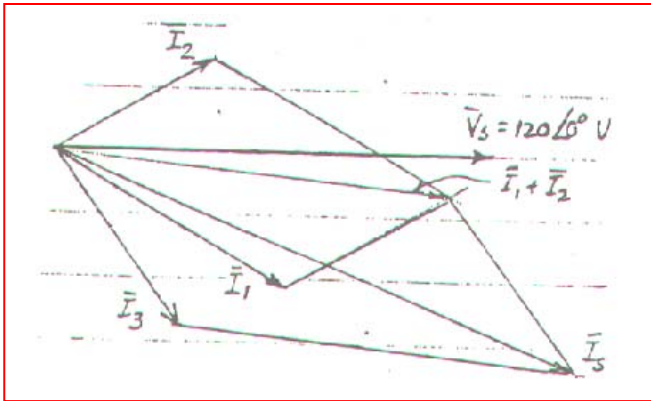
92.

$$(a) \quad \mathbf{I}_1 = \frac{120}{40 \angle 30^\circ} = 3 \angle -30^\circ \text{ A}$$

$$\mathbf{I}_2 = \frac{120}{50 - j30} = 2.058 \angle 30.96^\circ \text{ A}$$

$$\mathbf{I}_3 = \frac{120}{30 + j40} = 2.4 \angle -53.13^\circ \text{ A}$$

(b)



$$(c) \quad \mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$$

$$= 6.265 \angle -22.14^\circ \text{ A}$$

93.

$$|I_1| = 5A, |I_2| = 7A$$

$$I_1 + I_2 = 10\angle 0^\circ, I_1 \text{ lags } V, I_2 \text{ leads } V$$

$$I_1 \text{ lags } I_2. \text{ Use } 2.5A / in$$

$$[\text{Analytically: } 5\angle\alpha + 7\angle\beta = 10$$

$$= 5\cos\alpha + j5\sin\alpha + 7\cos\beta + j7\sin\beta$$

$$\therefore \sin\alpha = -1.4\sin\beta$$

$$\therefore 5\sqrt{1-1.4^2\sin^2\beta} + 7\sqrt{1-\sin^2\beta} = 10$$

$$\text{By SOLVE, } \alpha = -40.54^\circ, \beta = 27.66^\circ]$$

94. $\mathbf{V}_1 = 100\angle 0^\circ \text{ V}$, $|\mathbf{V}_2| = 140 \text{ V}$, $|\mathbf{V}_1 + \mathbf{V}_2| = 120 \text{ V}$.
Let $50 \text{ V} = 1 \text{ inch}$. From the sketch, for $\angle \mathbf{V}_2$ positive,
 $\mathbf{V}_2 = 140\angle 122.5^\circ$. We may also have $\mathbf{V}_2 = 140\angle -122.5^\circ \text{ V}$

[Analytically: $|100 + 140\angle \alpha| = 120$

so $|100 + 140 \cos \alpha + j140 \sin \alpha| = 120$

Using the "Solve" routine of a scientific calculator,

$\alpha = \pm 122.88^\circ$.]