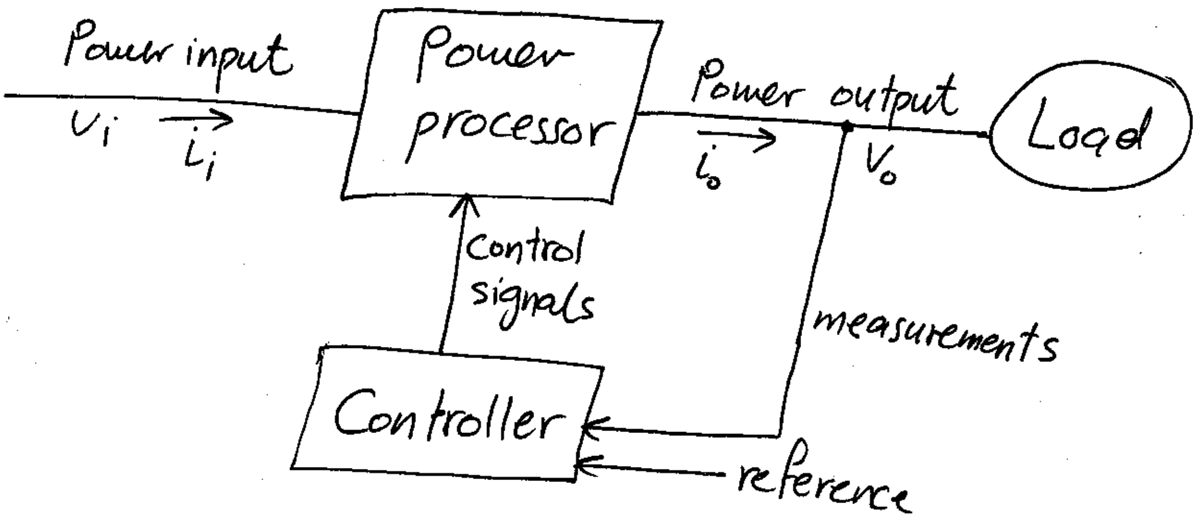
Power Electronics



Block diagram of a power electronic system

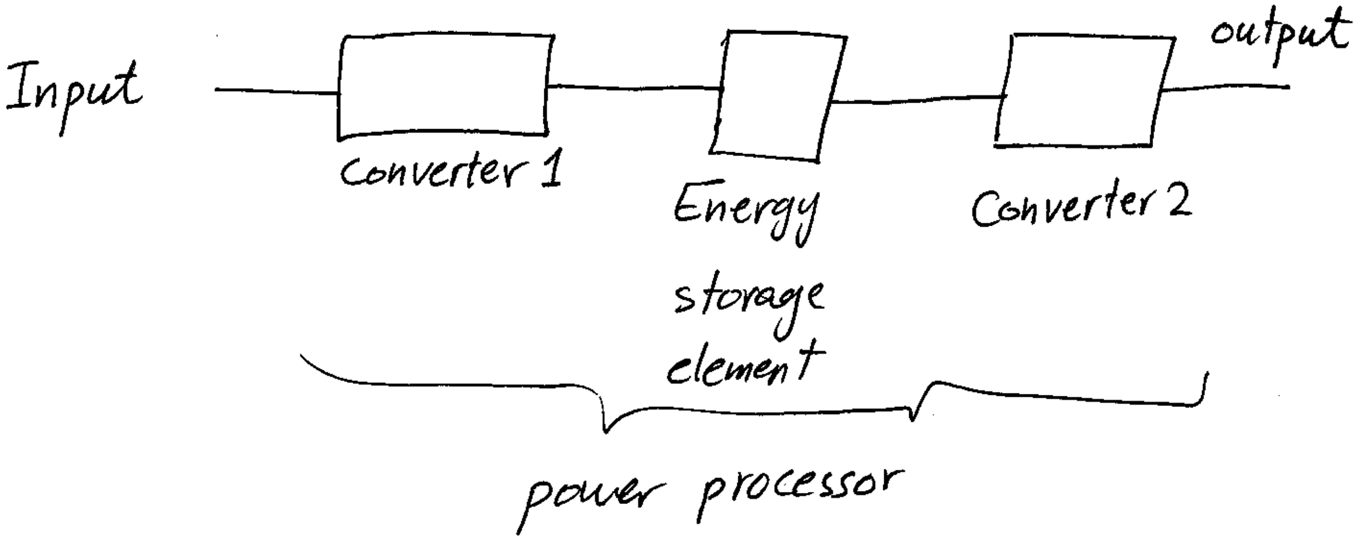
Classification of power processors and converters

Power processors

1. dc
   a. regulated (constant) magnitude
   b. adjustable magnitude

2. ac
   a. constant frequency, adjustable magnitude
   b. adjustable frequency and adjustable magnitude

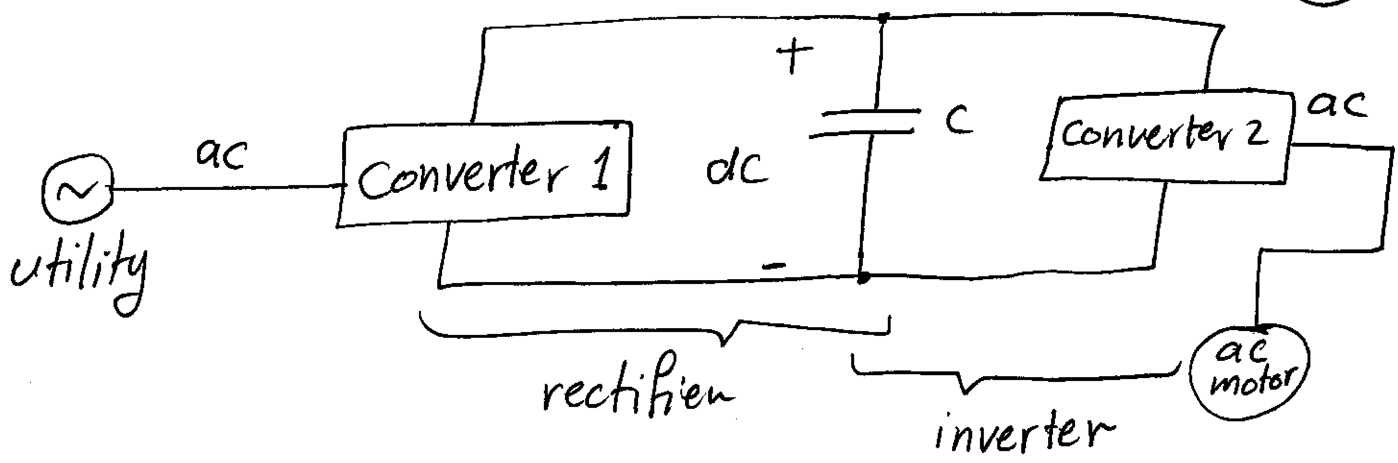
~~power converters~~



power converters

- Inverters
- rectifiers

1. ac to dc ⟶ rectifier
2. dc to ac ⟶ inverter
3. dc to dc ⟶ converters (buck or boost)
   step down   step up
4. ac to ac ⟶ rectifier + inverter



Block diagram of an ac motor drive

* The converters can be classified according to how the devices within the converter are switched :

1. Line frequency (naturally commutated) converters : the utility line voltages present at one side of the converter facilitate the turn-off of the power semiconductor devices. Similarly, the devices are turned on. Therefore, the devices are turned on and off at the line frequency of 50 or 60 Hz.

2. Switched (forced-commutated) converters : the controllable switches in the converter are turned on and off at frequencies that are higher compared to the line frequency.

③ Resonant and quasi-resonant converters: the controllable switches turn on and/or off at zero voltage and/or zero current.

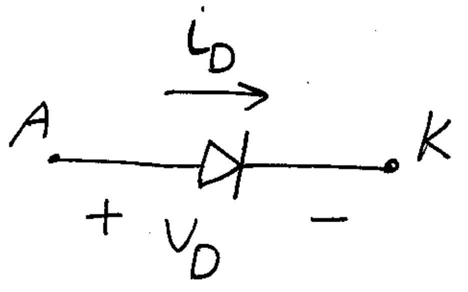
Ch. # 2   Mon.

Overview of Power Semiconductor switches

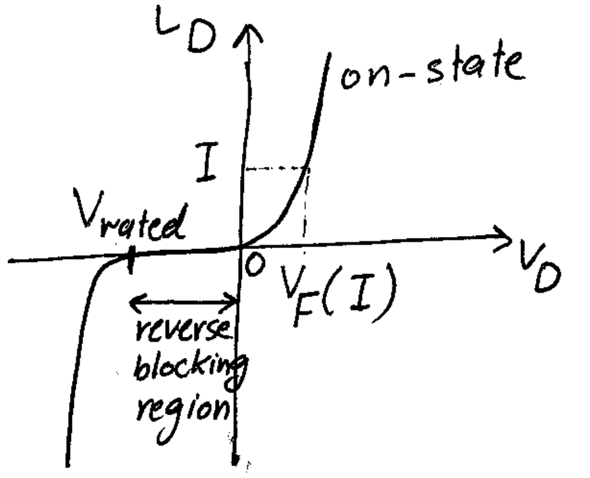
According to the degree of Controllability, power semiconductor devices can be classified into:

1. **Diodes**: On and off states are controlled by the power circuit.
2. **Thyristors**: Latched on by a control signal but must be turned off by the power circuit.
3. **Controllable switches**: turned on and off by control signals i.e. BJT, MOSFET, GTO, IGBT.

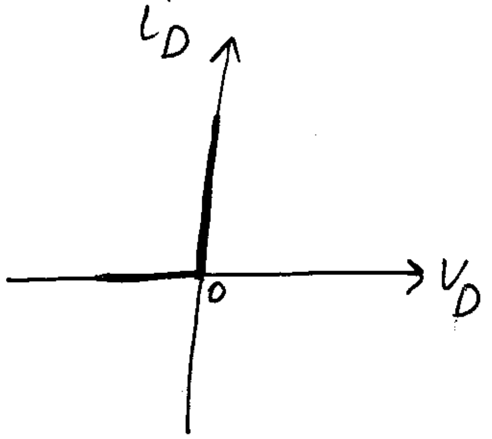
Diodes



Diode symbol



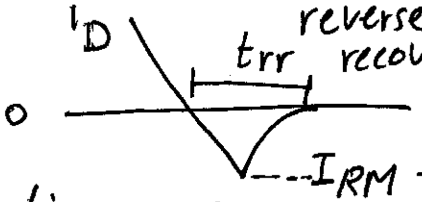
i-V characteristic



idealized characteristic

* At turn on, the diode can be considered an ideal switch because it turns on rapidly compared to the transients in the power circuit. However, at turn off, the diode current reverses for a reverse-recovery time t_{rr} before falling to zero. This reverse-recovery current can lead to overvoltages in inductive circuits. In most circuits, this reverse current does not affect the converter input/output characteristic and so the diode can also be considered as ideal during the

turn off transients.



→ R: reverse
M: Maximum

Depending on the applications requirements, various types of diodes are available:

① Schottky diodes: used in very low output voltage circuits as the forward voltage drop is low (typically 0.3V). The blocking voltage ~~is~~ ranges from 50 upto 100V.

application

② Fast-recovery diodes: used in high-frequency circuits in combination with controllable switches where a small reverse-recovery time is needed. At power levels of several hundred volts and several hundred amperes, such diodes have t_{rr} ratings of less than a few microseconds.

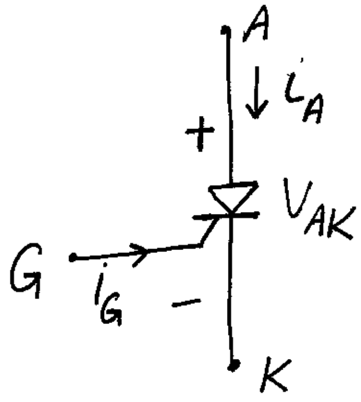
application

opposite

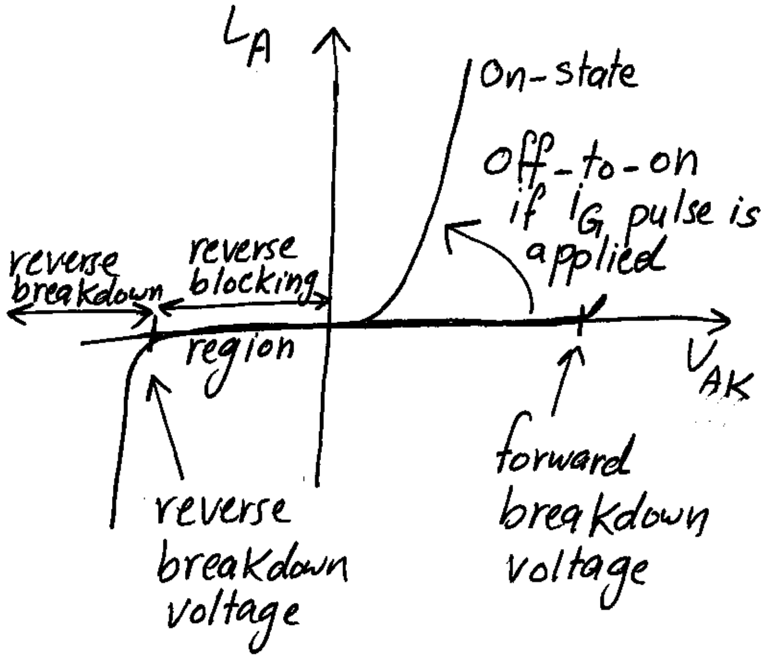
③ Line-frequency diodes: the on-state voltage of these diodes is designed to be as low as possible and as a consequence have larger t_{rr} which are acceptible for line frequency applications.

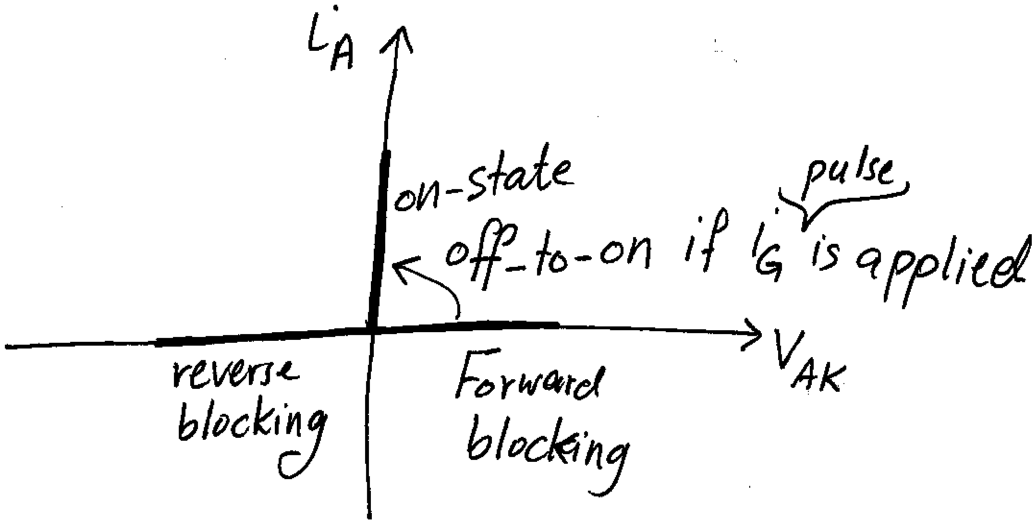
applications

Thyristors



Thyristor Symbol



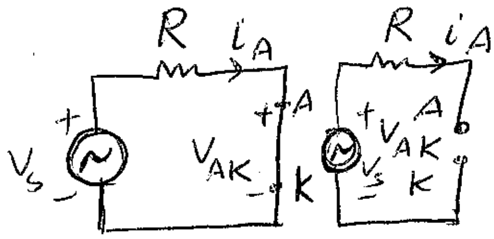


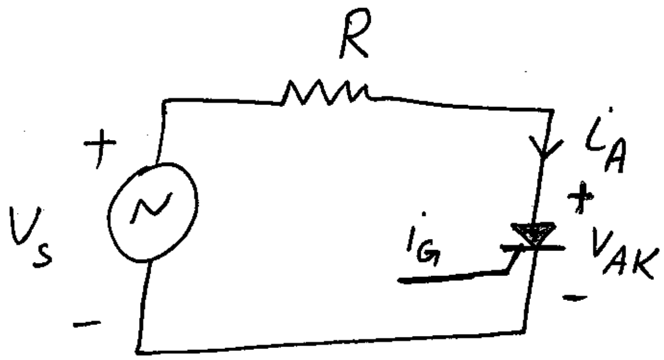
* The thyristor can be triggered into the on state by applying a pulse of positive gate current for a short duration provided that the device is in its forward-blocking state. Once the device begins to conduct, it is turned latched on and the gate current can be removed. The thyristor

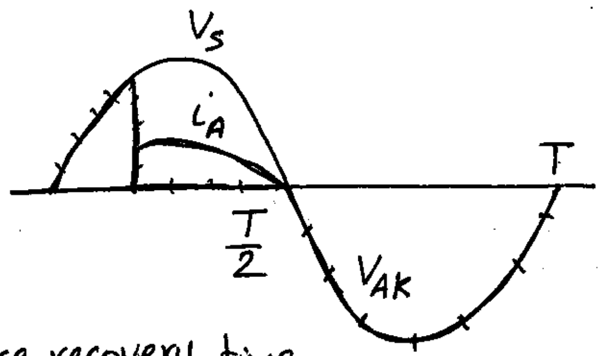
cannot be turned off by the gate and the thyristor conducts as a diode. When the anode current tries to go negative, the thyristor does turn off and the current goes to zero.

* In reverse bias at voltages below the reverse breakdown voltage, only a negligibly small leakage current flows in the thyristor.

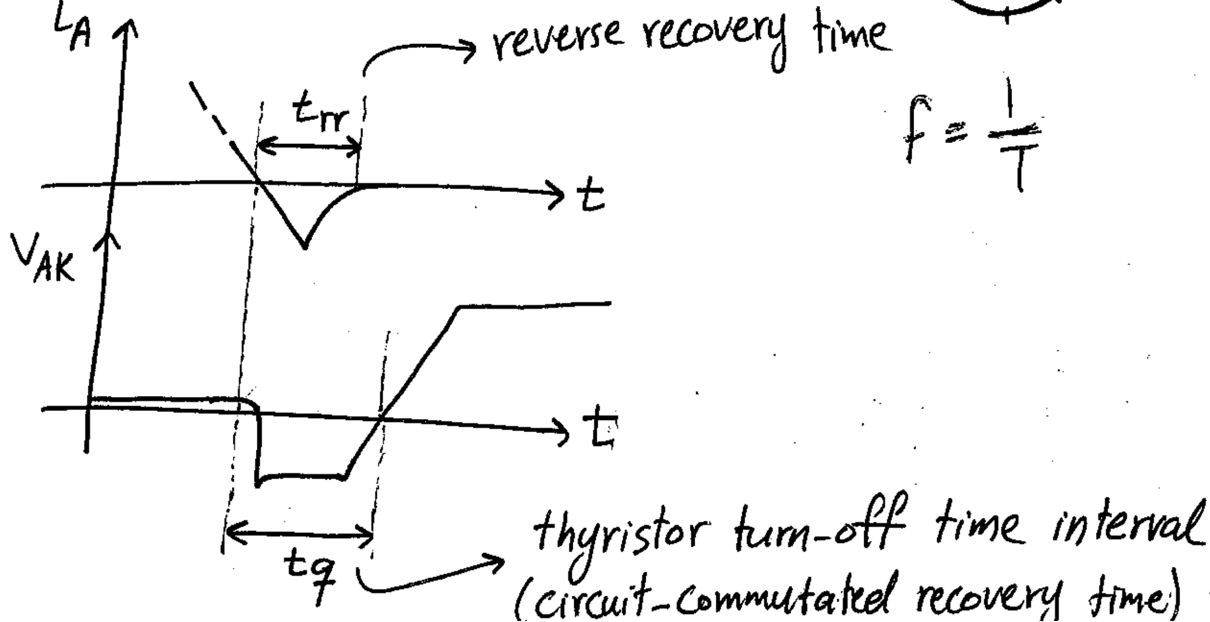
* Consider the following circuit:







$f = \frac{1}{T}$



* If a forward voltage is applied to the thyristor before t_q has passed, the device may turn on and damage to the device and/or circuit could result. Thyristor data sheets specify t_q with a specified reverse voltage applied during this interval as well as a specified wate of wise of voltage beyond this interval.

Types of thyristors

(1) phase-control thyristors (converter thyristors):
(50 or 60 Hz)
used in rectifying line-frequency voltages and currents for dc & ac motor drives and in high-voltage dc power transmission. It needs large voltage and current handling capabilities and it has low on-state voltage drop. (4000 A with blocking voltages of (5-7KV). On-state voltages wange from 1.5 V for 1000-V devices to 3.0V for the 5-7 KV devices.

② Inverter-grade thyristors: They have small turn-off time tq and low on-state voltage. 2500V and 1500 A. tq is in the range of a few microseconds to 100μs. Used in high frequency applications.

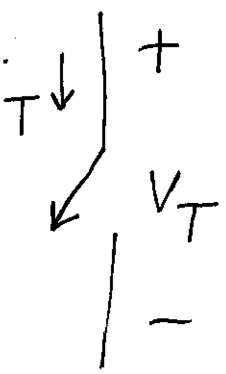
③ Light-activated thyristors: These can be triggered on by a pulse of light guided by optical fibers. They used in high-voltage applications such as high-voltage dc transmission lines where many thyristors are connected in series to make up a converter valve. 4KV & 3KA with on-state voltage of 2V is reported and light trigger power requirements of 5mW.

Desired Characteristics in Controllable Switches

BJTs, MOSFETs, GTOs and IGBTs can be turned on and off by control signals applied to the control terminal of the device. They are called "Controllable switches" and are represented in a generic manner by the circuit symbol shown below.

No current flows when the switch is off and when it is on, current can flow in the direction of the arrow only. The ideal controllable switch has the following characteristic:

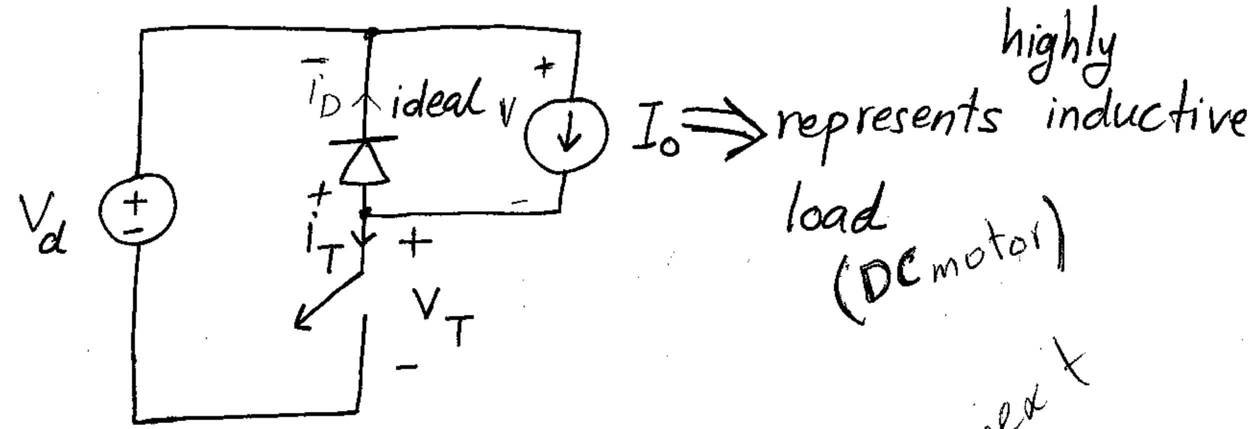
1. Block large forward and reverse voltages with zero current flow when off.



2. Conduct large currents with zero voltage drop when on.
3. Switch from on to off or vice versa instantaneously when triggered.
4. Vanishingly small power required from control Source to trigger the switch.

* However, real devices dissipate energy (don't have the above characteristics).

* Consider the following very commonly encountered situation in power electronics:



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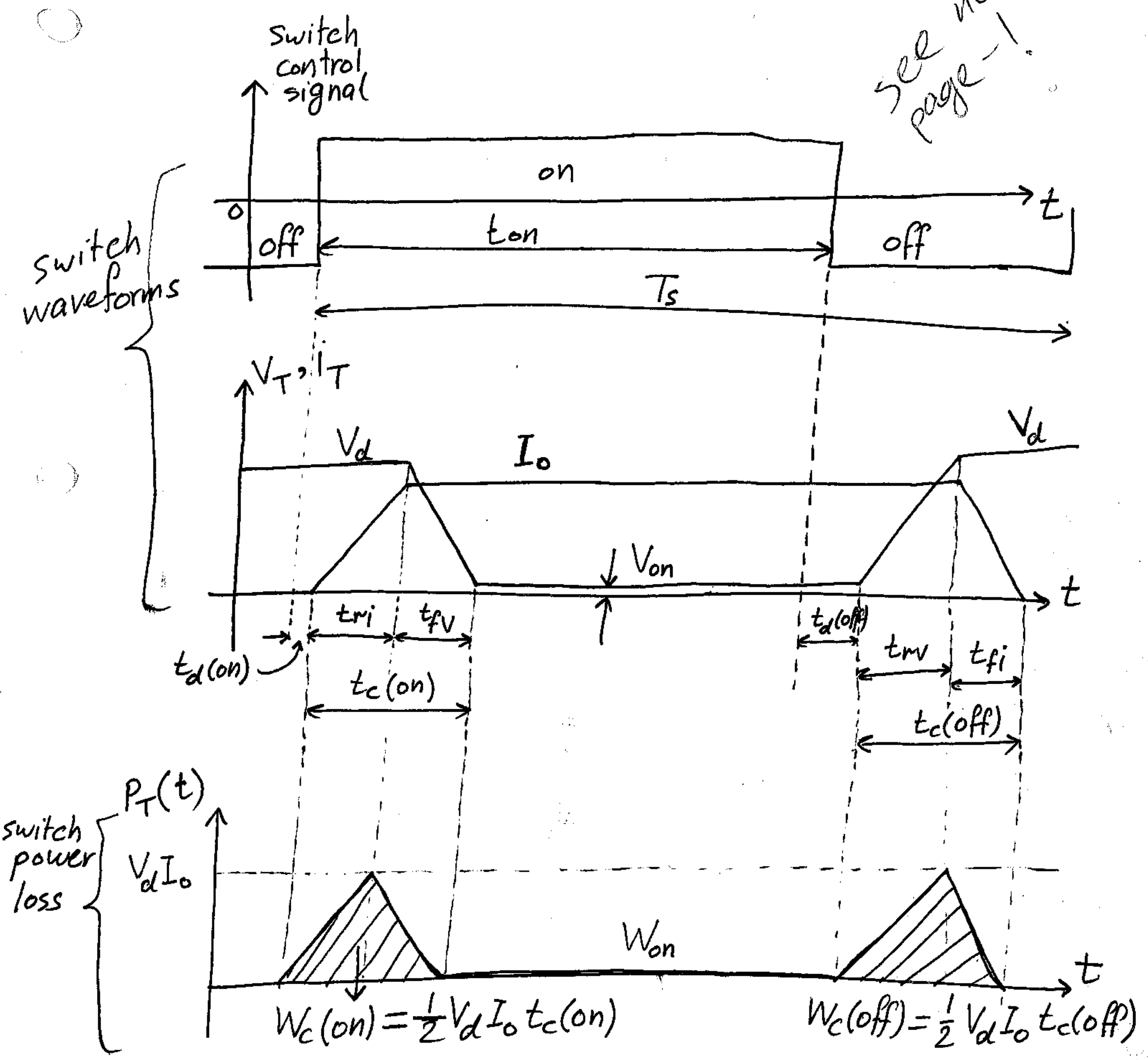
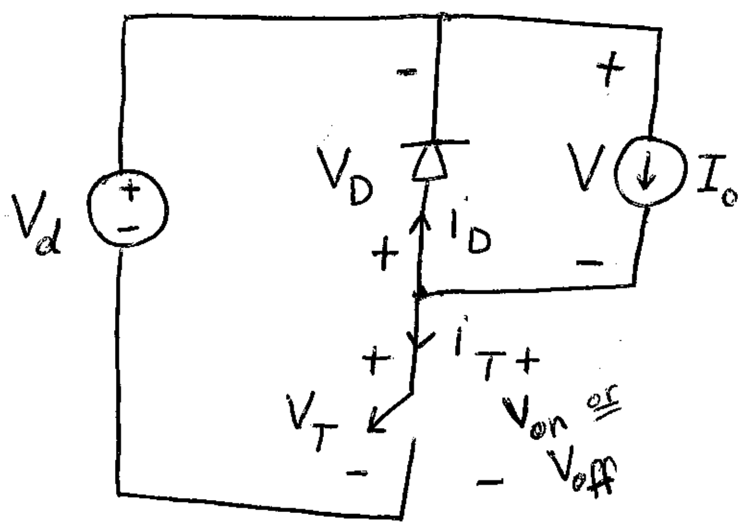




$$-V_d - V_D + V_T = 0$$

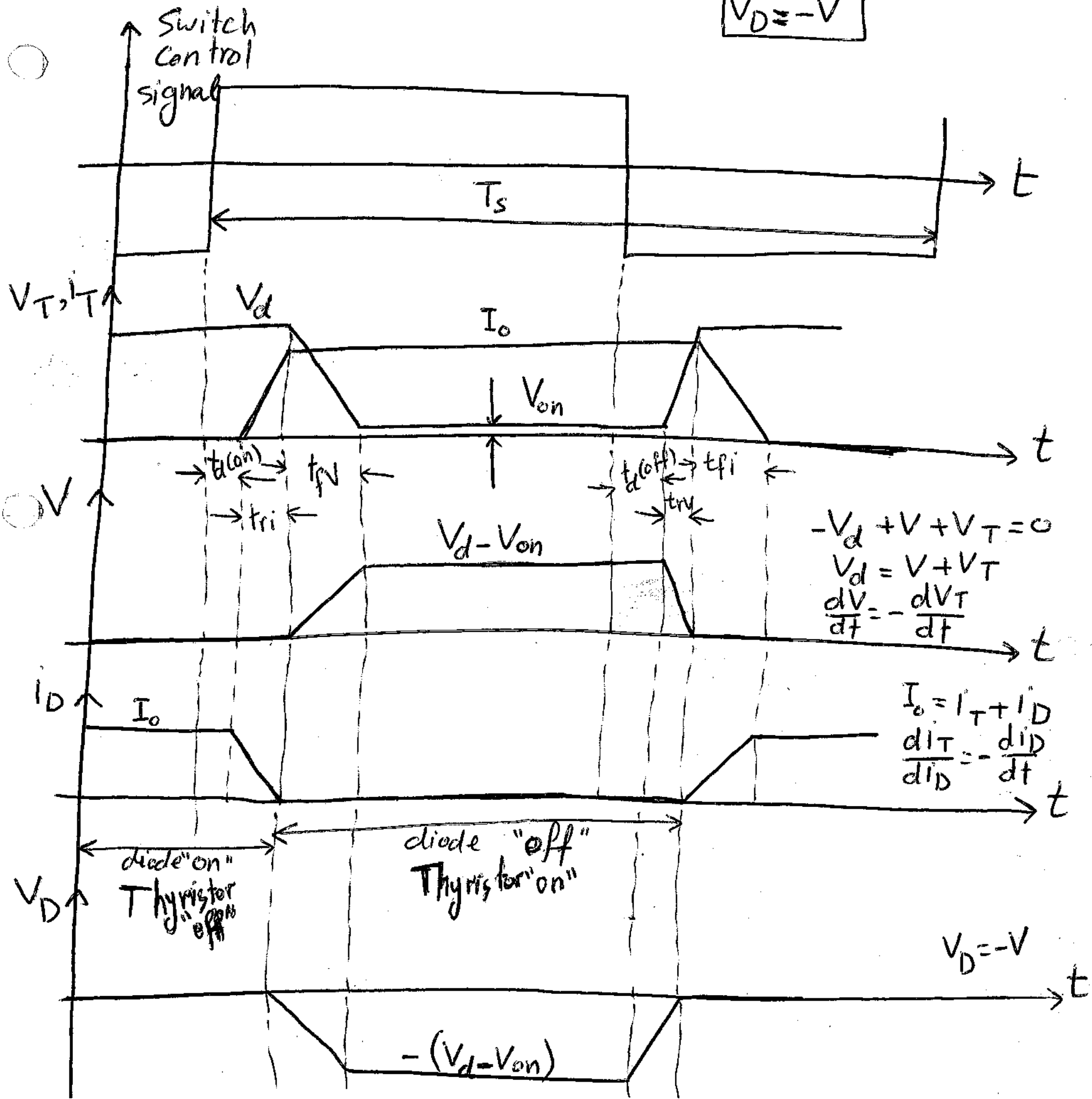
$$-V_D = V_d - V_T$$

$$\boxed{V_D = V_T - V_d}$$

or:

$$V_D + V = 0$$

$$\boxed{V_D = -V}$$



$$-V_d + V + V_T = 0$$

$$V_d = V + V_T$$

$$\frac{dV}{dt} = -\frac{dV_T}{dt}$$

$$I_o = i_T + i_D$$

$$\frac{di_T}{di_D} = -\frac{di_D}{dt}$$

t_{ri} : current rise time.

t_{fv} : voltage fall time.

$t_{d(on)}$: on delay time.

$t_{d(off)}$: off delay time.

$t_{c(on)}$: turn-on crossover interval.

$t_{c(off)}$: turn-off crossover interval.

V_{on} : switch on voltage.

$W_{c(on)}$: energy dissipated in the device during the turn-on crossover interval.

$W_{c(off)}$: energy dissipated in the device during the turn-off crossover interval.

$$W = \int P(t)dt$$

$$t_{c(on)} = t_{ri} + t_{fv}$$

$$W_{c(on)} = \frac{1}{2} V_d I_o t_{c(on)}$$

$$W_{on} = V_{on} I_d t_{on} \quad , \quad t_{on} \gg t_{c(on)} \;\&\; t_{c(off)}$$

$$t_{c(off)} = t_{rv} + t_{fi}$$

$$W_{c(off)} = \frac{1}{2} V_d I_o t_{c(off)}$$

* There are f_s turn-on and turn-off transitions per second. Hence, the average switching power loss P_s in the switch due to

these transitions can be approximated as:

$$P_s = \frac{1}{2} V_d I_o f_s \left(t_c(on) + t_c(off) \right) .$$

* The average power dissipated during the on-state P_{on} :

$$P_{on} = V_{on} I_{on} \frac{t_{on}}{T_s}$$

* From the above discussions, the following characteristics in a controllable switches are desirable:

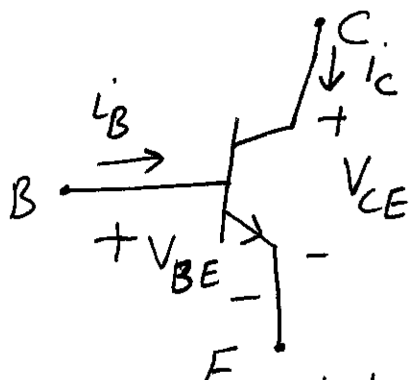
1. Small leakage current in the off state.
2. Small on-state voltage V_{on} to minimize the on-state power losses.
3. Short turn-on and turn-off times. This permits the device to be used at high switching frequencies.
4. Large forward- and reverse-voltage-blocking capability.
5. High on-state current rating.
6. Positive temperature coefficient of on-state resistance.
7. Small control power required to switch the device.

⑧ Capability to withstand rated voltage and rated current simultaneously while switching

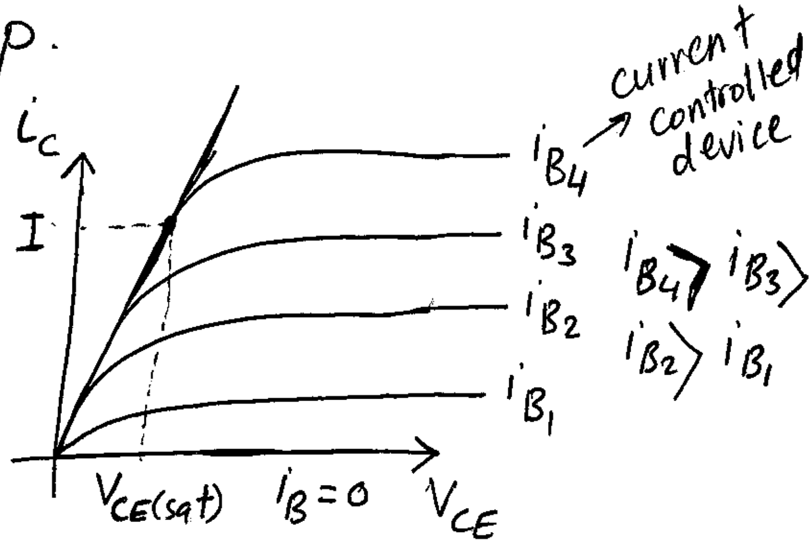
⑨ Large dV/dt and di/dt ratings.

Bipolar Junction Transistors and Monolithic Darlingtons (BJT) (MD)

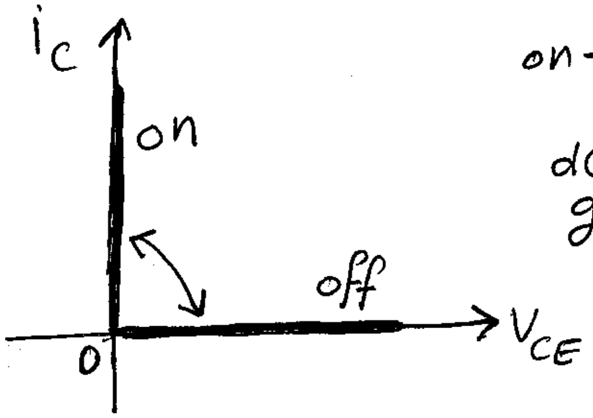
unit **Monolithic Darlingtons**: Darlington configuration on a single chip.



symbol



i-V characteristic



ideal characteristic

on-state voltage ← $V_{CE(sat)} = 1-2V$

dc current gain ← $h_{FE} = 5-10$ (relatively small)

$I_B > \frac{I_C}{h_{FE}}$

⇓

That is why Darlington config. is used

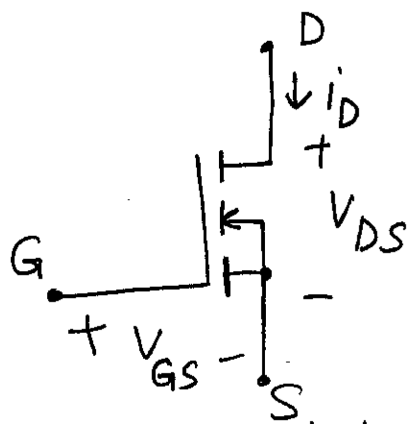
* switching time: few hundered nanoseconds to a few microseconds.

* With MDs, BJTs are available in voltage watings upto 1400V and current ratings of a few hundred ampers.

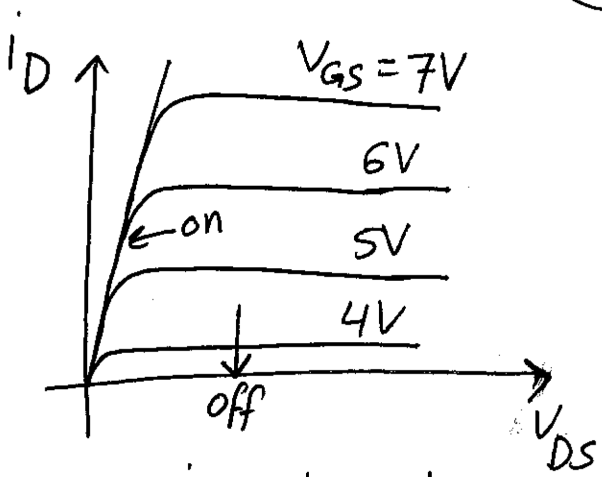
* BJT is a current controlled device.

Metal-Oxide-Semi Conductor Field Effect Transistors (MOSFET)

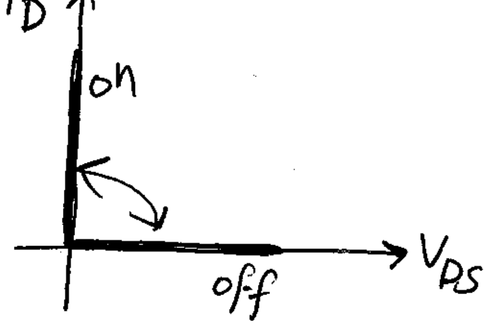
* MOSFET is a voltage controlled device. (V_{GS})



symbole



i-v characteristic



idealized characteristics

* The device is fully on and approximates a closed switch when the gate-source V_{GS} is below the threshold value $V_{GS(th)}$.

* MOSFETs require the continuous application of V_{GS} in order to be in the on state.

* There is no gate current in steady state operation. No gate current except during the transition from on to off or vice versa when the gate capacitance is being charged

* Switching time is very short ⟹ few tens of nanoseconds to a few hundred nanoseconds.

* $r_{DS(on)} = K B V_{DSS}^{2.5-2.7}$ (on-state resistance)

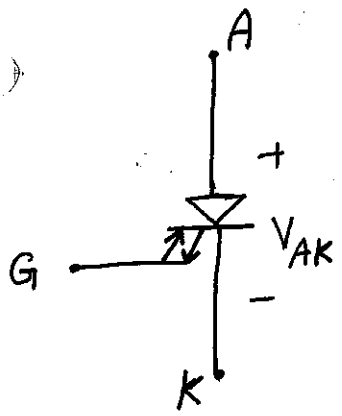
K: constant depends on the device geometry.

BV_{DSS}: Blocking voltage rating.

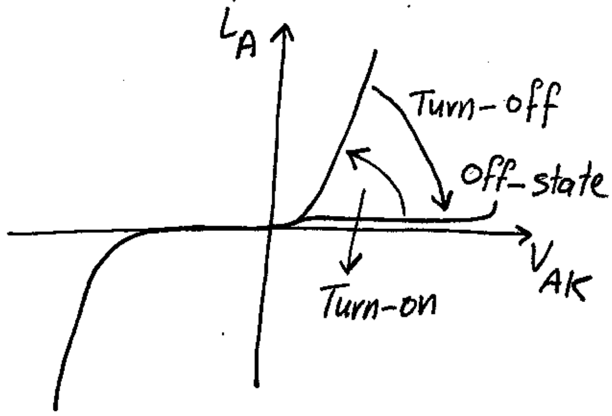
* MOSFETs are available in voltage ratings of 1000V and small current ratings, and with up to 100A at small voltage ratings.

Gate-Turn-Off Thyristors (GTO)

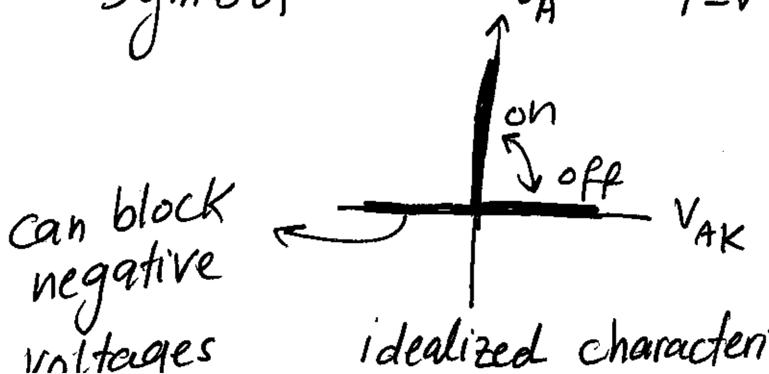
* Like the thyristor, the GTO can be ~~tuned~~ turned on by a short-duration gate current pulse, and once in the on-state, the GTO may stay on without any further gate current. However, unlike the thyristor, the GTO can be turned off by applying a negative gate-cathode voltage, therefore causing a sufficiently large negative gate current to flow. This negative gate current need only flow for a few microseconds (during the turn-off time), but it must have a very large magnitude.



Symbol

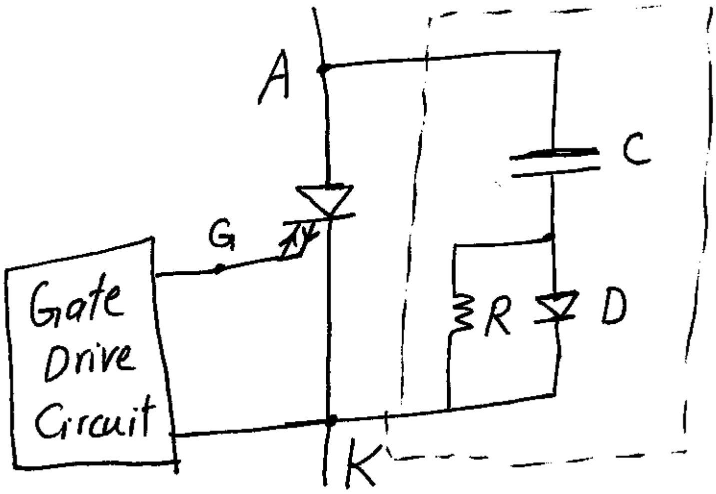


i-v characteristic



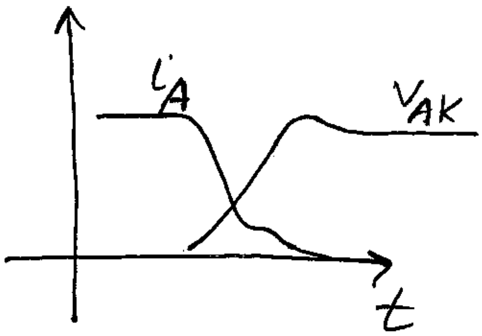
idealized characteristic

* The turn-off switching transient is different from that of MOSFET and BJT. A snubber circuit is used to reduce $\frac{dV}{dt}$ at turn-off as shown below:



Snubber circuit to reduce $\frac{dv}{dt}$ at turn-off

GTO + Snubber circuit



turn-off characteristic

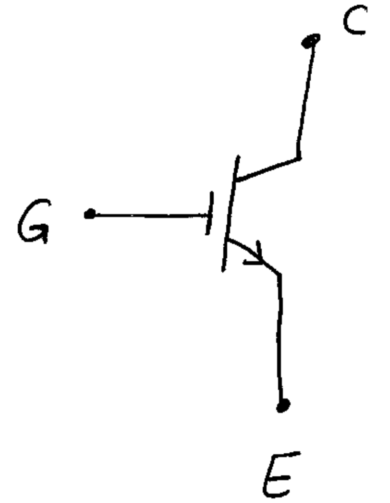
* on-state voltage of $\underbrace{\text{GTO}}_{}$ (2-3V) is slightly higher than those of thyristors.

* The switching speeds are in the range of a few microseconds to 25 μs.

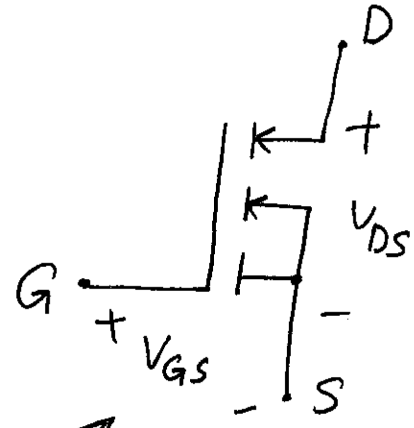
* Voltages up to 4.5 KV and currents up to few Kilo Ampere. frequency ≡ few hundred to 10 KHz.
(High voltage, high current and high frequency applications)

Insulated Gate Bipolar Transistors (IGBT)

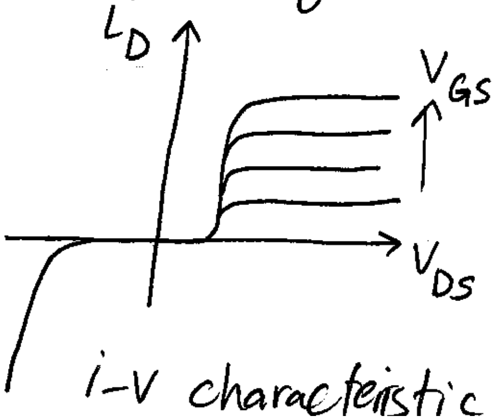
* IGBTs have some of the advantages of the MOSFET, the BJT and the GTO combined. Similar to the MOSFET, the IGBT has a high impedance gate, which requires only a small amount of energy to switch the device. Like the BJT, the IGBT has a small on-state voltage. Similar to the GTO, IGBTs can be designed to block negative voltages.



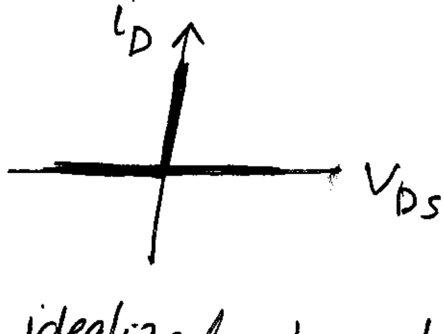
IGBT symbol



IGBT symbol



i-v characteristic



idealized characteristic

Comparison of Controllable Switches

Device	Power Capability	Switching Speed
BJT/MD	Medium	Medium
MOSFET	Low	Fast
GTO	High	Slow
IGBT	Medium	Medium
MCT	Medium	Medium

Justification for using Idealized Device Characteristics

How to choose the device:

1. On-state voltage or on-state resistance dictates the conduction losses in the device.
2. Switching times dictate the energy loss per transition and determine how high the operating frequency can be.
3. Voltage and current ratings determine the device power-handling capability.

4. The power required by the control circuit determines the ease of controlling the device.
5. The temperature coefficient of the device on-state resistance determines the ease of connecting them in parallel to handle large currents.
6. Device cost is a factor in its selection.

Why idealized Characteristics?!

1. on-state voltage can be ignored compared to the operating voltage.
2. Switching times are short compared to the period of the operating frequency.

problems

2-1 PP. 32

$t_{ri} = 100\,ns$ $\quad t_{fv} = 50\,ns$ $\quad t_{rv} = 100\,ns$ $\quad t_{fi} = 200\,ns$

$V_d = 300V$ $\quad I_o = 4A$

Calculate & Plot the switching power loss as a function of frequency in a range of 25-100KHz.

$$P_s = \frac{1}{2} V_d I_o f_s \left(t_{c(on)} + t_{c(off)} \right)$$

$$t_{c(on)} = 100n + 50n = 150ns$$

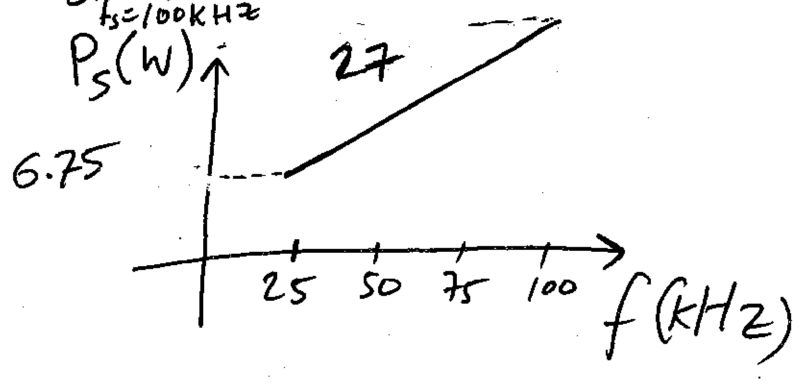
$$t_{c(off)} = 100 + 200 = 300ns$$

$$P_s = \left(\tfrac{1}{2}\right)(300)(4) f_s (150 + 300)$$

$$P_s = (600)(450 \times 10^{-9}) f_s = 2.7 \times 10^{-4} f_s$$

$$P_s\Big|_{f_s = 25KHz} = (2.7 \times 10^{-4})(25 \times 10^{3}) = 6.75W$$

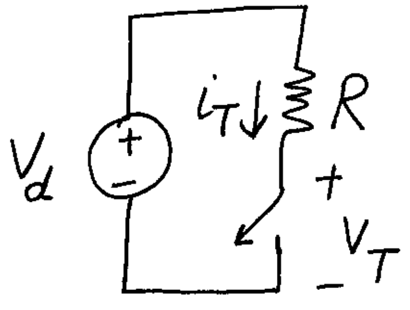
$$P_s\Big|_{f_s = 100KHz} = (2.7 \times 10^{-4})(100 \times 10^{3}) = 27W$$



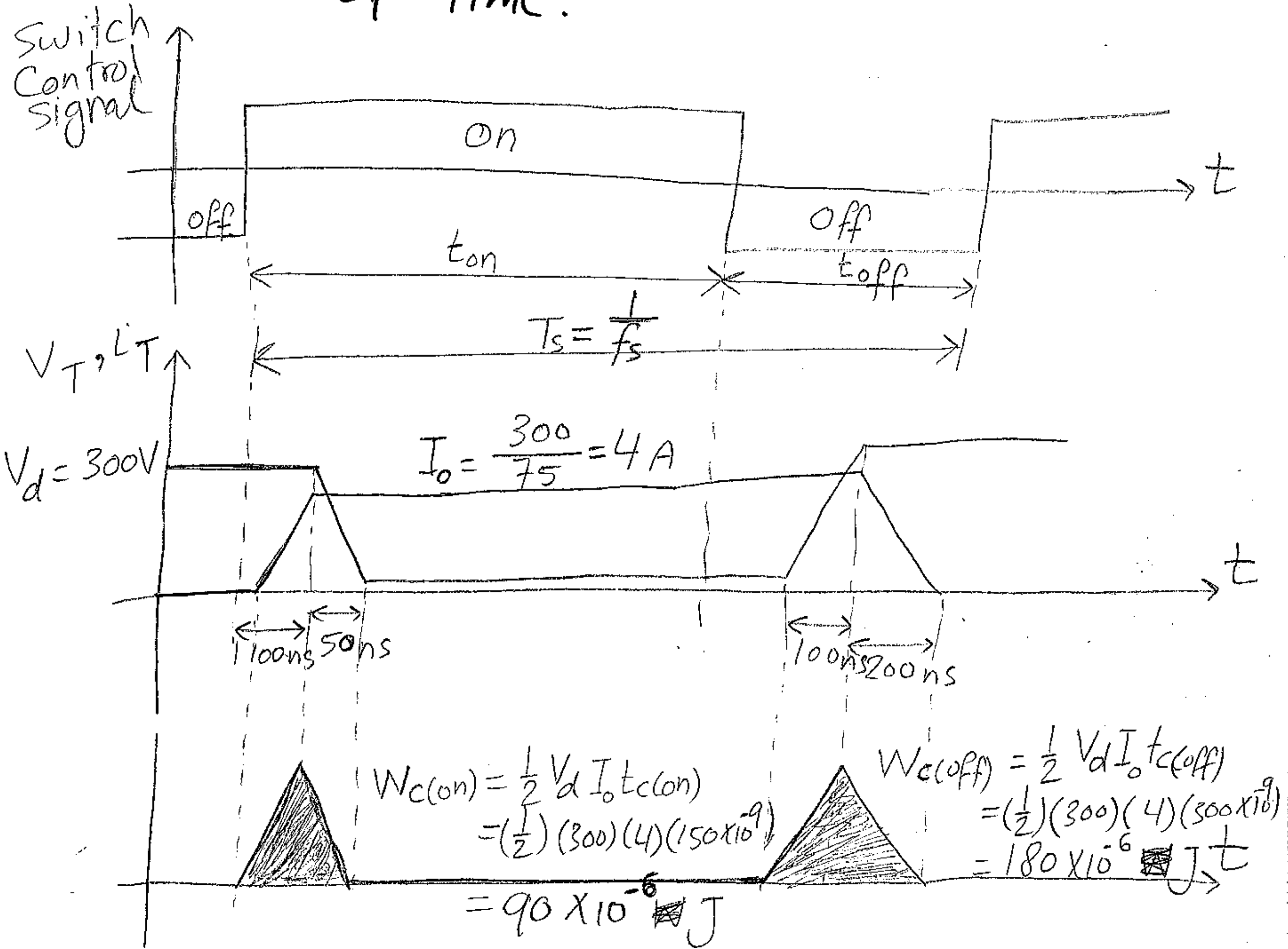
2-2 PP. 32

$V_d = 300V$ $f_s = 100\,kHz$ $R = 75\,\Omega$

$t_{ri} = 100ns$ $t_{fv} = 50ns$ $t_{rv} = 100ns$ $t_{fi} = 200ns$



Assuming linear voltage- and current-switching characteristics, plot the switch voltage and current and the switching power loss as a function of time.



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$$P_s = \frac{1}{2} V_d I_o f_s \left(t_{c(on)} + t_{c(off)}\right)$$

$$= \left(\frac{1}{2}\right)(300)(4)(100 \times 10^{3})(150+300) \times 10^{-9}$$

$$= 27 \quad W.$$

Review of Basic Electrical Circuit Concepts

Average power and rms current

$$p(t) = v\,i$$

if v & i are function of time and their waveform repeat with a time period T in steady state, then the average power flow can be calculated as:

$$P_{av} = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T v(t)\,i(t)\,dt$$

for pure resistive case:

$$P_{av} = \frac{1}{T}\int_0^T i^2 R\,dt = \frac{R}{T}\int_0^T i^2\,dt$$

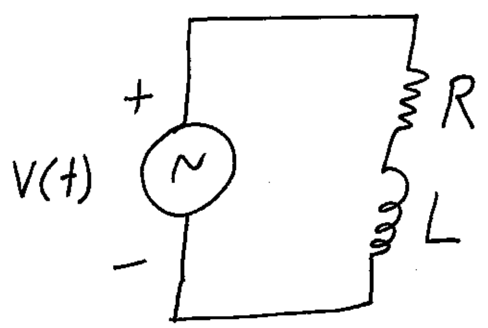
$P_{av} = RI^2$, I is the rms value of $i(t)$.

The rms value of $i(t)$ is:

$$I = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

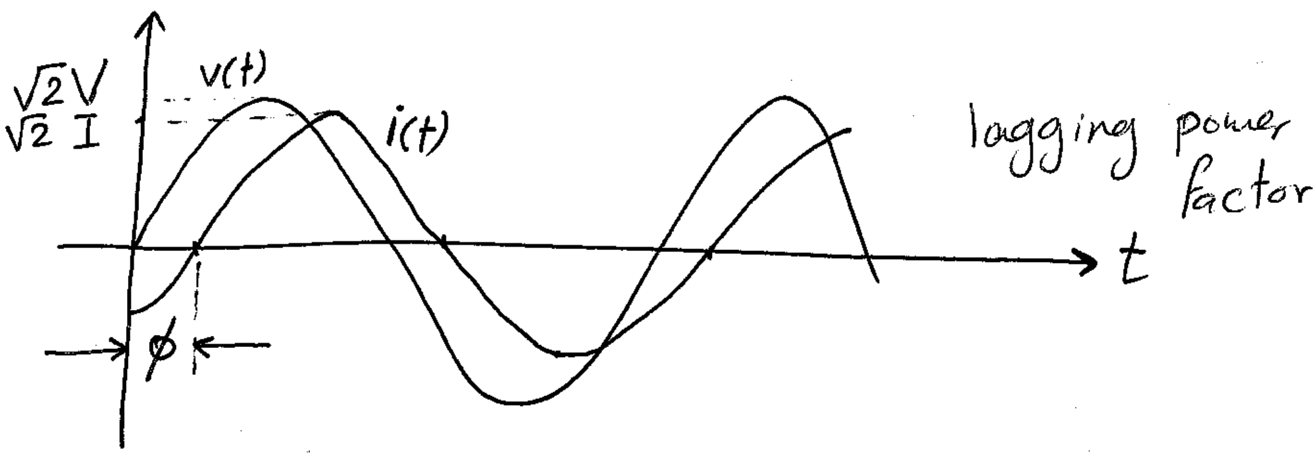
rms: root-mean-square

Steady-State ac Waveforms with Sinusoidal voltages and currents



$$v(t) = \sqrt{2}\, V \cos \omega t$$
$$i(t) = \sqrt{2}\, I \cos(\omega t - \phi)$$

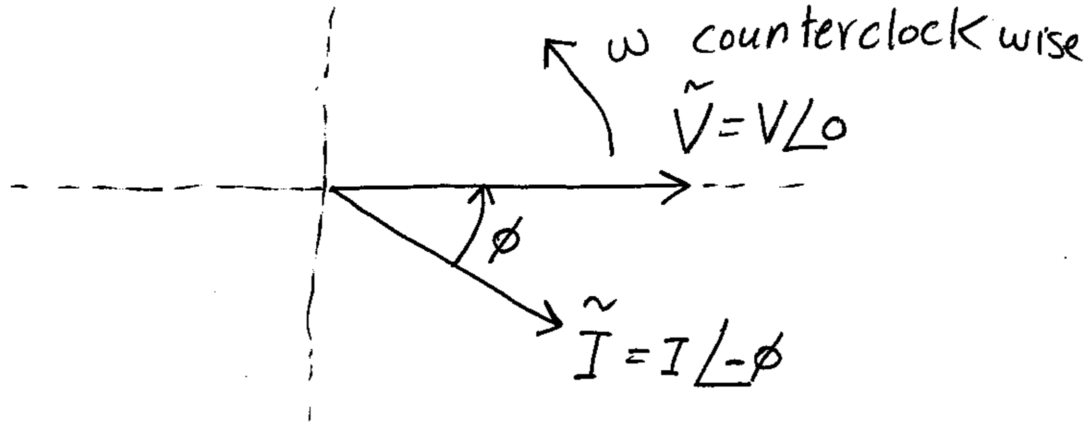
where V & I are the rms values.



Sinusoidal steady state waveforms of v & i

Phasor Representation

Since v & i are sinusoidals and have the same frequency, they can be represented in the complex plane by means of the projection of the rotating phasors to the horizontal real axis ~~and~~ as shown below:



rms $\qquad$ rms

$$\tilde{V} = Ve^{j0} \quad \text{and} \quad \tilde{I} = Ie^{-j\phi}$$

exponential form

$$\tilde{V} = Ve^{j0} = V(\cos 0 + j\sin 0) \longrightarrow \text{rectangular form}$$

$$\tilde{I} = Ie^{-j\phi} = I(\cos\phi - j\sin\phi)$$

$$\tilde{Z} = \frac{\tilde{V}}{\tilde{I}} = \frac{Ve^{j0}}{Ie^{-j\phi}} = \frac{V}{I}e^{j0+j\phi} = \frac{V}{I}e^{j\phi}$$

$$= \frac{V}{I}(\cos\phi + j\sin\phi)$$

let $\tilde{Z} = \dfrac{\tilde{V}}{\tilde{I}}$ $\qquad |Z| = \dfrac{|V|}{|I|}$

$$\tilde{Z} = Z(\cos\phi + j\sin\phi) = Z\cos\phi + jZ\sin\phi$$

$$\tilde{Z} = Ze^{j\phi} \Rightarrow \text{exponential form.}$$

$$\tilde{Z} = Z\cos\phi + jZ\sin\phi \Rightarrow \text{rectangular form.}$$

Power, Reactive Power, Power Factor & Apparent Power

The complex power $\tilde{S}$ is defined as:

$$\tilde{S} = \tilde{V}\tilde{I}^* = Ve^{j0}\cdot Ie^{j\phi} = VIe^{j\phi} = Se^{j\phi}$$

* $\underbrace{S = VI}_{\text{apparent power}} \Rightarrow$ magnitude of the complex power

* The real average power P is:

$$P = Re[S] = VI\cos\phi = \underbrace{S\cos\phi}_{\text{active power}}$$

$\underbrace{I_p = I\cos\phi}$, $\underbrace{I_q = I\sin\phi}$

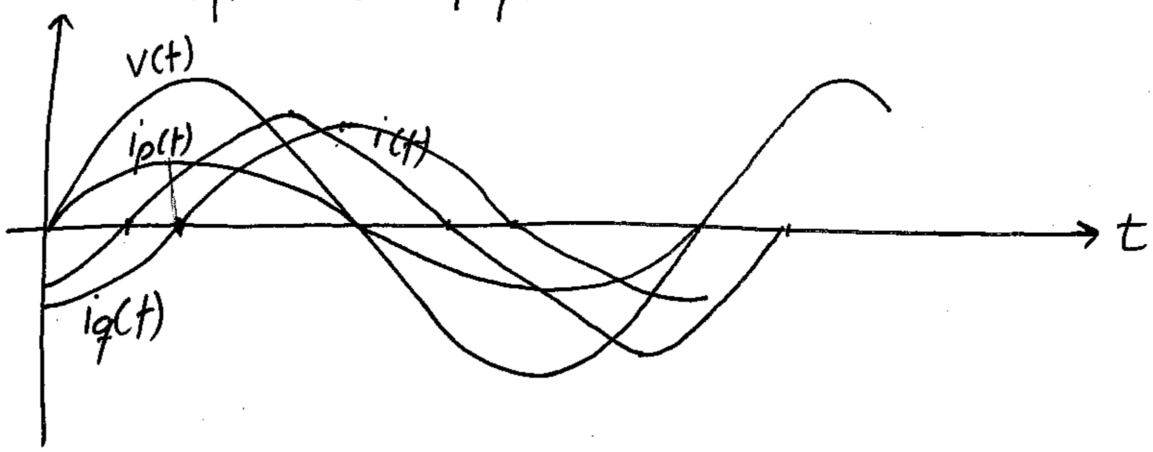
The current component which is in-phase with the voltage.

The current component which is out-of-phase with the voltage.

$$i_p(t) = \sqrt{2}\, I_p \cos\omega t = (\sqrt{2}\, I\cos\phi)\cos\omega t$$

$$i_q(t) = \sqrt{2}\, I_q \sin\omega t = (\sqrt{2}\, I\sin\phi)\sin\omega t$$

$$i(t) = \underbrace{i_p(t)}_{\text{in phase}} + \underbrace{i_q(t)}_{\text{out-of-phase}} .$$

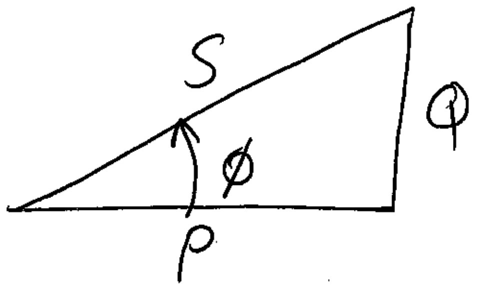


$$\tilde{S} = S\cos\phi + jS\sin\phi$$

$P = S\cos\phi \Longrightarrow$ active power (W)

$Q = S\sin\phi \Longrightarrow$ reactive power (VAR)

S : apparent power (VA)



Power factor $PF = \cos\phi$

$$= \cos\left[\tan^{-1}\frac{Q}{P}\right]$$

$$= \cos\left[\sin^{-1}\frac{Q}{S}\right]$$

$$= \cos\left[\cos^{-1}\frac{P}{S}\right]$$

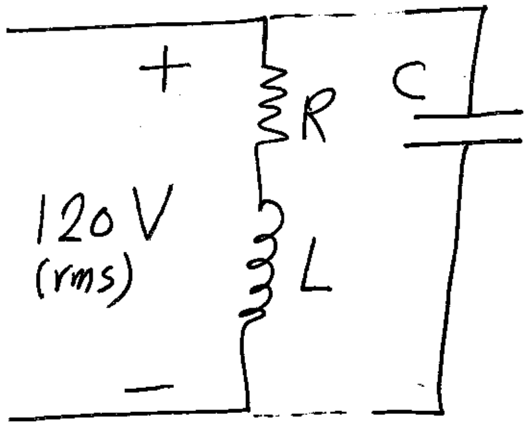
$$= \frac{P}{S}$$

* $\underbrace{PF = 1}_{\text{(unity)}}$ with pure resistive loads

* PF is lagging with inductive loads.

* PF is leading with capacitive loads.

Ex. An inductive load connected to a 120-V, 60 Hz ac source draws 1 kW at a power factor of 0.8. Calculate the capacitance required in parallel with the load in order to bring the combined power factor to 0.95 (lagging).

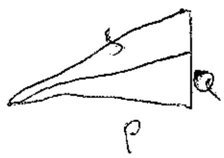


$f = 60\ Hz$

$\omega = 2\pi f$

$= (2)(\pi)(60)$

$= 377\ rad/s$



$P = 1\ kW$

$PF = 0.8$

$PF = \frac{P}{S} \Rightarrow S = \frac{P}{PF} = \frac{1000}{0.8} = 1250\ VA$

$Q = \sqrt{S^2 - P^2} = \sqrt{1250^2 - 1000^2} = 750\ VAR$ (lagging)

$S = 1000 + j750\ VA$

With capacitor C:

$S = P + jQ - jQ_c$

$$0.95 = \frac{P}{\sqrt{P^2 + (Q - Q_c)^2}} = \frac{1000}{\sqrt{1000^2 + (750 - Q_c)^2}}$$

$Q_c = 421.3$ VAR (leading)

$$Q_c = \frac{V^2}{X_c} = \frac{V^2}{\left(\frac{1}{\omega C}\right)} = V^2 \omega C$$

$$421.3 = (120)^2 (377) C$$

$$C = 77.6 \mu F.$$

$\tilde{Z} = R + j\omega L \Rightarrow$ inductive load

$\tilde{Z} = R - \frac{1}{j\omega C} \Rightarrow$ capacitive load

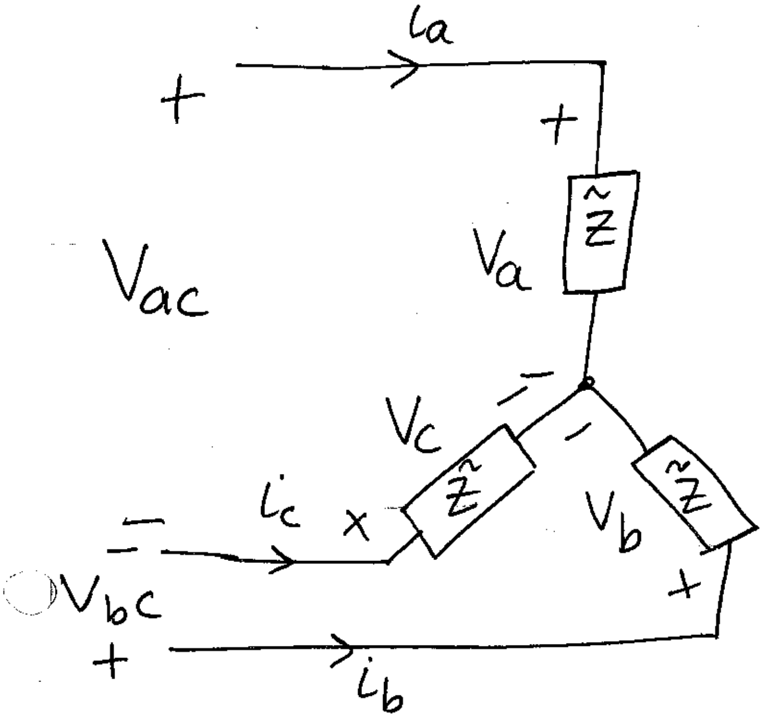
$\tilde{Z} = R \Rightarrow$ pure resistive load

$\tilde{Z} = j\omega L \Rightarrow$ pure inductive load

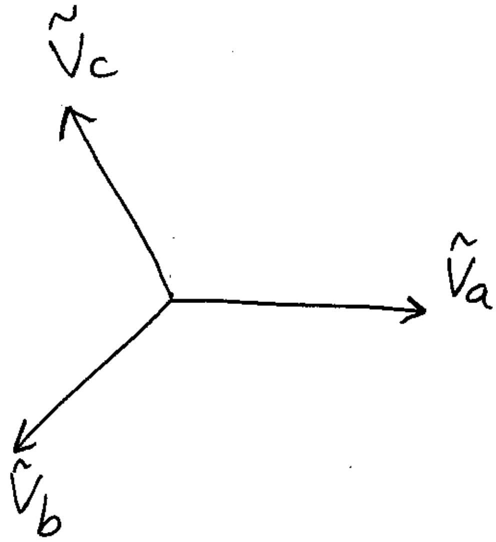
$\tilde{Z} = \frac{j}{\omega C} \Rightarrow$ pure capacitive load

$\omega = 2\pi f$ rad/s, f Hz.

Three-Phase Circuits



Balanced three-phase circuit



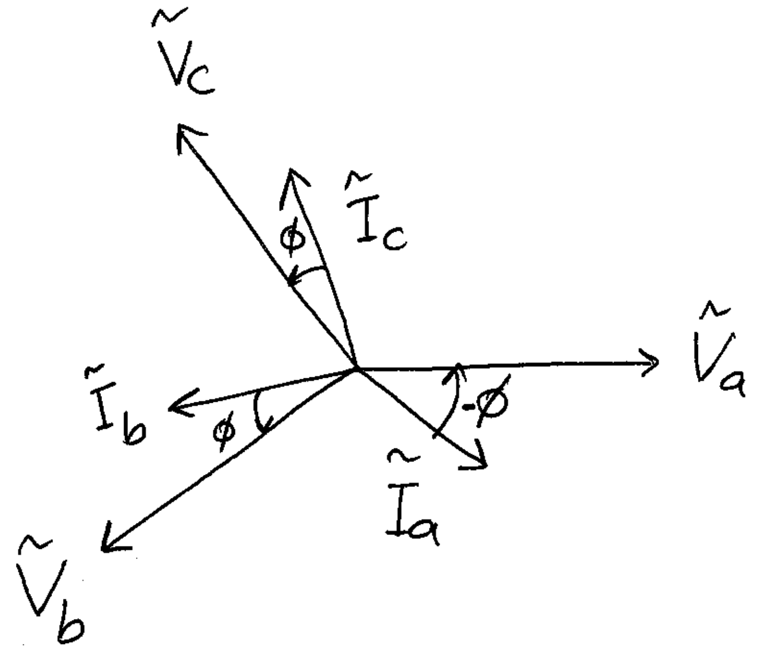
Balanced three-phase phasors

$$\tilde{I}_a = \frac{\tilde{V}_a}{\tilde{Z}} = \frac{V e^{j0}}{Z e^{j\phi}} = \frac{V}{Z} e^{-j\phi} = I e^{-j\phi}$$

$$\tilde{I}_b = \tilde{I}_a e^{-j2\pi/3} = I e^{-j(\phi + 2\pi/3)}$$

$$\tilde{I}_c = \tilde{I}_a e^{j2\pi/3} = I e^{-j(\phi - 2\pi/3)}$$

With an inductive load (+ve ϕ), the phase voltage and current phasors are:



* It is possible to calculate the line-to-line voltages from the phase voltages as:

$$\tilde{V}_{ab} = \tilde{V}_a - \tilde{V}_b = V\angle 0 - V\angle -120$$

$$= V - (V\cos 120 - jV\sin 120)$$

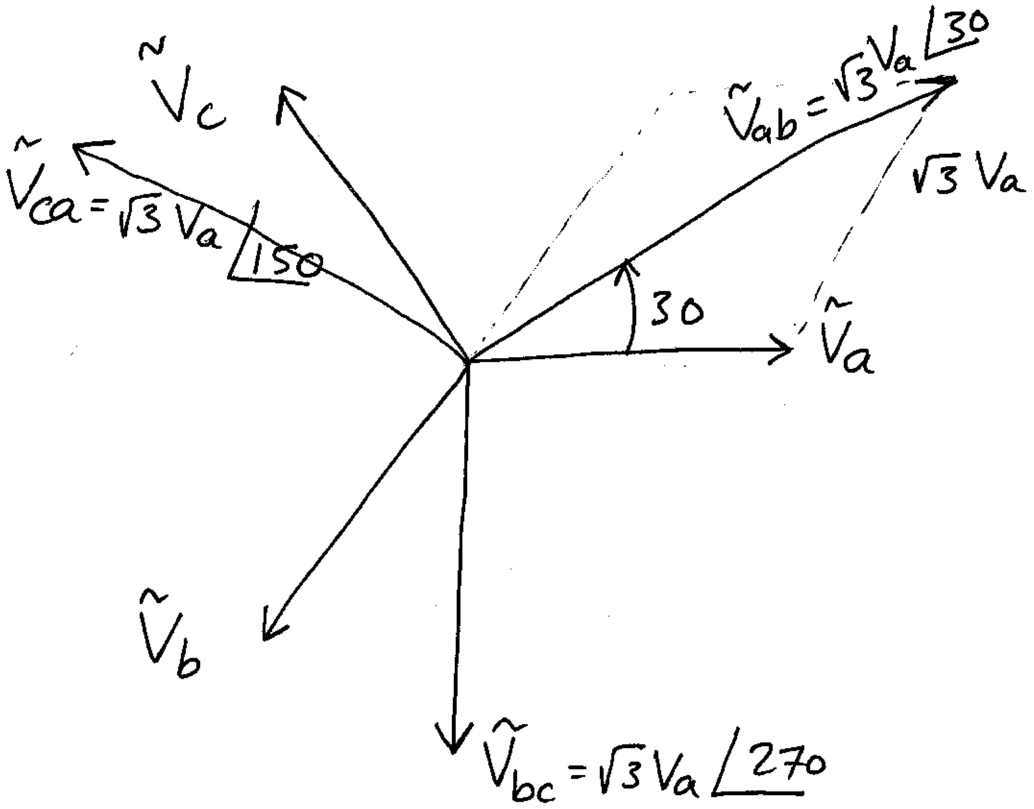
$$= V - V\cos 120 + jV\sin 120$$

$$= \sqrt{3}\, V\angle 30$$

$$\boxed{V_{LL} = \sqrt{3}\, V}$$ line-to-line voltage magnitude

$$S_{phase} = VI \quad , \quad P_{phase} = VI\cos\phi$$

$$S_{3\text{-}phase} = 3\, S_{phase} = 3VI = \sqrt{3}\, V_{LL} I$$



$$P_{3\text{-phase}} = 3 P_{phase} = 3 V I \cos\phi = \sqrt{3} V_{LL} I \cos\phi$$

* The three-phase circuit operates on the same power factor as the per-phase power factor.

Nonsinusoidal Waveforms in steady state

* The steady state voltages and currents of power electronic devices are usually periodic but not sinusoidal.

Fourier Analysis of Repetitive Waveforms

* a nonsinusoidal waveform $f(t)$ repeating with an angular frequency ω can be expressed as:

$$f(t) = F_0 + \sum_{h=1}^{\infty} f_h(t) = \frac{1}{2}a_0 + \sum_{h=1}^{\infty} \{a_h \cos(h\omega t)$$

$$+ b_h \sin(h\omega t)\}$$

where $F_0 = \frac{1}{2}a_0 \Rightarrow$ average value of $f(t)$.

$$a_h = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos(h\omega t)\,d(\omega t) \qquad h = 0, \ldots, \infty$$

and

$$b_h = \frac{1}{\pi}\int_0^{2\pi} f(t)\sin(h\omega t)\,d(\omega t) \qquad h = 1, \ldots, \infty$$

$$F_0 = \frac{1}{2}a_0$$

The average value is:

$$F_0 = \frac{1}{2}a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(t)\,d(\omega t) = \frac{1}{T}\int_0^{T} f(t)\,dt$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Each frequency component $f_h(t)$:

$$f_h(t) = a_h \cos(h\omega t) + b_h \sin(h\omega t)$$

$$\tilde{F}_h = F_h\, e^{j\phi_h}$$

$$F_h = \frac{\sqrt{a_h^2 + b_h^2}}{\sqrt{2}}$$ rms magnitude

and the phase ϕ_h is:

$$\tan \phi_h = \frac{-b_h}{a_h}$$

$$\phi_h = \tan^{-1}\left(\frac{-b_h}{a_h}\right)$$

forget it

forget it....!

* The rms value of $f(t)$ (F) can be expressed in terms of the rms values of its Fourier series components:

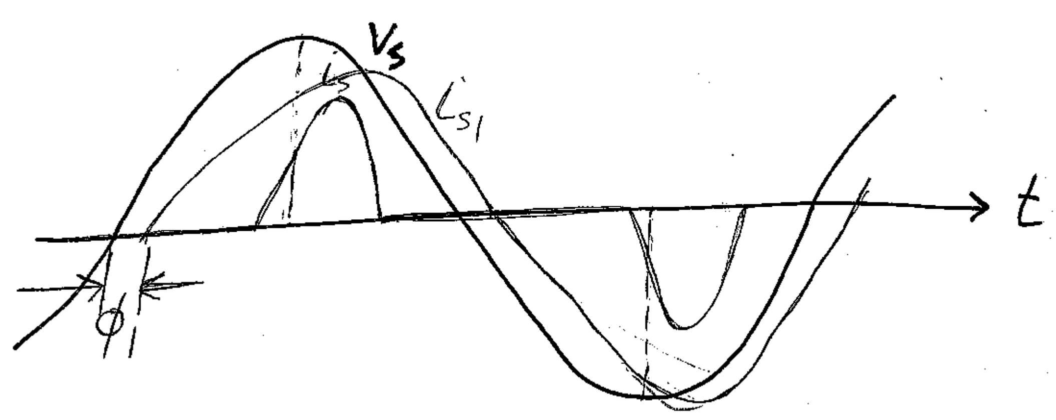
$$F = \left(F_0^2 + \sum_{h=1}^{\infty} F_h^2\right)^{1/2}$$

$$F = \sqrt{F_0^2 + F_1^2 + F_2^2 + \cdots + F_\infty^2}$$

* See table 3-1 pp. 41 for simplifying the calculations of a_h & b_h.

Line-Current Distortion

* The following is a line current i_s drawn from the utility by the power electronic equipment that is sinusoidal. To simplify our analysis, we will assume a pure sinusoidal input voltage v_s of the fundamental frequency of $\omega_1 = \omega$ and $f_1 = f$:



$$v_s = \sqrt{2} V_s \sin \omega_1 t$$

* The input current in steady state is the sum of its Fourier (harmonic) components as

$$i_s(t) = i_{s1}(t) + \sum_{h \neq 1} i_{sh}(t)$$

where i_{s1} : the fundamental component (the component with the line-frequency f_1)

i_{sh} : the component at the h harmonic frequency f_h $(= hf_1)$.

* i_{s1} and i_{sh} can be expressed as

$$i_s(t) = \sqrt{2}\, I_{s1} \sin(\omega_1 t - \phi_1) + \sum_{h \neq 1} \sqrt{2}\, I_{sh} \sin(\omega_h t - \phi_h)$$

where:

ϕ_1 : the phase angle between V_s and i_{s1}.

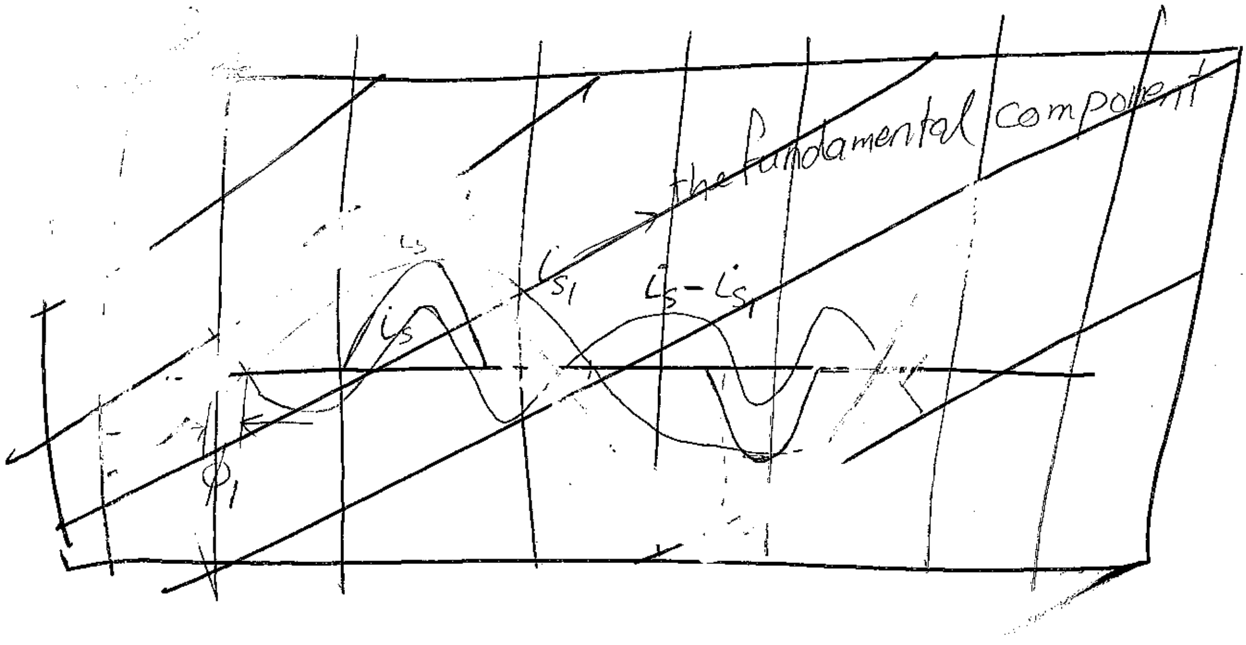
* The rms value I_s of i_s can be calculated as:

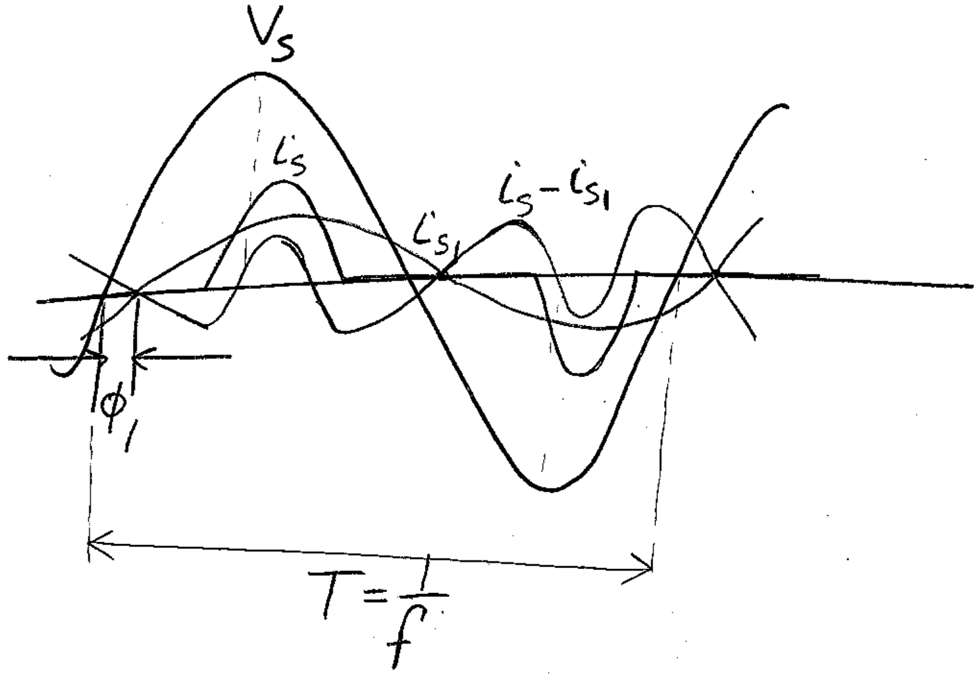
$$I_s = \left(\frac{1}{T_1} \int_0^{T_1} i_s^2(t)\, dt \right)^{1/2}$$

$$I_s = \left(I_{s1}^2 + \sum_{h \neq 1}^{\infty} I_{sh}^2 \right)^{1/2}$$

* The amount of distortion of the waveform is quantified by means of an index called the "total harmonic distortion" THD. The distortion component i_{dis} is:

$$i_{dis}(t) = i_s(t) - i_{s1}(t) = \sum_{h \neq 1}^{\infty} i_{sh}(t).$$





* The THD in the current is defined as:

$$\% THD = 100 \times \frac{I_{dis}}{I_{s_1}}$$

$$\boxed{THD\% = 100 \times \frac{\sqrt{I_s^2 - I_{s_1}^2}}{I_{s_1}}}$$ (try to use this)

$$= 100 \times \sqrt{\sum_{h \neq 1} \left(\frac{I_{sh}}{I_{s_1}}\right)^2}$$

I_s : rms of i_s.

I_{s_1} : rms of i_{s_1}.

I_{dis} : rms of i_{dis} , $i_{dis} = i_s(t) - i_{s_1}(t)$

$$I_{dis} = \sqrt{I_s^2 - I_{s_1}^2} = \left(\sum_{h \neq 1} I_{sh}^2\right)^{1/2}$$

Power and Power Factor

$$P = \frac{1}{T_1}\int_0^{T_1} p(t)\,dt = \frac{1}{T_1}\int_0^{T_1} v_s(t)\, i_s(t)\,dt$$ ⇒ fundamental component

$$= \frac{1}{T_1}\int_0^{T_1} \sqrt{2}V \sin\omega_1 t \left[\left(\sqrt{2} I_{s_1} \sin(\omega_1 t - \phi_1)\right) + \sum_{h \neq 1}^{\infty} \sqrt{2} I_{sh} \sin(\omega_h t - \phi_h)\right] dt$$

$$P=\frac{1}{T_1}\int_0^{T_1}\sqrt{2}V\sin\omega_1 t\cdot\sqrt{2}\,I_{s_1}\sin(\omega_1 t-\phi_1)\,dt \Rightarrow \text{because}$$

$$\int_0^{2\pi}\sin(\omega t)\sin(h\omega t)\,d(\omega t)=0$$

for $h=2,3,\dots\infty$

$$P=V_s\,I_{s_1}\cos\phi_1 .$$

*Note that the current components at harmonic frequencies do not contribute to the average (real) power drawn from the sinusoidal voltage source V_s.

The apparent power S :

$$S=\overset{rms}{V_s}\,\overset{rms}{I_s}\Rightarrow \text{definition}$$

$$PF=\frac{P}{S}$$

$$PF=\frac{V_s\,I_{s_1}\cos\phi_1}{V_s\,I_s}=\frac{I_{s1}}{I_s}\cos\phi_1$$

* The displacement Power Factor (DPF) which is the same as the power factor in linear circuits with sinusoidal voltages and currents is defined as the cosine of the angle ϕ_1:

$$DPF=\cos\phi_1$$

Therefore:

$$PF=\frac{I_{s1}}{I_s}\,DPF .$$

* In terms of THD:

$$PF = \frac{1}{\sqrt{1+THD^2}} DPF$$

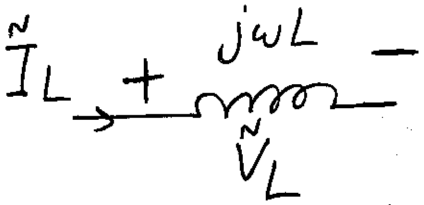
Inductor and Capacitor Responses

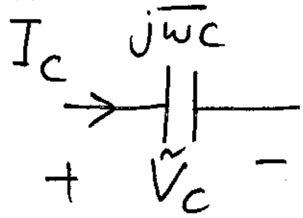
* The currents lags the voltage by 90° in an inductor and leads the voltage by 90° in a capacitor.

$$\tilde{I}_L = \frac{\tilde{V}_L}{j\omega L} = \left(\frac{\tilde{V}_L}{\omega L}\right) e^{-j\pi/2} \quad \text{in an inductor}$$

and

$$\tilde{I}_c = j\omega C \tilde{V}_c = (\omega C \tilde{V}_c) e^{j\pi/2} \quad \text{in a capacitor}$$

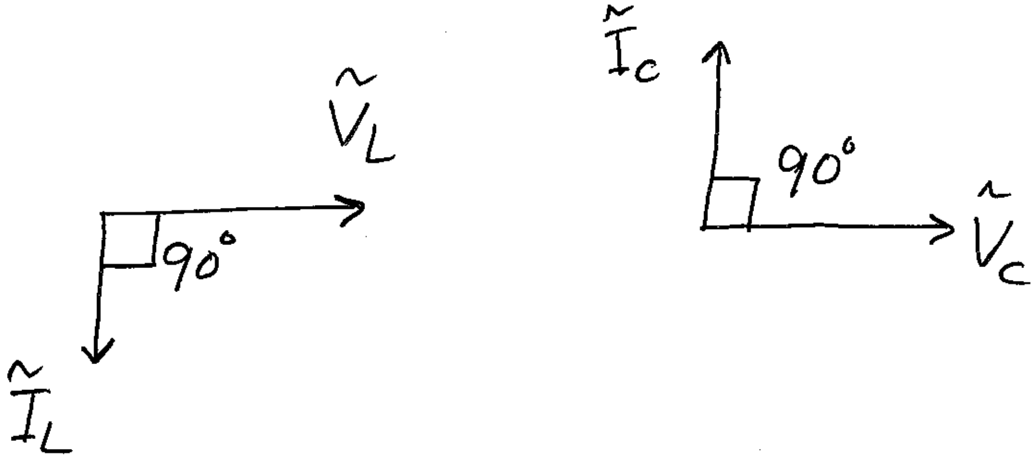




$$\tilde{V}_L = (j\omega L)\tilde{I}_L \qquad \tilde{I}_c = (j\omega C)\tilde{V}_c$$

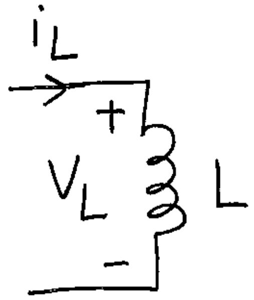
$$V_L = L\frac{di(t)}{dt} \qquad i_c = C\frac{dV(t)}{dt}$$

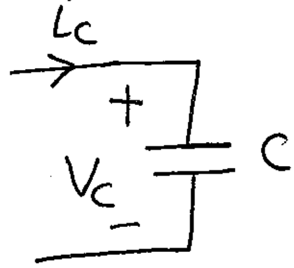
$$i(t) = \frac{1}{L}\int V_L(t)\,dt + K \qquad V(t) = \frac{1}{C}\int i_c(t)\,dt + K$$



phasor representation of inductor and capacitor

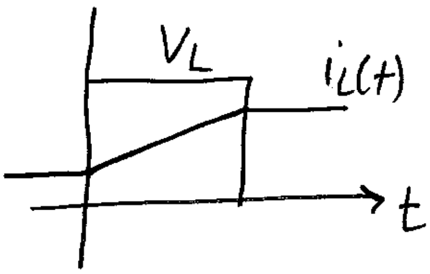
Average V_L and I_c in steady state

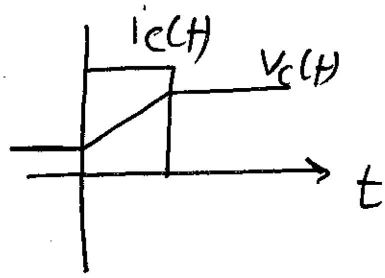




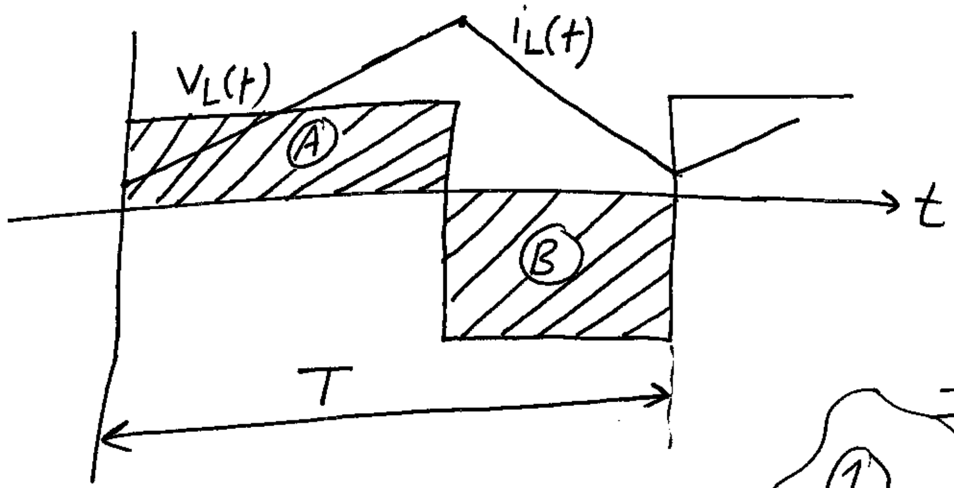
$$V_L(t) = L\frac{di_L(t)}{dt}$$

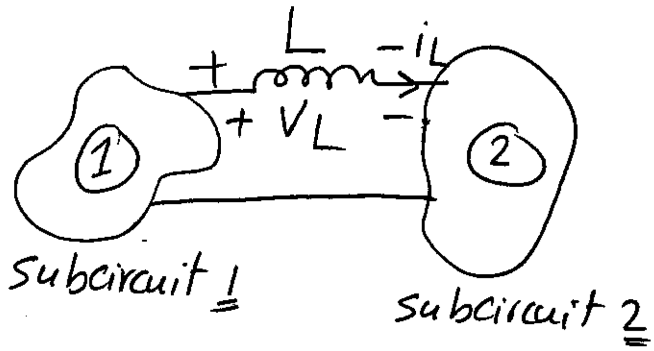
$$i_c(t) = C\frac{dV_c(t)}{dt}$$



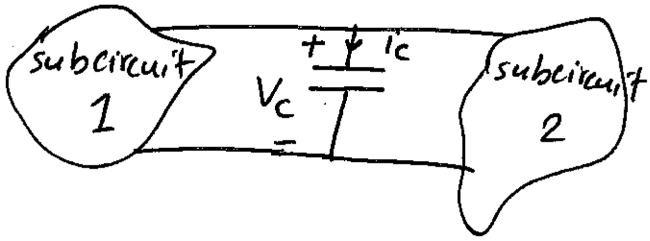


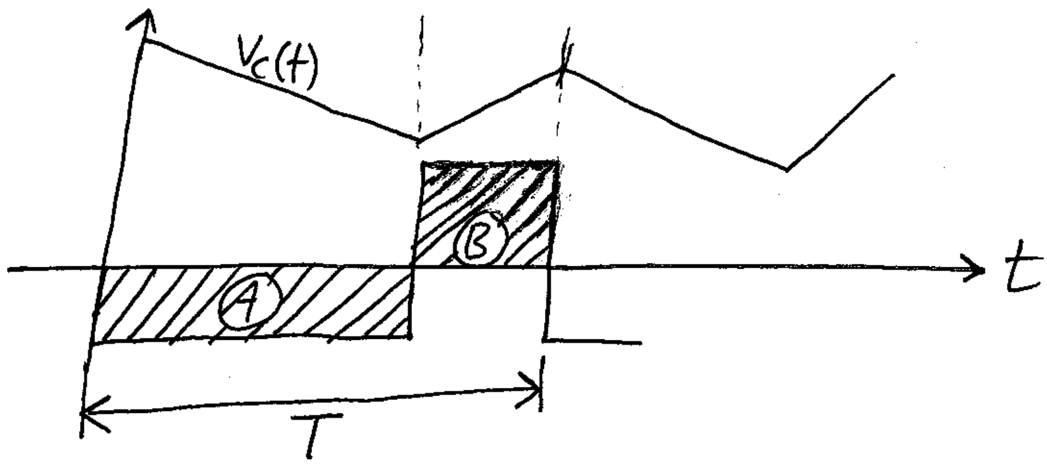
Inductor and capacitor response





* In steady state, the average inductor voltage (average over one time period) must be zero, i.e the area $\boxed{A = B}$.





* In steady state, the average capacitor current (average over one time period) must be zero i.e. $\boxed{A = B}$

$p = vi$

$E = \int p$

$= \int L\frac{di}{dt} i$

$= L$

$p = vi$ $E = P.t$

$p = \frac{E}{t}$

$p = \frac{dE}{dt}$

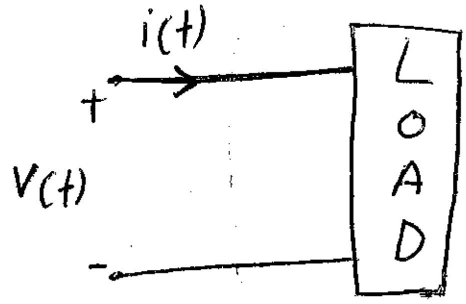
$E = \int p\,dt = \int vi\,dt$

$= \int i L\frac{di}{dt}\,dt$

$\int L\frac{di}{dt}\,dt$

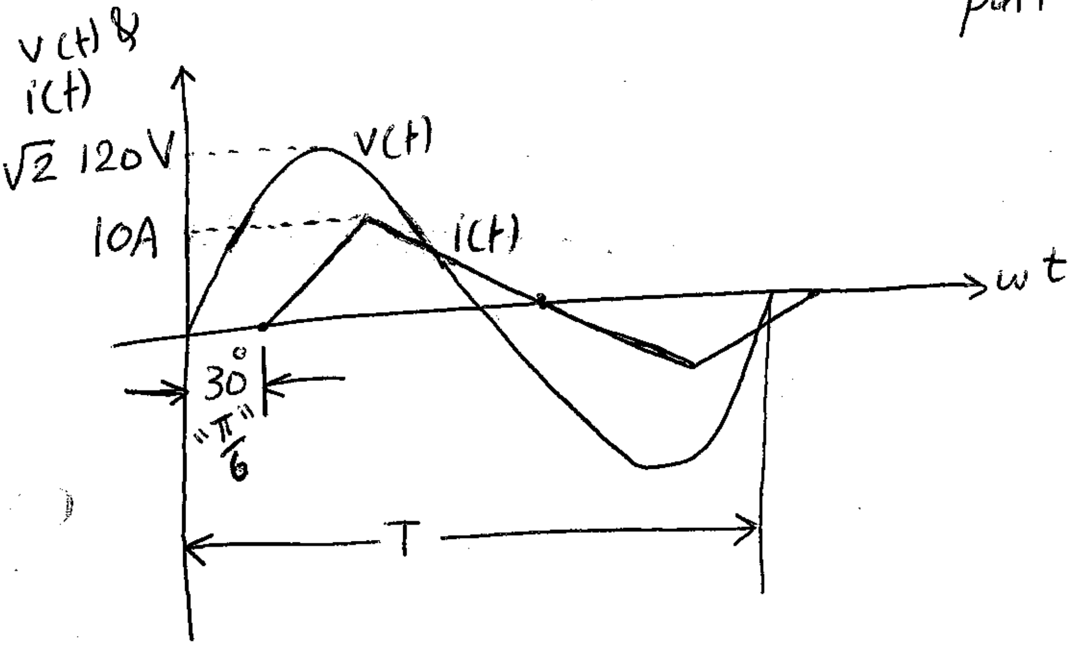
problem 3-6 PP. 58

* $V(t) = \sqrt{2} V \sin\omega t$
* $i(t)$ is triangular wave with amplitude $A = 10A$.
* $i(t)$ lags $v(t)$ by $\phi = 30°$.



Calculate P , DPF, THD, PF.

$V = 120V$, $A = 10A$, $\phi = 30°$ (The waveform given in part e)



$$P = V I_1 \cos\phi_1$$

$$i(t) = \frac{1}{2} a_0 + \sum_{h=1}^{\infty} \{a_h \cos(h\omega t) + b_h \sin(h\omega t)\}$$

$a_0 = 0$

$a_h = 0$ (odd function)

$$b_h = \frac{1}{\pi}\int_0^{2\pi} f(t)\sin(h\omega t)\,d(\omega t)$$

$b_h = 0$ for even h

$$f(t) = \begin{cases} \frac{40t}{T} & 0 < t < \frac{T}{4} \\ \frac{-40t}{T} + 20 & \frac{T}{4} < t < \frac{3T}{4} \\ \frac{40t}{T} - 40 & \frac{3T}{4} < t < T \end{cases}$$

$$b_h = \frac{1}{\pi}\left[\int_0^{\pi/2} \frac{40t}{T}\sin(h\omega t)\,d(\omega t) + \int_{\pi/2}^{3\pi/2} \left(\frac{-40t}{T} + 20\right)\sin(h\omega t)\,d(\omega t)\right.$$

$$\left. + \int_{3\pi/2}^{2\pi} \left(\frac{40t}{T} - 40\right)\sin(h\omega t)\,d(\omega t)\right]$$

Carrying out this integration yields:

$b_1 = 0.8106 * 10 = 8.106$.

$b_3 = -0.09006 * 10 = -0.9006$.

$i_1(t) = 8.106 \sin \omega t$

$I_1 = \frac{8.106}{\sqrt{2}} = 5.7318\ A$

$P = (120)(5.7318)\cos 30^\circ = 595.7\ W$.

$$DPF = \cos\phi_1 = \cos 30^\circ = 0.866$$

$$\% THD = 100 * \frac{\sqrt{I^2 - I_1^2}}{I_1}$$

$$I = \sqrt{\frac{1}{T}\int_0^T (i(t))^2 dt} = \left[\frac{1}{T}\left[\int_0^{T/4} \left(\frac{40t}{T}\right)^2 dt + \int_{T/4}^{3T/4} \left(\frac{-40t}{T} + 20\right)^2 dt + \int_{3T/4}^{T} \left(\frac{40t}{T} - 40\right)^2 dt\right]\right]^{1/2}$$

$I = 5.7736$ (For triangular wave $rms = \frac{A}{\sqrt{3}}$)

$$\% THD = 100 * \frac{\sqrt{(5.7736)^2 - (5.7318)^2}}{5.7318}$$

$$= 12.099$$

$$PF = \frac{I_1}{I} DPF = \frac{5.7318}{5.7736} * 0.866 = 0.8597$$

Do the same for all waveforms of this question.

How to Integrate Simbolically in MATLAB

@

```
>> syms a x
>> A= int(a*sin(x),x)  ↵
```

$\Rightarrow \int a \sin x \, dx$

```
A =
   -a*cos(x)
>> AA = int (a*sin(x),a) ↵
```

$\Rightarrow \int a \sin x \, da$

```
AA =
   1/2*a^2 *sin(x)
>> AA= subs(AA, x, ( pi/2)) ↵
AA =
   (1/2)*a^2
>> AA = subs(AA, , 2) ↵
AA =
   2
>> f= a^2 * exp(x) ; ↵
>> R= int(f, x, 1,2) ↵
```

$\Rightarrow \int_1^2 a^2 e^x dx$

```
R =
   a^2*exp(2)-a^2*exp(1)
>> vpa(R,5) ↵
ans =
   4.6708*a^2 .
```

(b)

>> Z = int(sin(x), x, 0, (Pi/2)) ↵

Z =

1

Returning back to our case of Fourier Analysis:

$$b_h = \frac{1}{\pi}\left[\int_0^{\pi/2} \frac{40t}{T}\sin(h\underbrace{\omega t}_{\theta})\, d\underbrace{\omega t}_{\theta} + \int_{\pi/2}^{3\pi/2}\left(\frac{-40t}{T}+20\right)\sin(h\omega t)\, d\omega t + \int_{3\pi/2}^{2\pi}\left(\frac{40t}{T}-40\right)\sin(h\omega t)\, d\omega t\right]$$

$$\theta = \omega t \Rightarrow t = \frac{\theta}{\omega} = \frac{\theta}{2\pi f} = \frac{\theta T}{2\pi}$$

$$b_h = \frac{1}{\pi}\left[\int_0^{\pi/2} \frac{40\,\theta T}{2\pi T}\sin(h\theta)\, d\theta + \int_{\pi/2}^{3\pi/2}\left(\frac{-40\,\theta T}{2\pi T}+20\right)\sin(h\theta)\, d\theta + \int_{3\pi/2}^{2\pi}\left(\frac{40\,\theta T}{2\pi T}-40\right)\sin(h\theta)\, d\theta\right]$$

$$b_h = \frac{1}{\pi}\left[\int_0^{\pi/2} \frac{40\theta}{2\pi}\sin(h\theta)\, d\theta + \int_{\pi/2}^{3\pi/2}\left(\frac{-40\theta}{2\pi}+20\right)\sin(h\theta)\, d\theta + \int_{3\pi/2}^{2\pi}\left(\frac{40\theta}{2\pi}-40\right)\sin(h\theta)\, d\theta\right]$$

b&f

c

```
>> syms th h
>> A = int(((40*th)/(2*pi))*sin(h*th),th,0,(pi/2))

A =

    ---------------

>> B= int((((40*th)/(2*pi)+20)*sin(h*th),th,(pi/2),(3*pi/2))

B =

    ----

>> C= int((((40*th)/(2*pi)-40)*sin(h*th),th,(3*pi/4),(2*pi))

C =

    --

>> bh=(1/π)*(A+B+C)

bh =

    ------

>> b1 = subs(bh,h,1)
b1 =

>> b2 = subs(bh,h,2)
b2 =
```

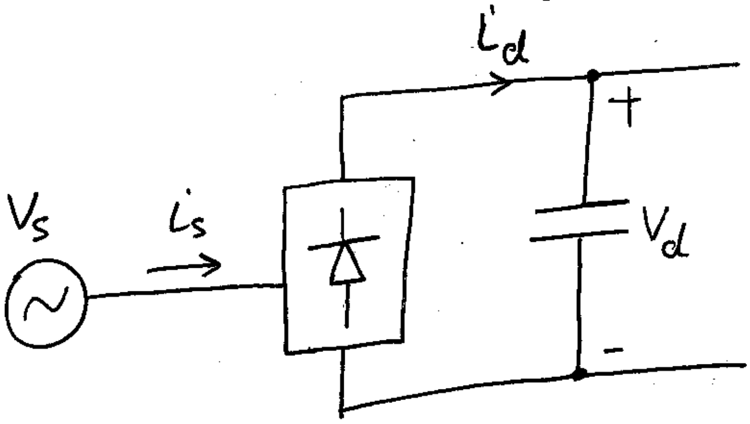
Line-Frequency Diode Rectifiers

Line-Frequency ac → Uncontrolled dc

Line-frequency = 50 or 60 Hz.

Rectifiers: circuits which change from AC to DC.

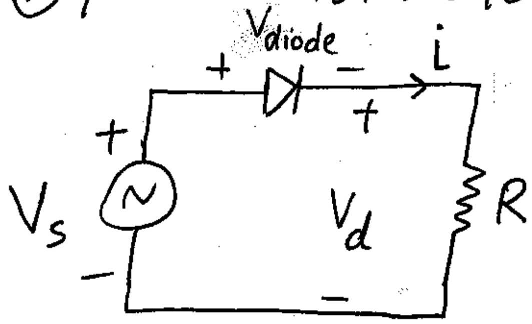
uncontrolled: The devices are turned on & off by the line voltage.

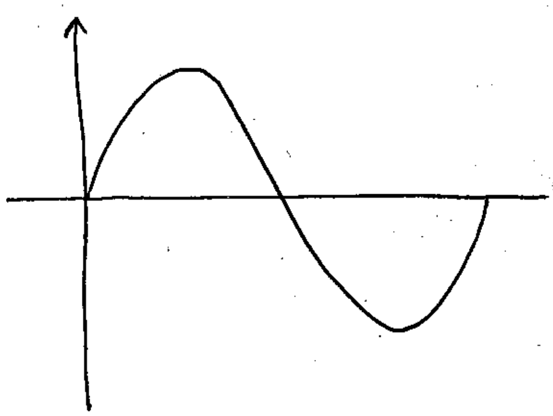


Block diagram of a rectifier

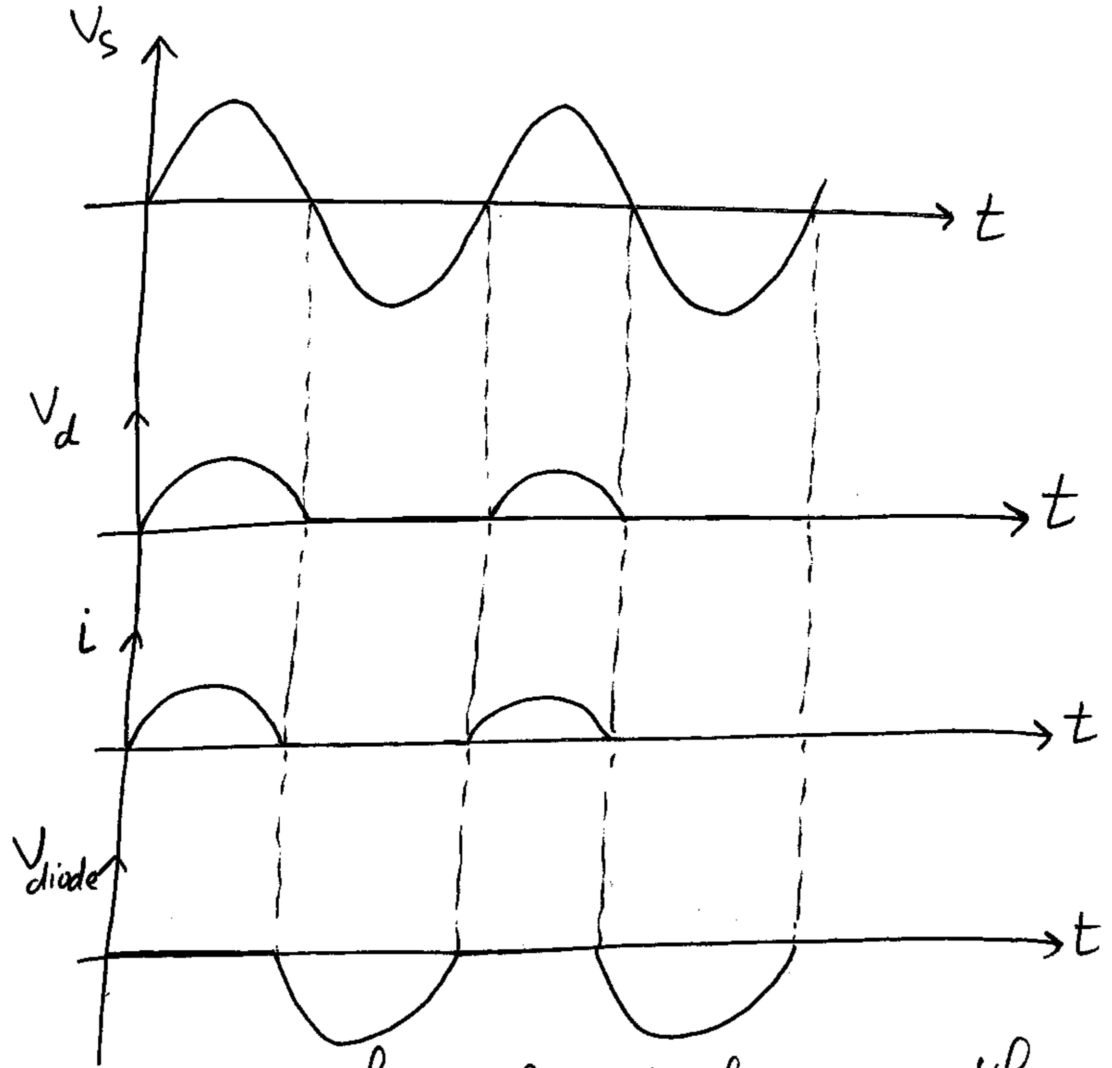
Basic Rectifier Concepts

① pure resistive load





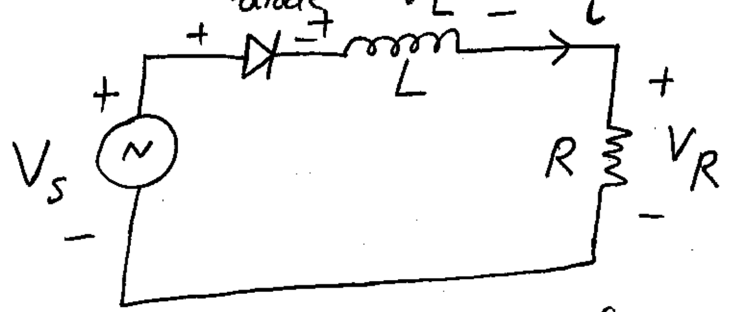
Half-wave rectifier with pure resistive load



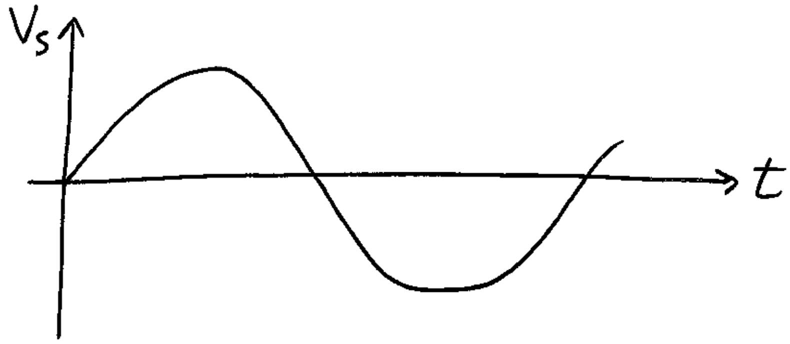
The waveforms of the half-wave rectifier with pure resistive loads

* Because of the large ripple in V_d and i, this circuit is of little practical signifcance.

② Inductive load

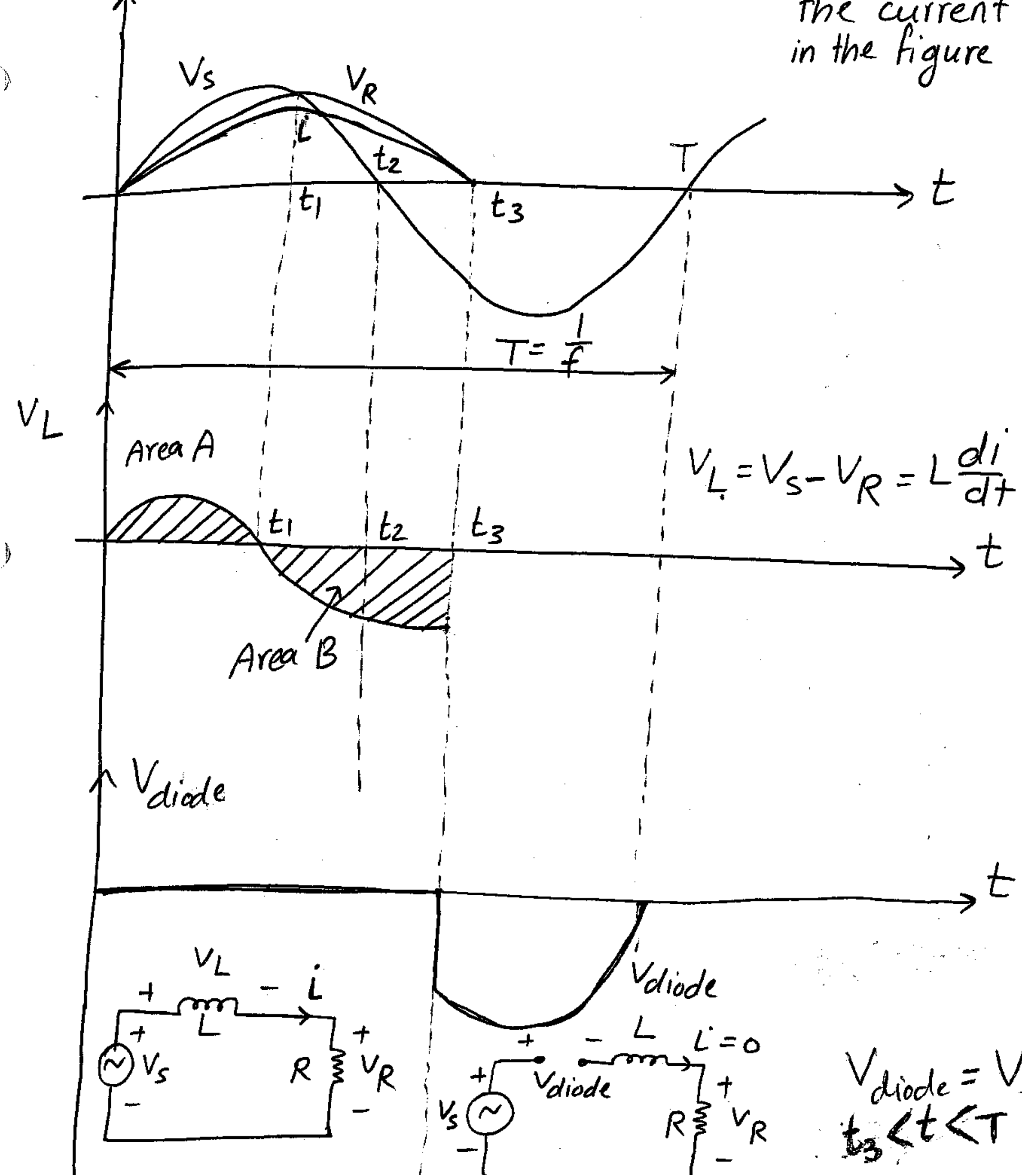


Half-wave rectifier with inductive load



* For the first half cycle, the diode is on. The equation is:

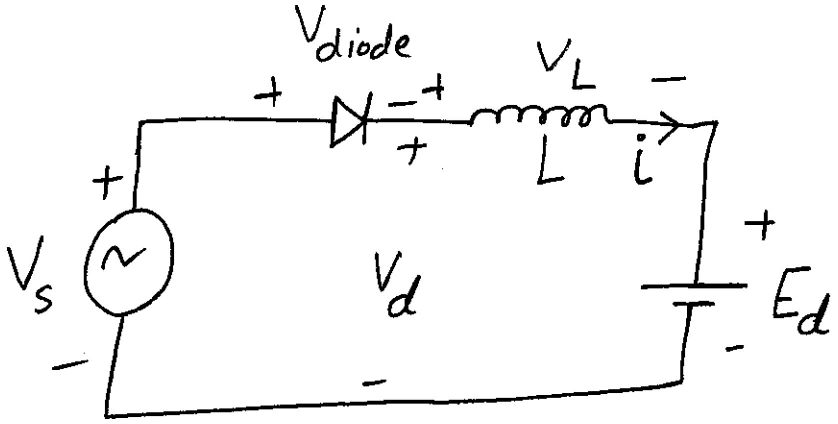
$$V_s = Ri + L\frac{di}{dt} = V_R + V_L \Rightarrow$$ solving for i, yields the current i shown in the figure



$$V_L = V_s - V_R = L\frac{di}{dt}$$

$$V_{diode} = V_s$$
$$t_3 < t < T$$

③ Load with an internal dc voltage

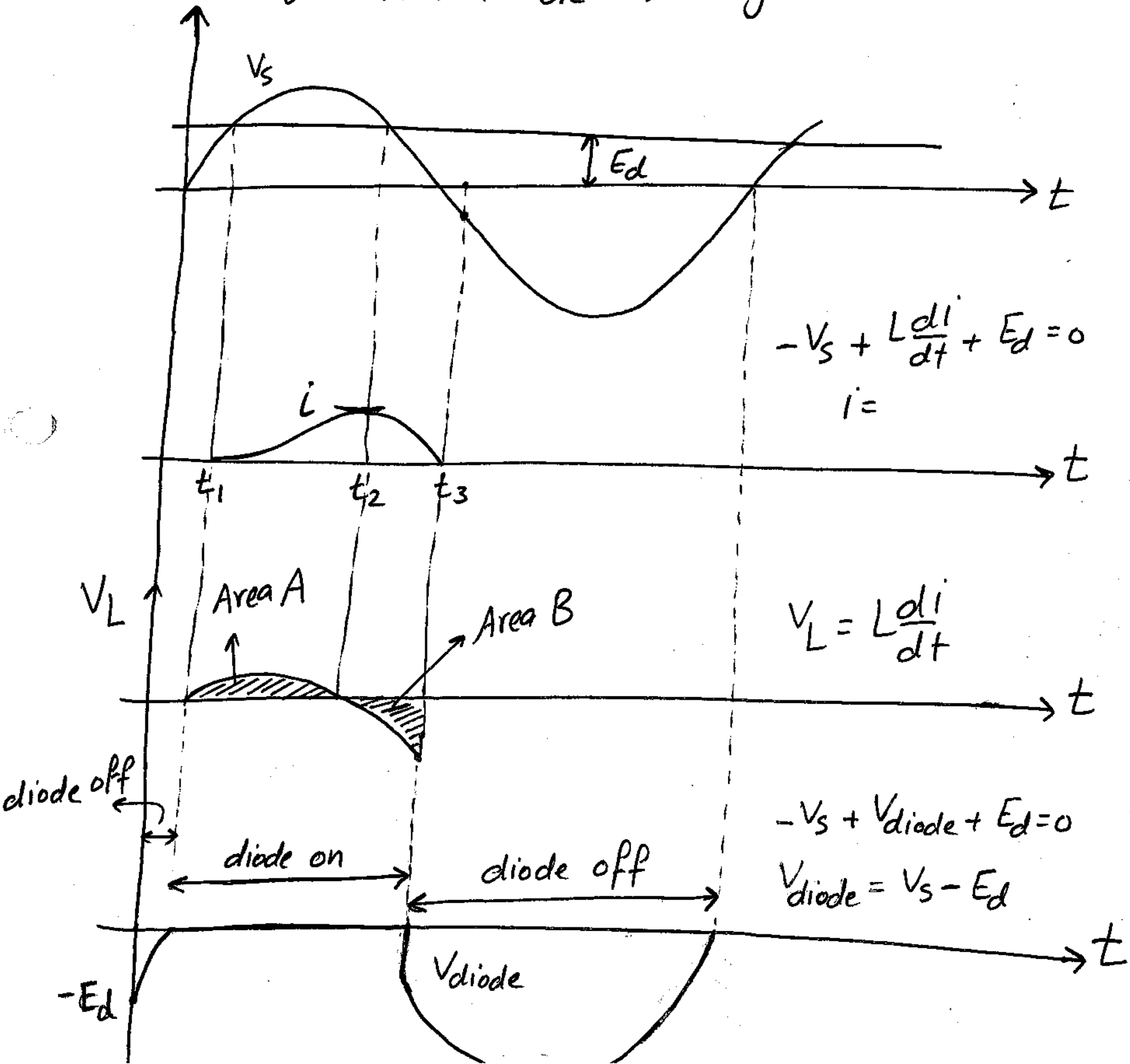


* The diode conducts when $V_s > V_d$

$$V_d = L\frac{di}{dt} + E_d$$

$$V_d = V_L + E_d$$

Half-wave rectifier with pure resistive load and internal dc voltage



$$-V_s + L\frac{di}{dt} + E_d = 0$$

$$i =$$

$$V_L = L\frac{di}{dt}$$

$$-V_s + V_{diode} + E_d = 0$$

$$V_{diode} = V_s - E_d$$

first 07.03.2016

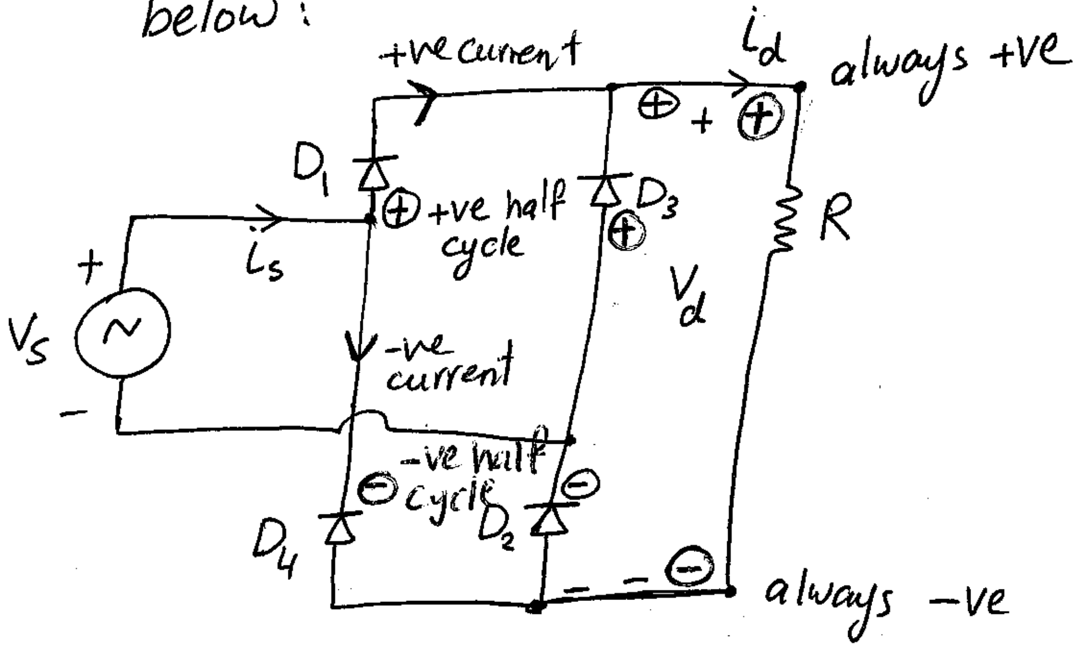
First 03.11.2016

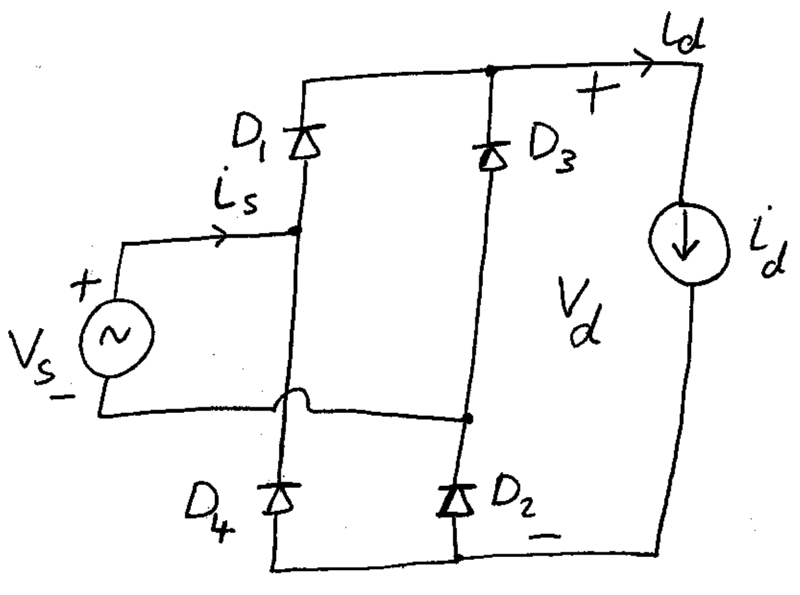
Single-Phase Diode Bridge Rectifiers

(Full-wave rectifier)

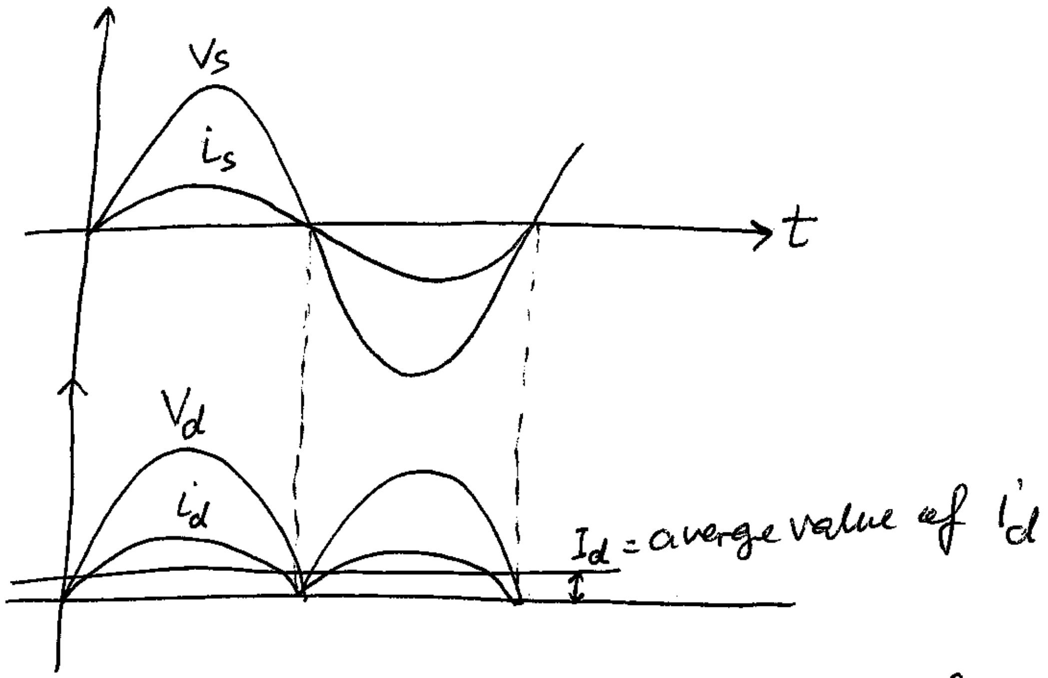
① Idealized circuit with $L_s = 0$

* Replace the dc side of the rectifier by a resistance R or a constant dc current source I_d as shown below:

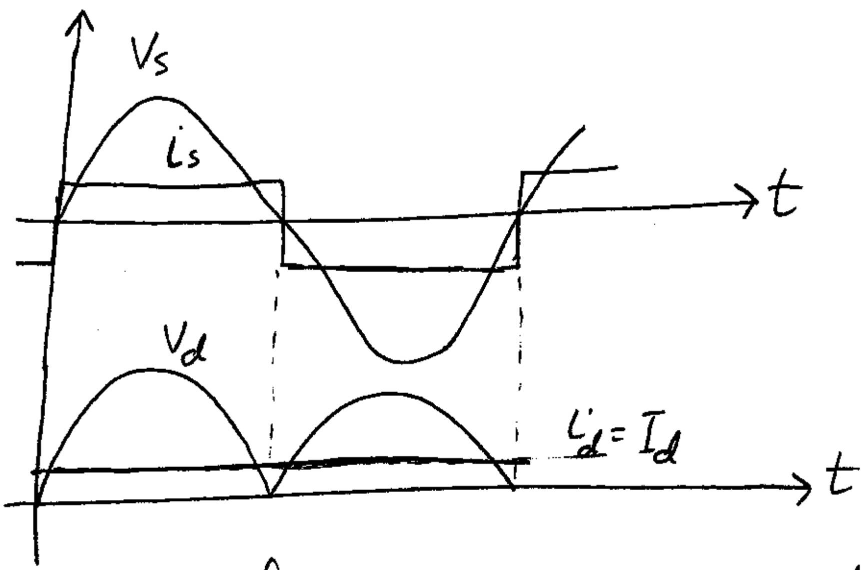




Full-wave rectifier (Bridge rectifier) without L_s and with pure resistive load or constant load current



Waveforms with pure resistive load (full-wave rectifier)



Waveforms with constant current i_d (full-wave rectifier)

* At any time, the dc-side output voltage of the diode rectifier can be expressed as:

$$V_d(t) = |V_s|$$

Similarly, the ac-side current can be expressed as

$$i_s = \begin{cases} i_d & \text{if } V_s > 0 \\ -i_d & \text{if } V_s < 0 \end{cases}$$

58

2 because it is half period

$$V_{do} = \frac{2}{T}\int_0^{T/2} \sqrt{2}V_s \sin\omega t\, dt = \frac{1}{\omega T/2}\sqrt{2}V_s \cos\omega t \Big|_{T/2}^{0} =$$

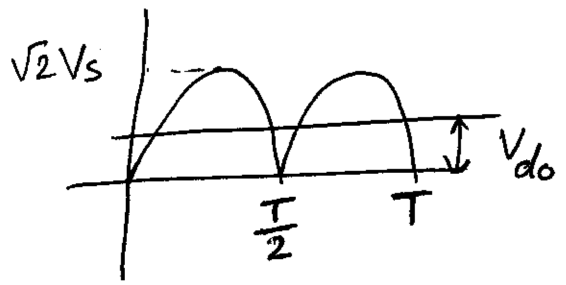
rms value of the input voltage

$$\frac{2}{2\pi}\int_0^{\pi}\sqrt{2}V_s \sin\omega t\, d\omega t$$

$$= \frac{2}{\pi}\sqrt{2}V_s = 0.9V_s$$

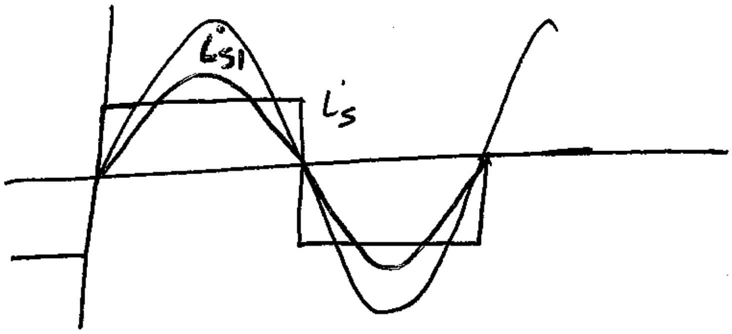
$$V_{do} = \frac{1}{T}\int_0^{T/2}\sqrt{2}V_s\sin\omega t\,dt + \frac{1}{T}\int_{T/2}^{T}\sqrt{2}V_s\sin\omega t\,dt$$

average output voltage where subscript "o" stands for the idealized case with $L_s = 0$.



V_s: rms value of the input voltage

* For the case of a constant current on the load side:



$I_s = I_d$

I_s: rms value of source current.

$$I_{s_1} = \frac{2}{\pi}\sqrt{2}I_d \quad \text{(rms value of the fundamental component)}$$

$$= 0.9\, I_d$$

$$I_{sh} = \begin{cases} 0 & \text{for even values of } h \\ I_{s_1}/h & \text{for odd values of } h \end{cases} \quad \text{(rms value of the components)}$$

$$THD = 100 \times \frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}}$$

$$= 100 \times \frac{\sqrt{I_d^2 - (0.9 I_d)^2}}{0.9 I_d} = 100 \times \frac{\sqrt{1 - 0.9^2}}{0.9}$$

$$= 100 \times \frac{\sqrt{1 - 0.81}}{0.9} = 48.43\%$$

* i_{s1} is in phase with the V_s waveform. Therefore,

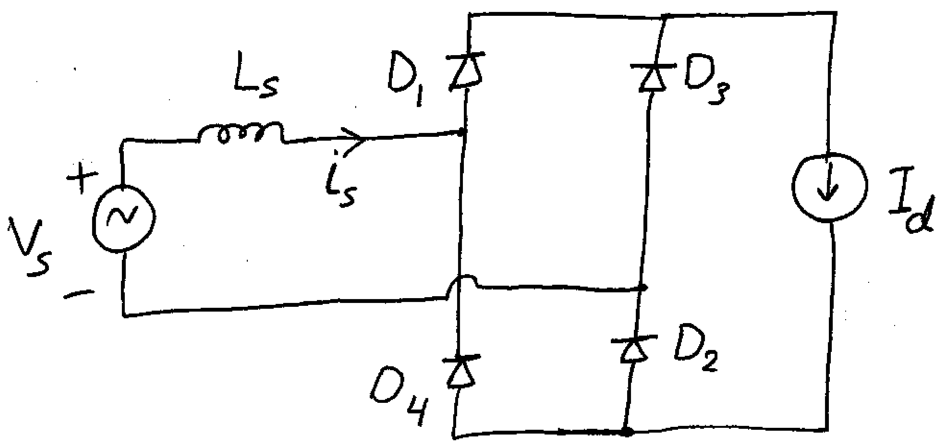
$$DPF = 1.0$$

and $$PF = DPF \frac{I_{s1}}{I_s} = (1.0) \frac{0.9 I_d}{I_d} = 0.9.$$

Effect of L_s on Current Commutation

start with what is available on the ~~next~~ page

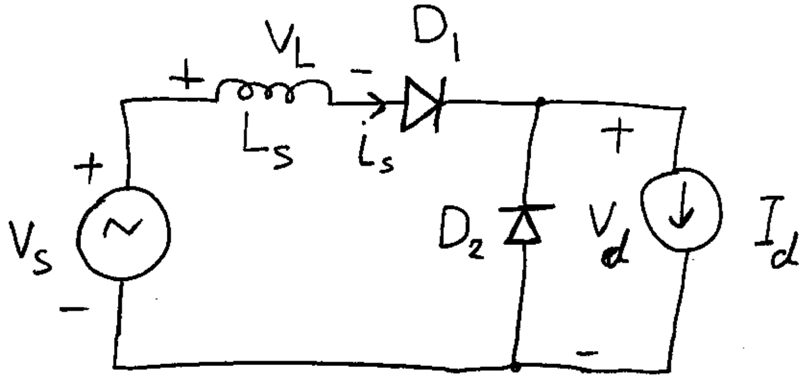
* The dc side is represented by a constant dc current I_d and there is a finite inductance L_s on the ac-side



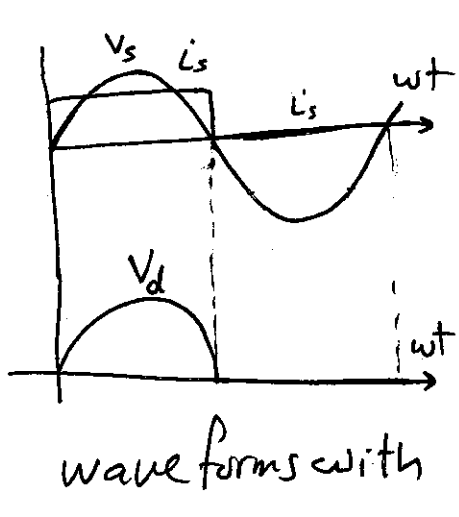
* Due to the finite L_s, the transition of the ac-side current i_s from a value of $+I_d$ to $-I_d$ (or vice versa) will not instantaneous.

* The finite time interval required for the transition from $+I_d$ to $-I_d$ (or vice versa) is called the current Commutation time, or the commutation interval u, and this process where the current conduction shifts from one diode (or a set of diodes) to the other is called the commutation process.

* Consider the following circuit:



with $L_s = 0$



waveforms with $L_s=0$.

* When $\underline{\omega t < 0}$

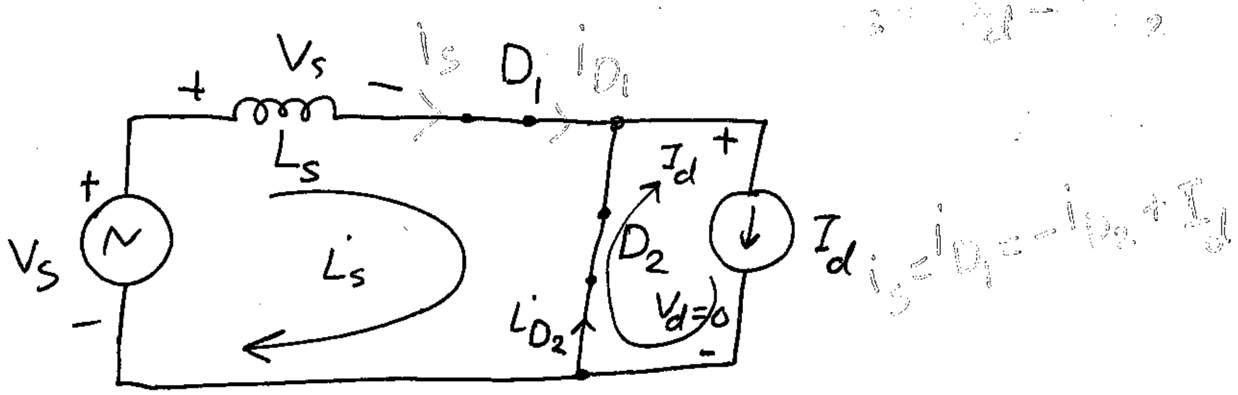
$V_s < 0$, I_d is circulating through D_2,

$V_d = 0$ and $i_s = 0$.

* when $\omega t = 0$, V_s becomes positive,

D_1 conducts (on), i_s starts building up

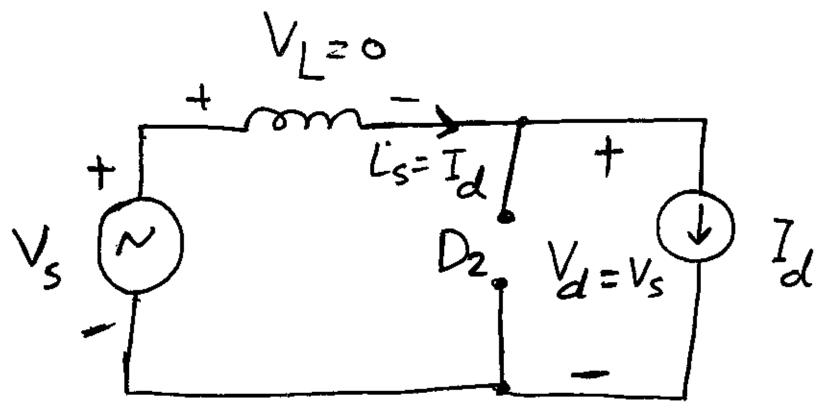
For the interval when $0 < i_s < I_d$, we have the following circuit:



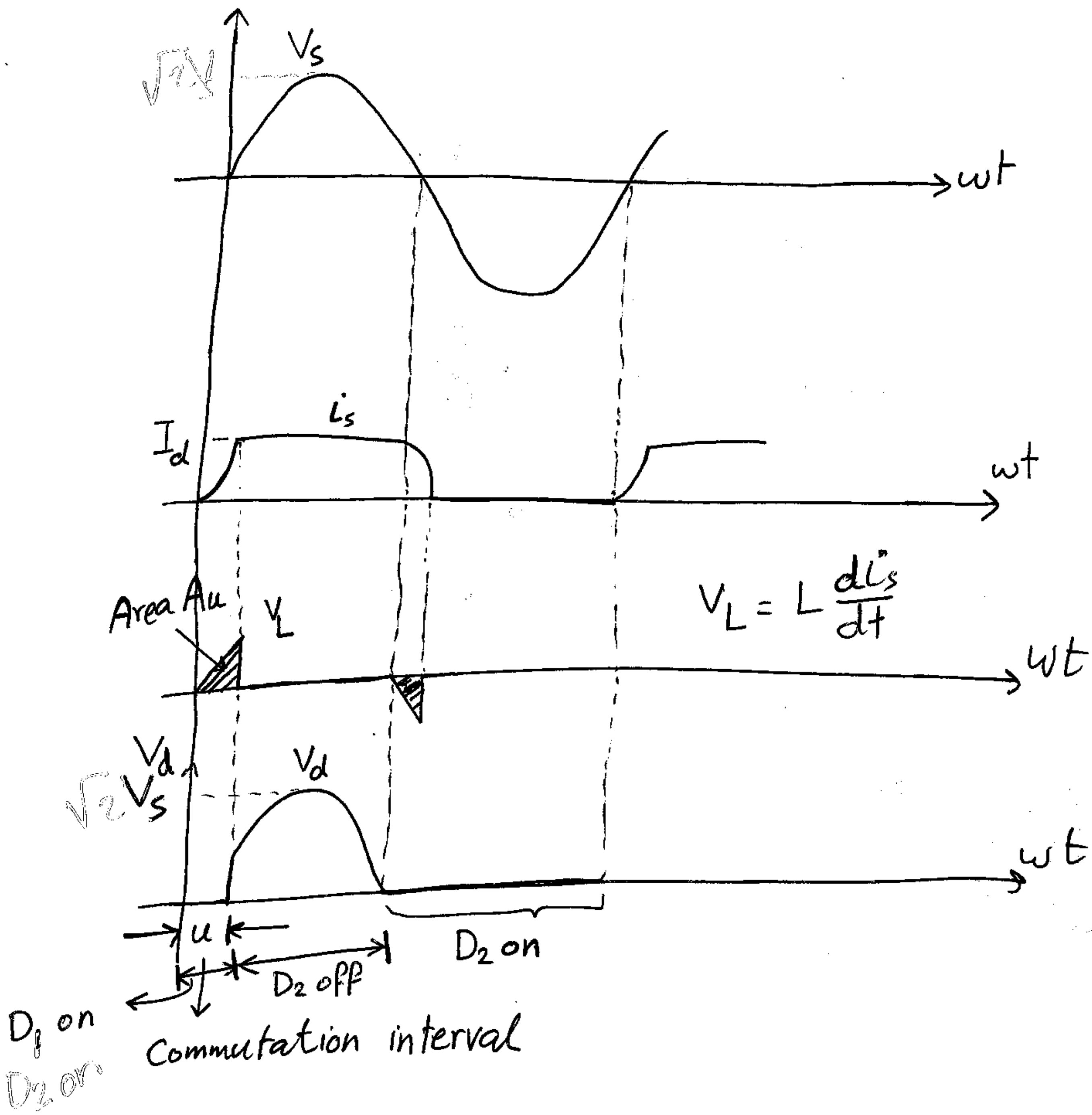
Circuit during the commutation $0 < i_s < I_d$
(starts when $\omega t = 0$ and ends when $i_s = I_d$)

$$i_{D_2} = I_d - i_s \qquad \frac{di_{D_2}}{dt} = -\frac{di_s}{dt}$$

* i_s starts building up till $i_s = I_d$. At this instant $i_{D_2} = 0$ and we will have the following circuit:



$V_L = 0$
+
−
$i_s = I_d$
+
V_s
−
D_2
$V_d = V_s$
−
I_d



$\sqrt{2}V_s$
V_s
ωt
I_d
i_s
ωt
Area A_u
V_L
$V_L = L\frac{di_s}{dt}$
ωt
V_d
$\sqrt{2}V_s$
V_d
ωt
u
D_2 off
D_2 on
D_1 on
D_2 on
Commutation interval

$V_L = \sqrt{2}\, V_s \sin\omega t = L_s \frac{di_s}{dt}$ during the commutation interval $0 < \omega t < u$

$L_s \frac{di_s}{dt} = \omega L_s \frac{di_s}{d(\omega t)}$

$\sqrt{2}\, V_s \sin\omega t \quad = \omega L_s \frac{di_s}{d(\omega t)}$

$$\sqrt{2} V_s \sin\omega t \; d(\omega t) = \omega L_s \, di_s$$

$$\int_0^u \underbrace{\sqrt{2} V_s \sin\omega t}_{V_L \text{ in this interval} = A_u} d(\omega t) = \omega L_s \int_0^{I_d} di_s = \omega L_s I_d$$

$$A_u = \int_0^u \sqrt{2} V_s \sin\omega t \; d(\omega t) = \sqrt{2} V_s \quad \cos\omega t \Big]_u^0$$

$$= \sqrt{2} V_s (1 - \cos u) = \omega L_s I_d$$

$$\cos u = 1 - \frac{\omega L_s I_d}{\sqrt{2} V_s}$$

if $L_s = 0$, then $\cos u = 1$ and therefore $u = 0$.

* The commutation interval u increases with L_s and I_d and decreases with V_s.

* The finite commutation interval reduces the average value of the output voltage. The average value V_{do} of V_d if $L_s = 0$ is:

average value with $L_s=0$

$$V_{do} = \frac{1}{2\pi}\int_0^{\pi} \sqrt{2}V_s \sin\omega t \, d(\omega t) = \frac{2\sqrt{2}}{2\pi}V_s = 0.45V_s$$

with a finite L_s and hence a nonzero u,

$$V_d = \frac{1}{2\pi}\int_u^{\pi} \sqrt{2}V_s \sin\omega t \, d(\omega t)$$

$$= \frac{\sqrt{2}V_s}{2\pi} \cos\omega t \Big]_{\pi}^{u} = \frac{\sqrt{2}V_s}{2\pi}\cos u - \frac{\sqrt{2}V_s}{2\pi}\cos\pi$$

$$\cos u = 1 - \frac{\omega L_s I_d}{\sqrt{2}V_s}$$

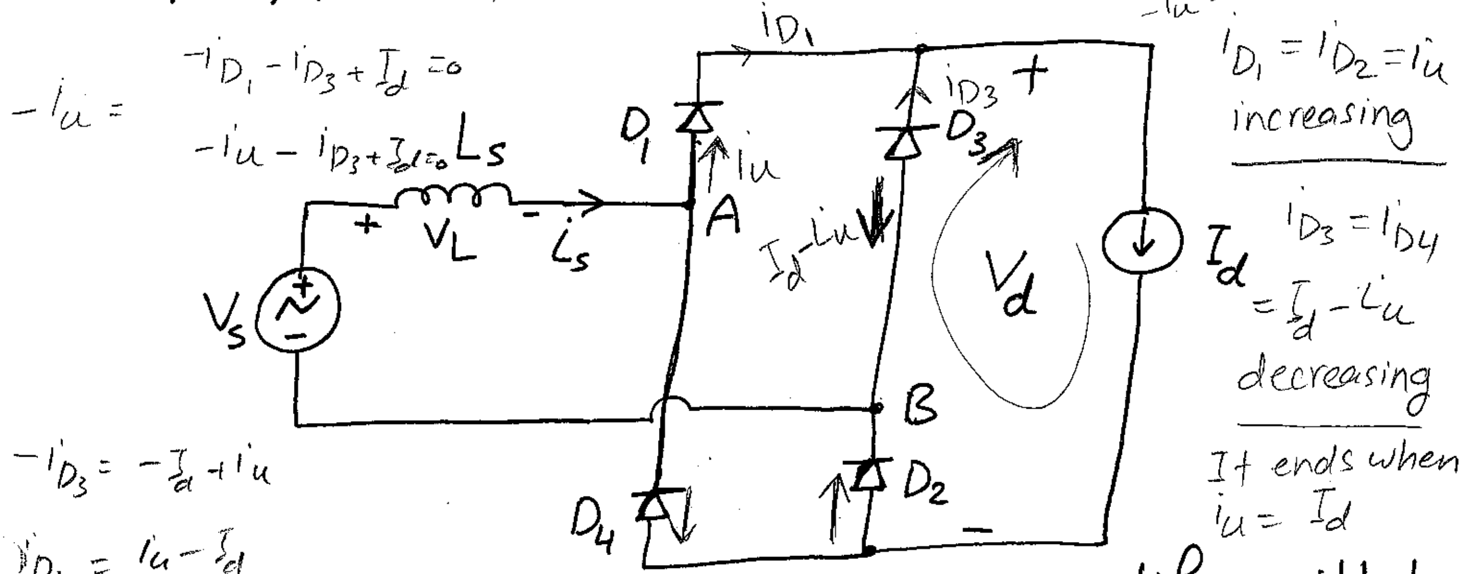
$$= \frac{\sqrt{2}V_s}{2\pi}\cos u + \frac{\sqrt{2}V_s}{2\pi}$$

$$= \frac{\sqrt{2}V_s}{2\pi} + \frac{\sqrt{2}V_s}{2\pi}\left(1 - \frac{\omega L_s I_d}{\sqrt{2}V_s}\right)$$

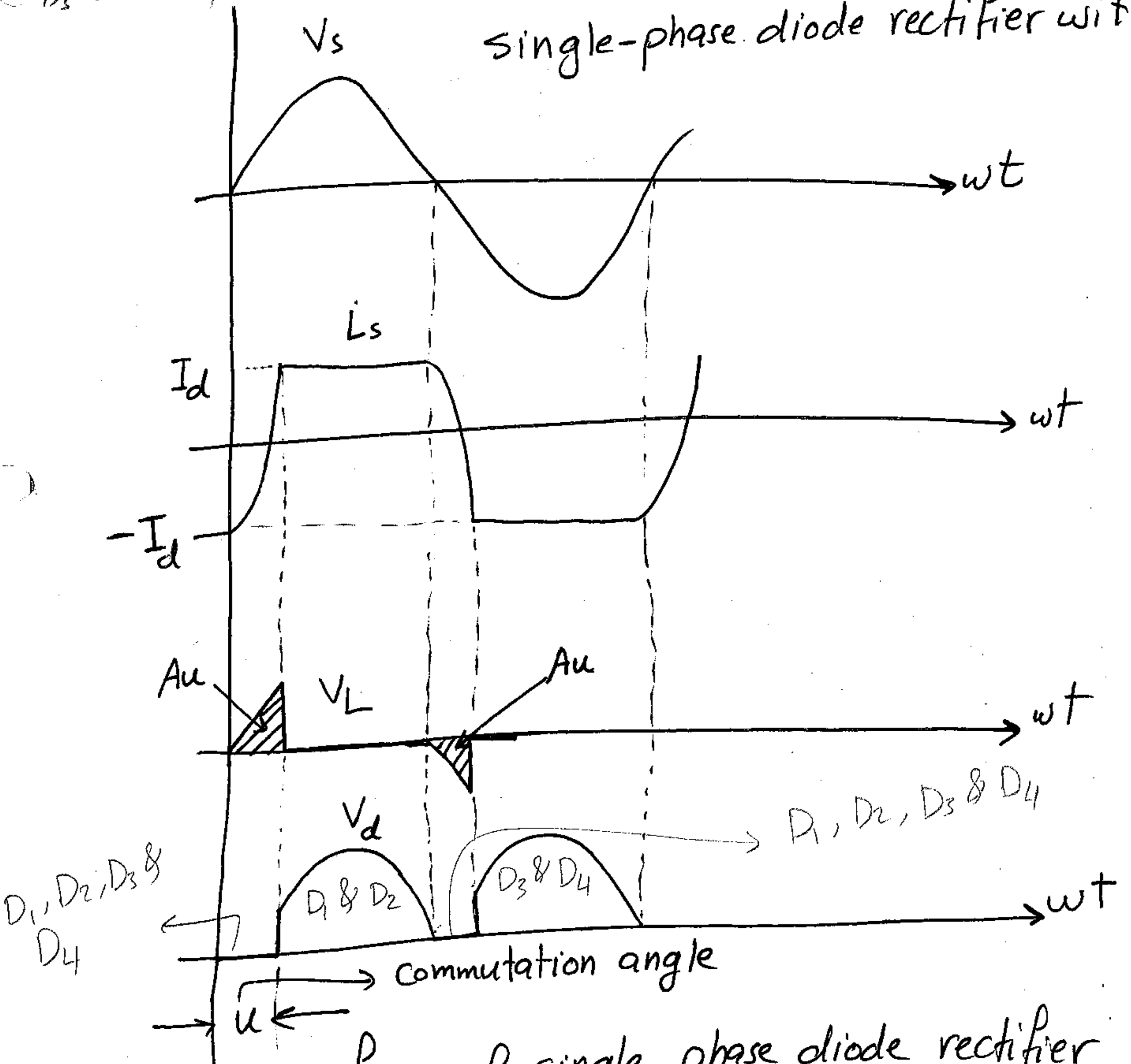
$$= \frac{\sqrt{2}V_s}{2\pi} + \frac{\sqrt{2}V_s}{2\pi} - \frac{\sqrt{2}V_s}{2\pi}\frac{\omega L_s I_d}{\sqrt{2}V_s}$$

$$\boxed{V_d = \frac{2\sqrt{2}V_s}{2\pi} - \frac{\omega L_s I_d}{2\pi} = 0.45V_s - \frac{\omega L_s I_d}{2\pi}}$$

* Now we will extend the analysis to the full-wave rectifier:



single-phase diode rectifier with L_s



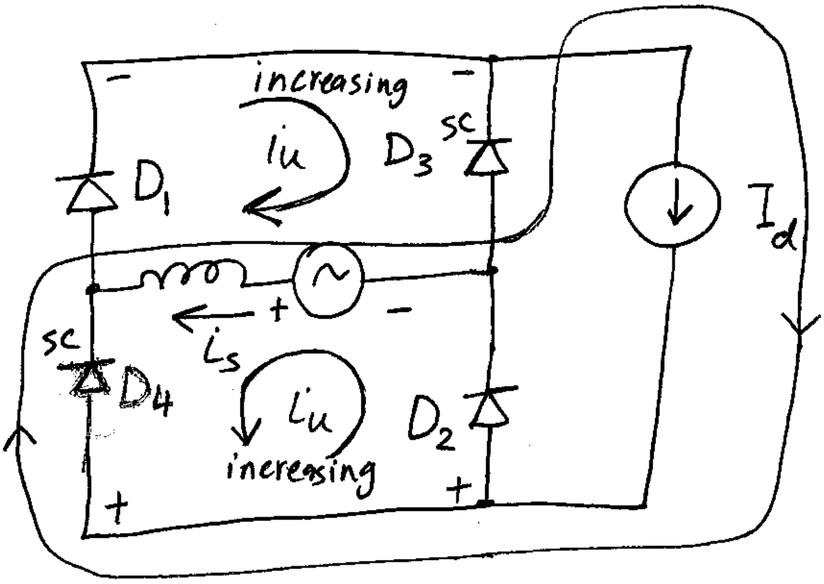
waveforms of single-phase diode rectifier with L_s

$0 < \omega t < u$

D_3 & D_4 are conducting.

$i_s = -I_d$ at $\omega t = 0$ (the beginning of the commutation process)

and we have the following equivalent circuit:



The circuit during the current commutation
$0 < \omega t < u$

diodes D_1 & D_2 are forward biased because of the short-circuit path provided by the conducting diodes 3 and 4 i.e. All diodes are forward biased (short circuits) which means that $V_d = 0$. Therefore,

$i_{D_1} = i_{D_2} = i_u$, $i_{D_3} = i_{D_4} = I_d - i_u$

and $i_s = -I_d + 2i_u$

* i_u builds up from zero at $\omega t = 0$ to I_d at the end of the commutation interval. Therefore, at $\omega t = u$, $i_{D_1} = i_{D_2} = I_d$ and $i_s = I_d$.

* During the commutation of current from diodes 3 and 4 to diodes 1 and 2, the current through the inductor L_s "i_s" changes from $-I_d$ to I_d.

* Coming back to the waveforms:

$$A_u = \int_0^u \sqrt{2} V_s \sin \omega t \, d(\omega t) \quad \text{Volt-radian}$$

$$= \int_0^u L_s \frac{di_s}{dt} d(\omega t) = \omega \int_{-I_d}^{I_d} L_s di_s \frac{dt}{dt}$$

$$= \omega \int_{-I_d}^{I_d} L_s di_s = \omega L_s \int_{-I_d}^{I_d} di_s = \omega L_s 2 I_d$$

$$= 2\omega L_s I_d$$

$$A_u = \int_0^u \sqrt{2} V_s \sin \omega t \, d(\omega t) = \sqrt{2} V_s (1 - \cos u)$$

$$\sqrt{2} V_s (1 - \cos u) = 2\omega L_s I_d \Rightarrow \cos u = 1 - \frac{2\omega L_s}{\sqrt{2} V_s} I_d$$

* The average value with $L_s = 0$ was

$$V_{d_o} = 0.9 V_s$$

with L_s, the ideal case when $L_s = 0$

$$V_d = V_{d_o} - \frac{Au}{\pi} = 0.9V_s - \frac{2\omega L_s I_d}{\pi}$$

or

$$V_d = \frac{2}{2\pi}\int_u^{\pi} \sqrt{2}V_s \sin\omega t \, d(\omega t) \Rightarrow \text{This for the two periods}$$

$$= \frac{2}{2\pi}\sqrt{2}V_s \left(-\cos\omega t\right)\Big]_u^{\pi}$$

$$= \frac{-2}{2\pi}\sqrt{2}V_s\left[\cos\pi - \cos u\right] = \frac{2\sqrt{2}V_s}{2\pi}\left(1+\cos u\right)$$

$$V_d = -2\frac{\sqrt{2}V_s}{2\pi}\left(-1-\cos u\right) = \frac{2\sqrt{2}V_s}{2\pi}\left(1+\cos u\right)$$

$$= \frac{2\sqrt{2}V_s}{2\pi} + \frac{2\sqrt{2}V_s}{2\pi}\left(1 - \frac{2\omega L_s}{\sqrt{2}V_s}I_d\right)$$

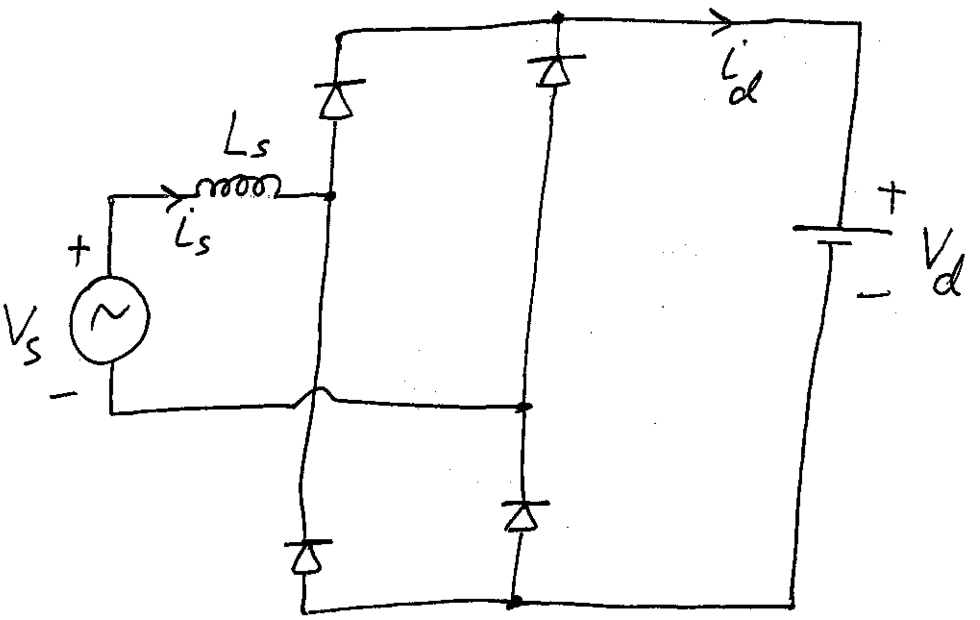
$$= \frac{2\sqrt{2}V_s}{2\pi} + \frac{2\sqrt{2}V_s}{2\pi} - \frac{2\sqrt{2}V_s}{2\pi}\,\frac{2\omega L_s}{\sqrt{2}V_s}I_d$$

$$V_d = \frac{4\sqrt{2}V_s}{2\pi} - \frac{4\omega L_s}{2\pi}I_d = \frac{2\sqrt{2}V_s}{\pi} - \frac{2\omega L_s}{\pi}I_d$$

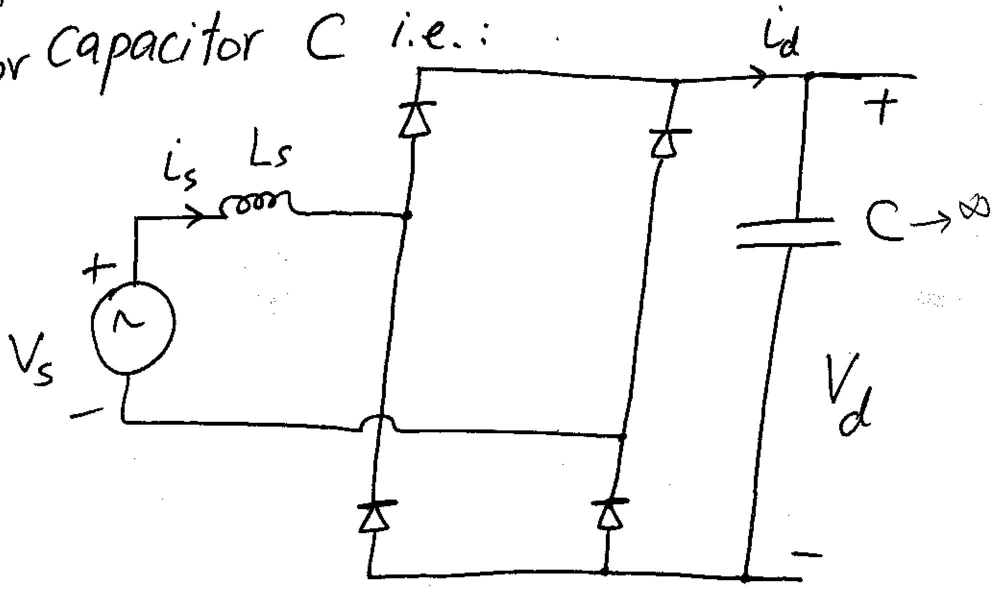
$$= 0.9V_s - \frac{2\omega L_s}{\pi}I_d$$

Constant dc– side Voltage $V_d(t) = V_d$

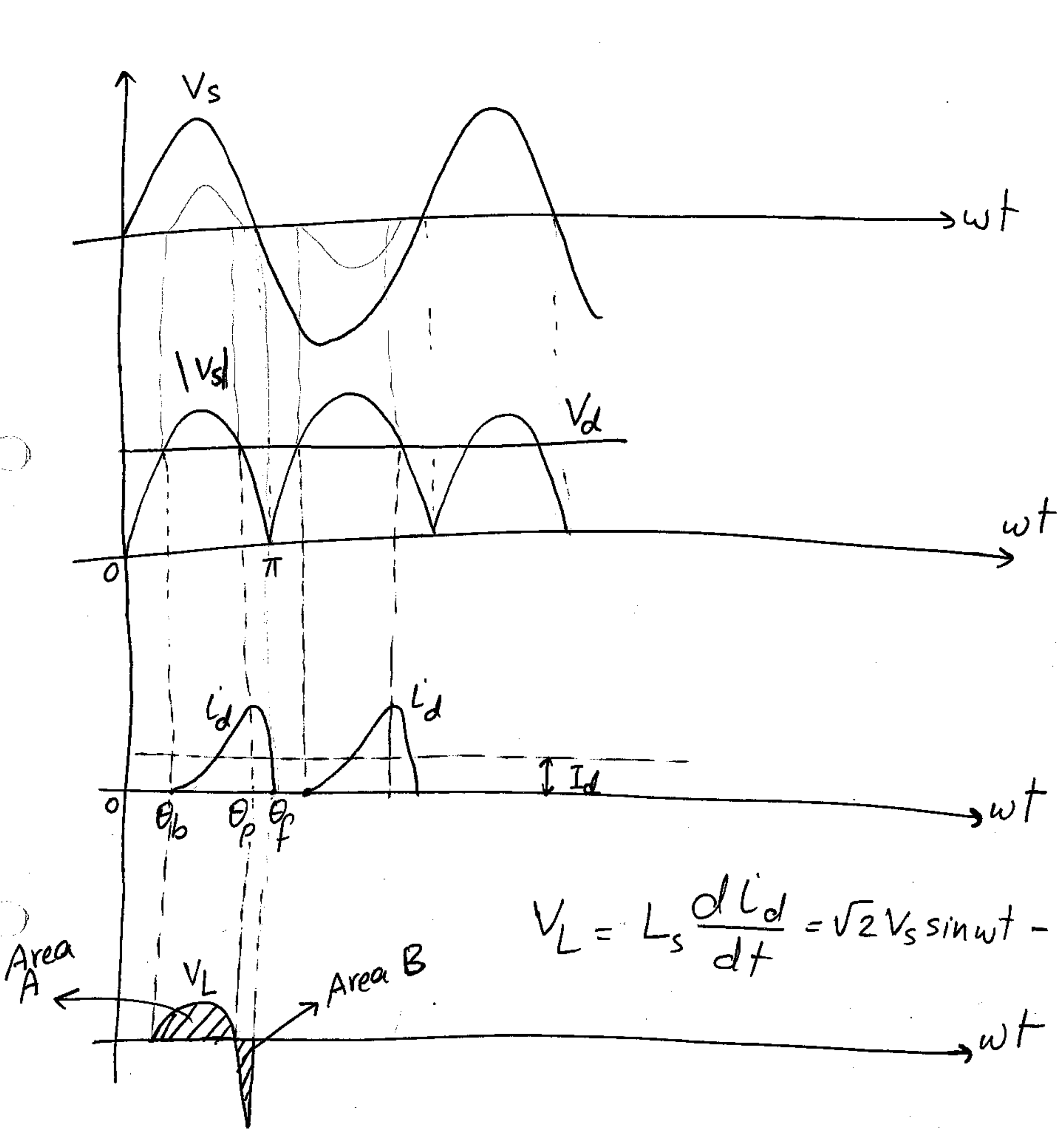
Now, we will consider the following circuit:



Constant V_d is equivalent to the case if there is no constant voltage source but with a large value for capacitor C i.e.:



The waveforms of this circuit are:



$$V_L = L_s \frac{d i_d}{dt} = \sqrt{2} V_s \sin \omega t - V_d$$

Now, we want to find the average value of i_d

I_d is:

$$I_d = \frac{1}{\pi} \int_{\theta_b}^{\theta_f} i_d(\theta)\, d\theta$$

θ_b can be calculated as:

$$V_d = \sqrt{2} V_s \sin \theta_b$$

$$\theta_b = \sin^{-1}\left(\frac{V_d}{\sqrt{2} V_s}\right), \quad \theta_{b_1}, \theta_{b_2}, \theta_{b_3}, \ldots$$

$$\theta_b, \theta_p, \theta_b + \pi, \ldots$$

and $\theta_b + \theta_p = \pi$

$$\theta_p = \pi - \theta_b$$

V_L can be expressed as:

$$V_L = L_s \frac{di_d}{dt} = \sqrt{2} V_s \sin\omega t - V_d \quad \text{because}$$

$$V_L + V_d = V_s$$

$$\int V_L d(\omega t) = \int L_s \frac{di_d}{dt} d(\omega t)$$

$$\int_{\theta_b}^{\theta} (\sqrt{2} V_s \sin\omega t - V_d)\, d\omega t = \omega L_s \int_{\theta_b}^{\theta} di_d \,, \quad \theta > \theta_b$$

$$\int_{\theta_b}^{\theta} (\sqrt{2} V_s \sin\omega t - V_d)\, d\omega t = \omega L_s \left[i_d(\theta) - \underbrace{i_d(\theta_b)}_{\text{Zero}} \right]$$

but $i_d(\theta_b) = 0$

$$\therefore \quad i_d(\theta) = \frac{1}{\omega L_s}\int_{\theta_b}^{\theta} \underbrace{(\sqrt{2}\,V_s \sin\omega t - V_d)}_{V_L}\, d\omega t$$

Now, $i_d(\theta_f) = 0$

$$\therefore \quad i_d(\theta_f) = 0 = \int_{\theta_b}^{\theta_f} (\sqrt{2}\,V_s \sin\omega t - V_d)\, d\omega t \Longrightarrow \theta_f \text{ can be calculated}$$

Now, θ_b & θ_f are known. Then

$$I_d = \frac{1}{\pi}\int_{\theta_b}^{\theta_f} i_d(\theta)\, d\theta$$

$$I_d = \frac{1}{\pi}\int_{\theta_b}^{\theta_f}\left[\frac{1}{\omega L_s}\int_{\theta_b}^{\theta}(\sqrt{2}\,V_s \sin\omega t - V_d)\, d\omega t\right] d\theta$$

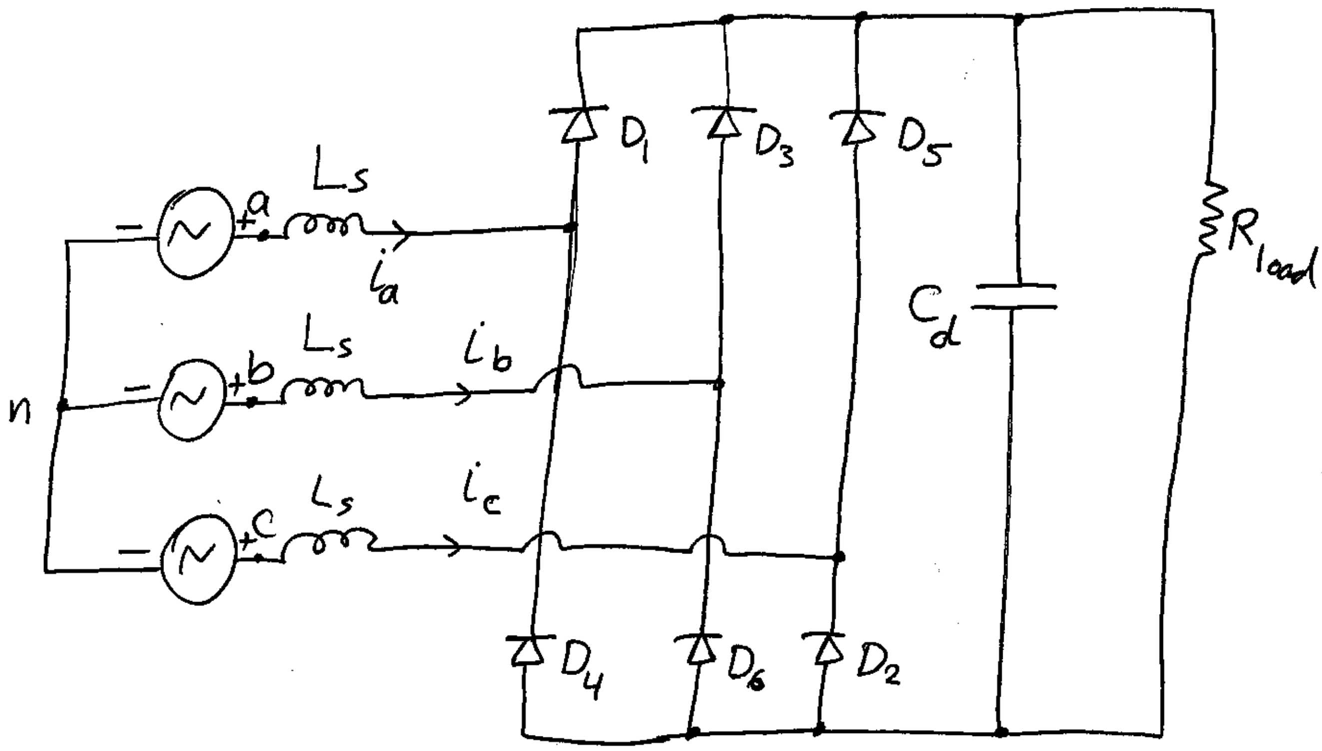
Assignment:

Get Fig. 5-17.

Three-phase, Full-bridge Rectifiers

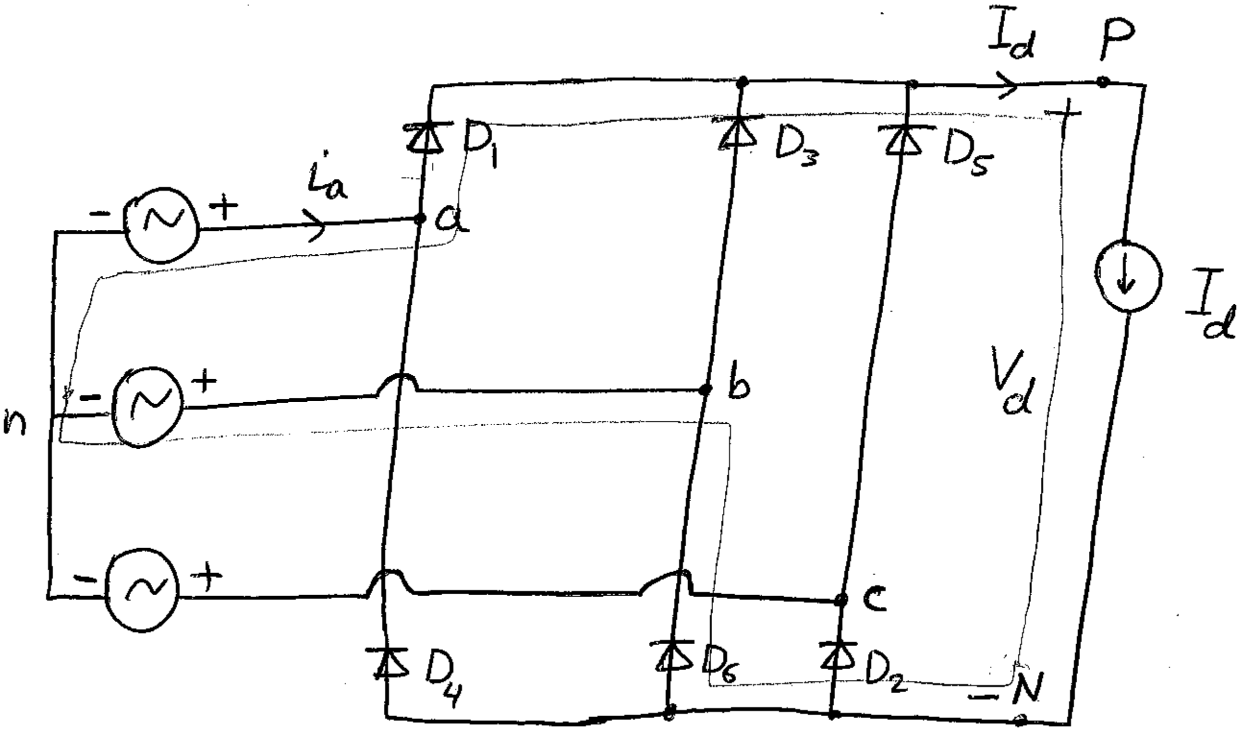
Three-phase rectifier circuits have, compared to single-phase rectifiers:

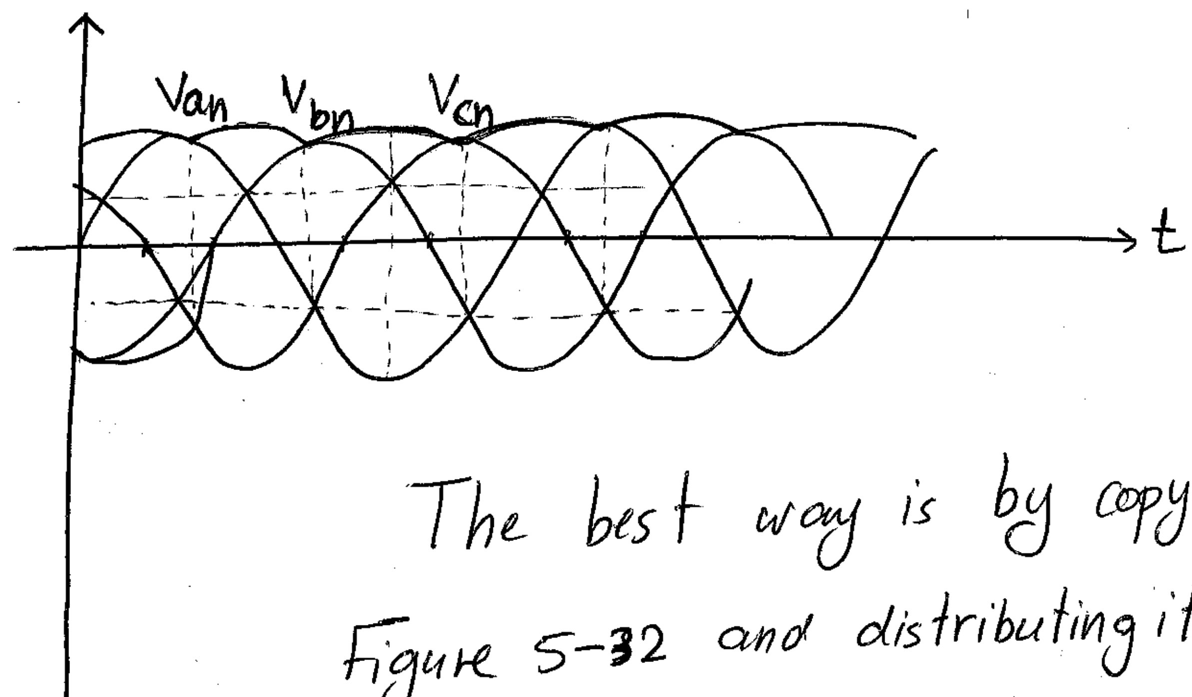
1. lower ripple content in the waveforms.
2. higher power-handling capability.



Three-phase six-pulse full-bridge diode rectifier

* For the idealized case with $L_s = 0$ and a constant dc current I_d instead of load resistance.





The best way is by copying Figure 5-32 and distributing it for students.

* Six-pulse rectifier because the instanteneous waveform of V_d consists of six segments per cycle. of line frequency.

$$i_a = \begin{cases} I_d & \text{when diode 1 is conducting} \\ -I_d & \text{when diode 4 is conducting} \\ 0 & \text{when neither diode 1 or 4 is conducting} \end{cases}$$

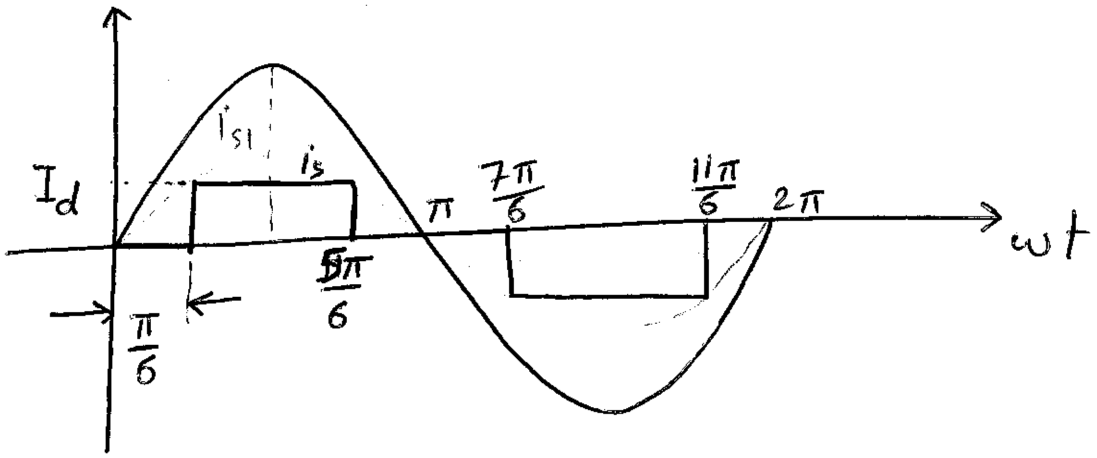
* The commutation of current from one diode to the next is instantaneous based on the assumption of $L_s = 0$.

* from Figure 5-32 :

$$V_d(t) = V_{ab} = \sqrt{2}\, \underset{\substack{\downarrow \\ \text{rms value of line-to-line} \\ \text{Voltages}}}{V_{LL}} \cos \omega t \qquad -\frac{\pi}{6} < \omega t < \frac{\pi}{6}$$

$$V_{do} = \frac{1}{\pi/3} \int_{-\pi/6}^{\pi/6} \sqrt{2}\, V_{LL} \cos \omega t \, d(\omega t) = \frac{3}{\pi} \sqrt{2}\, V_{LL} = 1.35\, V_{LL}$$

*one of the phase voltages and the corresponding phase current are shown below:



$\frac{12\pi}{6} - \frac{\pi}{6}$

$\frac{11}{6}$

$$I_s = \sqrt{\frac{1}{T}\int_0^T (i_s(t))^2\,dt}$$ rms value of i_s

$\frac{6\pi}{6} - \frac{\pi}{6}$

$\frac{6\pi - 1}{6}$ $\frac{\pi}{6}$

$$= \sqrt{\frac{1}{2\pi}\int_{\pi/6}^{5\pi/6} I_d^2\,d\omega t + \frac{1}{2\pi}\int_{\frac{7\pi}{6}}^{11\pi/6} I_d^2\,d\omega t}$$

$$= \sqrt{\frac{2}{3}}\, I_d = 0.816\, I_d$$

As for the Fourier analysis:

$$i_s(t) = I_o + \sum_{h=1}^{\infty}\left\{a_h\cos(h\omega t) + b_h\sin(h\omega t)\right\}$$

$$I_o = \frac{1}{2}a_o = 0$$

$a_h = 0$ for all h.

$$b_h = \frac{1}{\pi}\int_0^{2\pi} i_s(\omega t)\sin(h\omega t)\,d\omega t$$

$$= \frac{1}{\pi}\int_{\pi/6}^{5\pi/6} I_d \sin(h\omega t)\,d\omega t + \frac{1}{\pi}\int_{7\pi/6}^{11\pi/6} -I_d \sin(h\omega t)\,d\omega t$$

$$= \frac{I_d}{h\pi}\left[\cos(h\omega t)\Big|_{5\pi/6}^{\pi/6} + \cos(h\omega t)\Big|_{7\pi/6}^{11\pi/6}\right]$$

$$= \frac{I_d}{h\pi}\left[\cos\left(h\frac{\pi}{6}\right) - \cos\left(h\frac{5\pi}{6}\right) + \cos\left(h\frac{11\pi}{6}\right) - \cos\left(h\frac{7\pi}{6}\right)\right]$$

$b_1 = 1.1027\, I_d$

$b_2 = 0$

$b_3 = 0$

$b_4 = 0$

$b_5 = -0.22053\, I_d$

$b_6 = 0$

$b_7 = -0.15753\, I_d$

* $I_{sh} = \frac{I_{s1}}{h}$, where $h = 5, 7, 11, 13, \ldots$

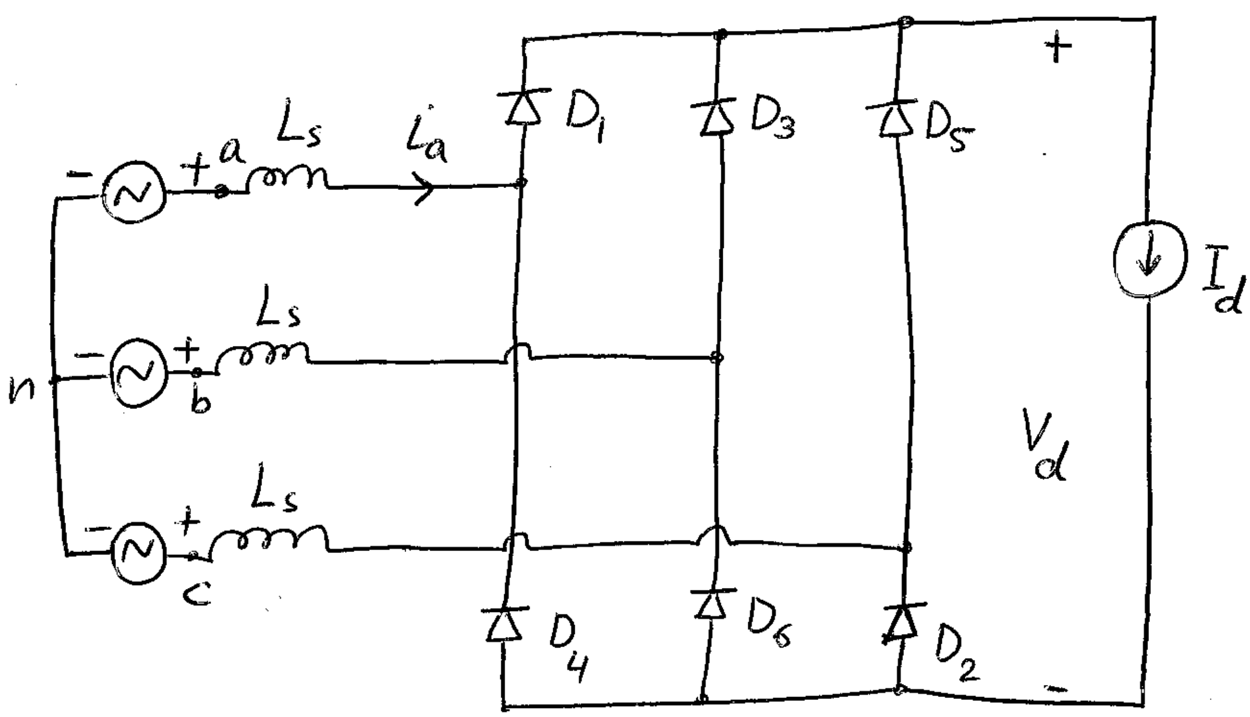
* Since i_{s_1} is in phase with its utility voltage

$$DPF = 1.0$$

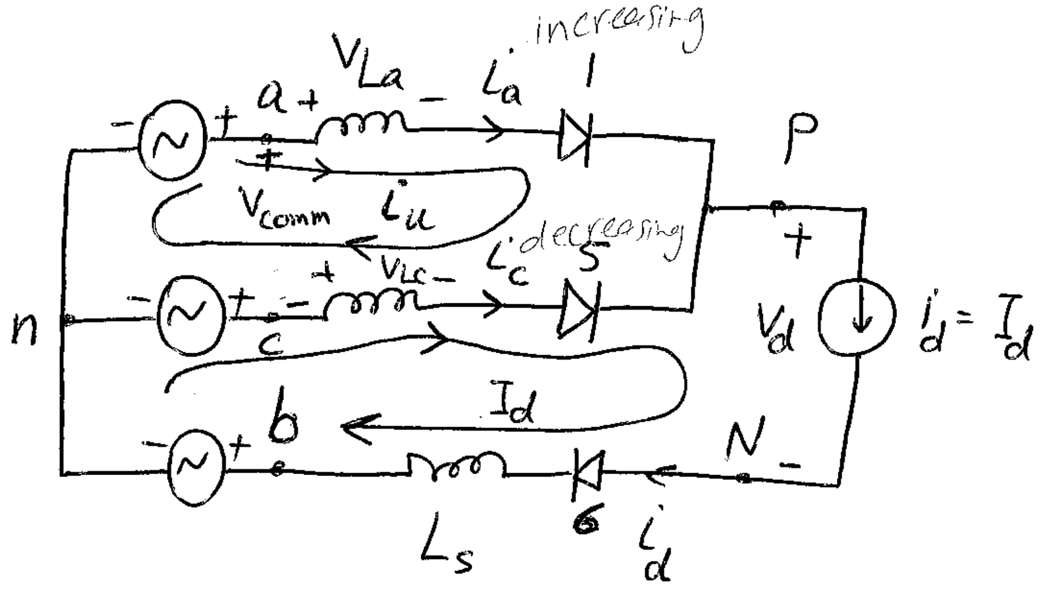
$$PF = \frac{I_{s1}}{I_s} DPF = \frac{1.1027/\sqrt{2}\, I_d}{0.816\, I_d}$$

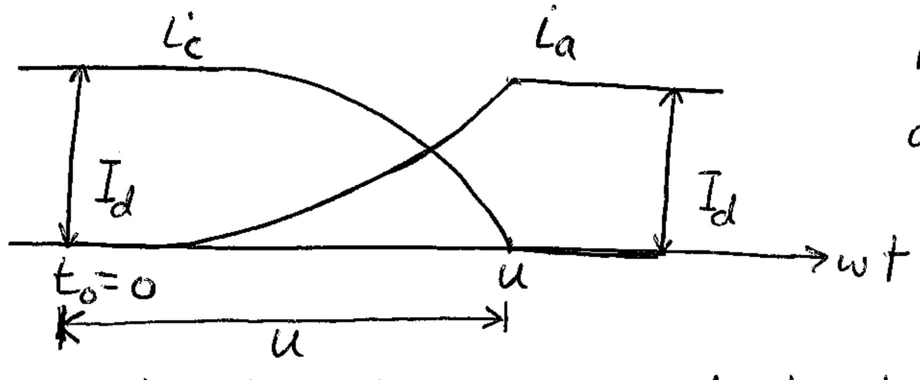
$$= \frac{0.782}{0.816} = 0.955$$

Effect of L_s on Current Commutation



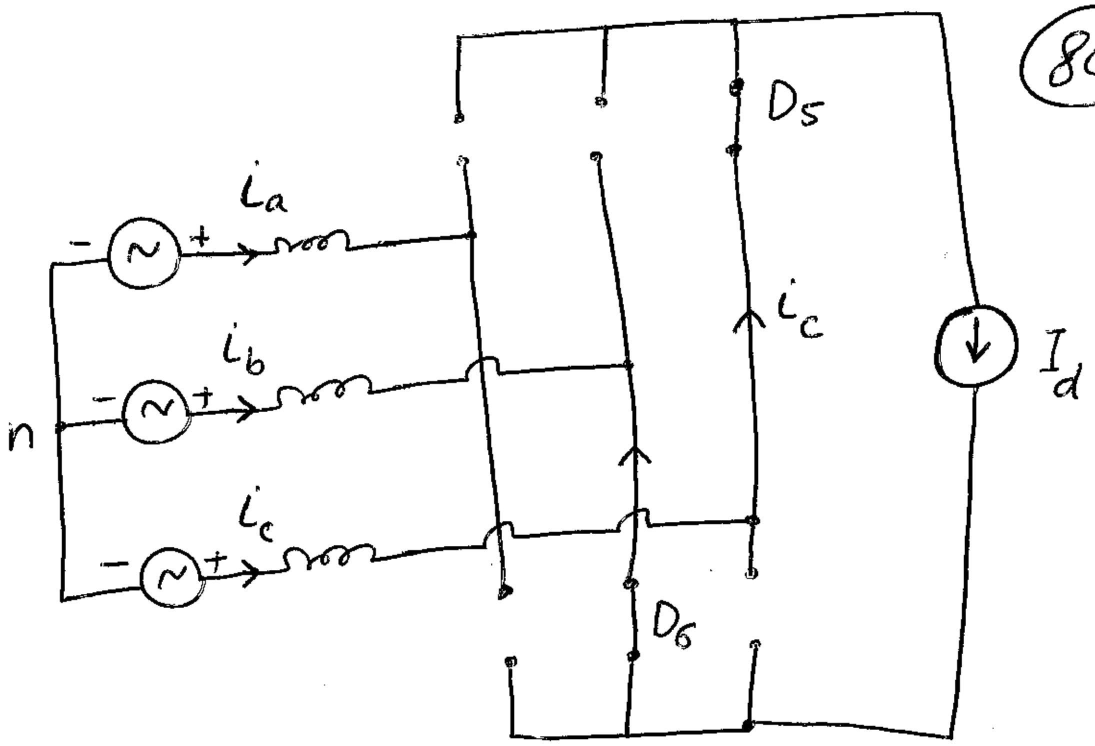
* Consider the commutation of current from diode 5 to diode 1 at $t=0$ or $\omega t = 0$. Before this i_d is flowing through diodes 5 and 6:





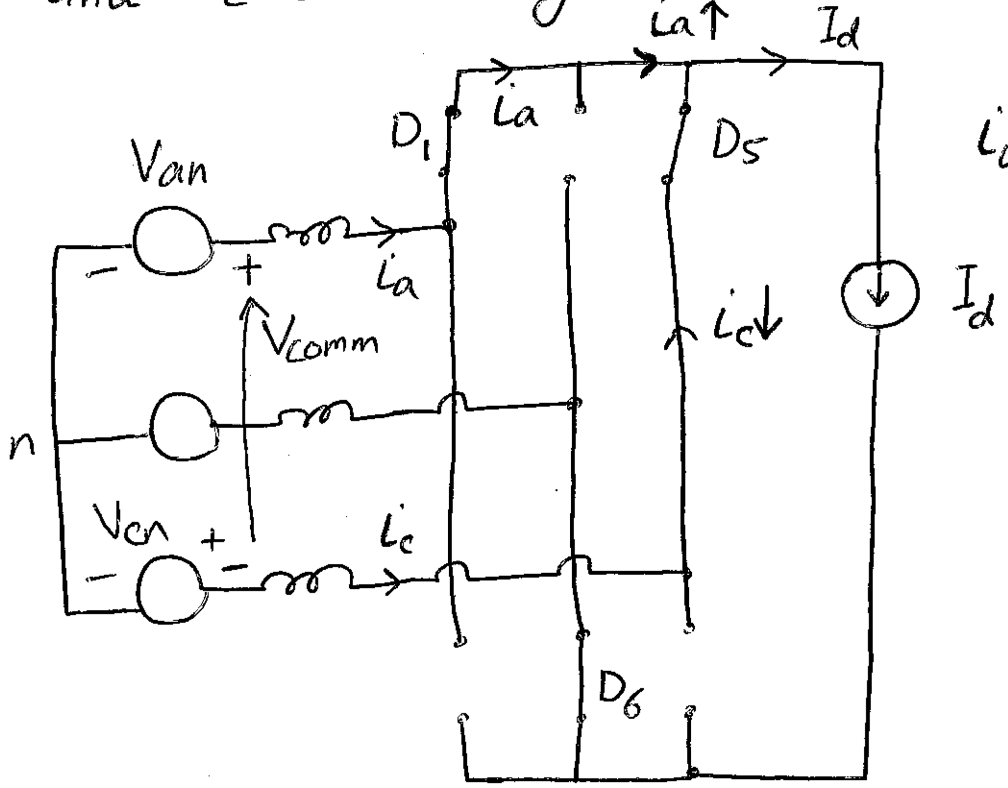
i_c start decreasing & i_a start increasing

At $t_o \Rightarrow$ i_c start decreasing and i_a start increasing (Before this during the steady state, i_c was flowing through D_5 and D_6 as shown below):



steady state circuit

During the commutation, i_a is increasing and i_c is decreasing as shown below:



$i_u = i_a$

$i_a = i_u$

The commutation circuit

During the commutation :

$$-V_{comm} + V_{an} - V_{cn} = 0$$

$$V_{comm} = V_{an} - V_{cn}$$

$$i_a = i_u \quad \&$$

$$i_c = I_d - i_u$$

i_u is building up from zero up to I_d at the end of the commutation interval $\omega t_u = u$.

$$V_{La} = L_s \frac{di_a}{dt} = L_s \frac{di_u}{dt}$$

and

$$V_{Lc} = L_s \frac{di_c}{dt} = -L_s \frac{di_u}{dt}$$

$$V_{comm} = V_{an} - V_{cn} = V_{La} - V_{Lc}$$

$$= L_s \frac{di_u}{dt} + L_s \frac{di_u}{dt} = V_{an} - V_{cn}$$

$$V_{comm} = 2L_s \frac{di_u}{dt} = V_{an} - V_{cn}$$

$$L_s \frac{di_u}{dt} = \frac{V_{an} - V_{cn}}{2}$$

$$\omega L_s \frac{di_u}{dt} = \omega \; \frac{V_{an} - V_{cn}}{2}$$

$$\omega L_s \, di_u = \frac{V_{an} - V_{cn}}{2} \omega \, dt$$

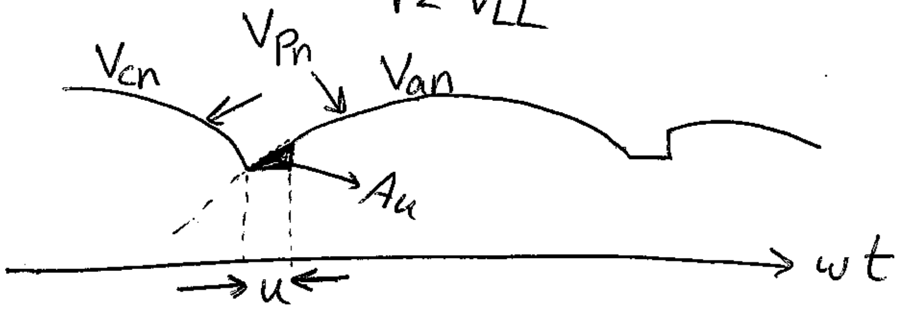
$$\int_0^{I_d} \omega L_s \, di_u = \int_0^{u} \frac{V_{an} - V_{cn}}{2} d(\omega t)$$

$$\omega L_s \int_0^{I_d} di_u = \int_0^{u} \frac{1}{2} \sqrt{2} V_{LL} \sin \omega t \; d(\omega t)$$

$$\omega L_s I_d = \frac{\sqrt{2}}{2} V_{LL} \cos(\omega t) \Big]_u^{0}$$

$$\omega L_s I_d = \frac{\sqrt{2}}{2} V_{LL} \left[\cos(0) - \cos u \right]$$

$$\cos u \doteq 1 - \frac{2 \omega L_s I_d}{\sqrt{2} V_{LL}}$$



$$V_{pn} = V_{an} - L_s \frac{di_u}{dt} = \frac{V_{an} + V_{cn}}{2}$$

$V_d = V_{do} - \Delta V_d \Rightarrow$ The average value of V_d

V_{do} : the average value with $L_s = 0$ (no commutation)

ΔV_d : the voltage drop due to commutation

$$A_u = \int_0^u \frac{V_{an} - V_{cn}}{2} \, d(\omega t) = \omega L_s I_d$$

$$\Delta V_d = \omega L_s I_d / (\pi/3)$$

$$\Delta V_d = \frac{3}{\pi} \omega L_s I_d$$

$$V_d = 1.35 V_{LL} - \frac{3}{\pi} \omega L_s I_d$$

problems

5-3 PP. 114

The voltage $v(t)$ across a load and the current $i(t)$ into the +ve terminal are as:

$$v(t) = V_d + \sqrt{2}\, V_1 \cos(\omega_1 t) + \sqrt{2}\, V_1 \sin(\omega_1 t) + \sqrt{2}\, V_3 \cos(\omega_3 t) \quad V$$

$$i(t) = I_d + \sqrt{2}\, I_1 \cos(\omega_1 t) + \sqrt{2}\, I_3 \cos(\omega_3 t - \phi_3) \; A$$

(a) $P \Rightarrow ?$ the average power supplied to the load

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T v(t)\, i(t)\,dt$$

$$= \frac{1}{T}\int_0^T \left[V_d + \sqrt{2}\, V_1 \cos(\omega_1 t) + \sqrt{2}\, V_1 \sin(\omega_1 t) + \sqrt{2}\, V_3 \cos(\omega_3 t)\right]\left[I_d + \sqrt{2}\, I_1 \cos(\omega_1 t) + \sqrt{2}\, I_3 \cos(\omega_3 t - \phi_3)\right] dt$$

$$P = V_d I_d + V_1 I_1 + V_3 I_3 \cos\phi_3$$

(b) $V = \sqrt{V_d^2 + V_1^2 + V_1^2 + V_3^2}$

$I = \sqrt{I_d^2 + I_1^2 + I_3^2}$

(c) $S = VI$

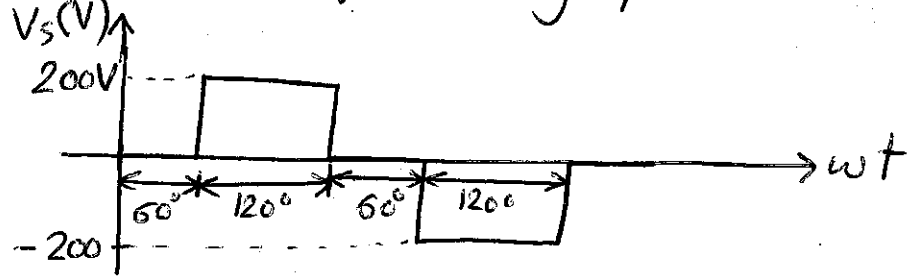
$P = S\ PF$

$$PF = \frac{P}{S} = \frac{V_d I_d + V_1 I_1 + V_3 I_3 \cos\phi_3}{\sqrt{V_d^2 + V_1^2 + V_1^2 + V_3^2}\sqrt{I_d^2 + I_1^2 + I_3^2}}$$

problem 5-4 For the following circuit, $I_d = 10A$ Calculate P if:

(a) V_s is sinusoidal with $V_s = 120V$ & $60Hz$

(b) V_s is the following pulse



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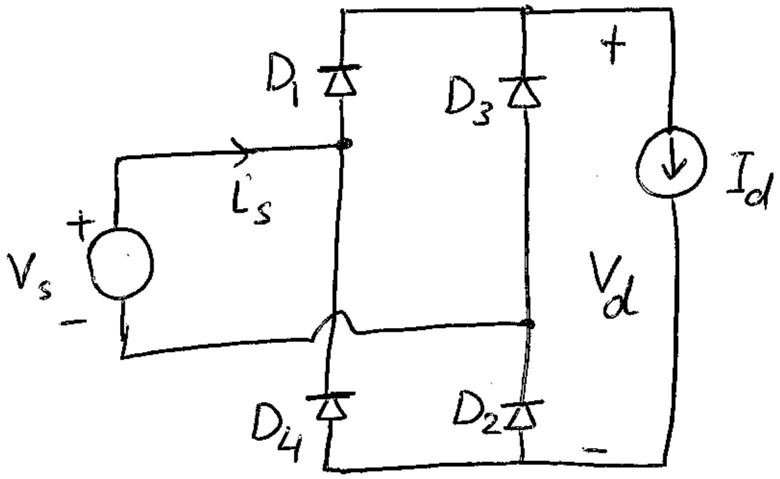
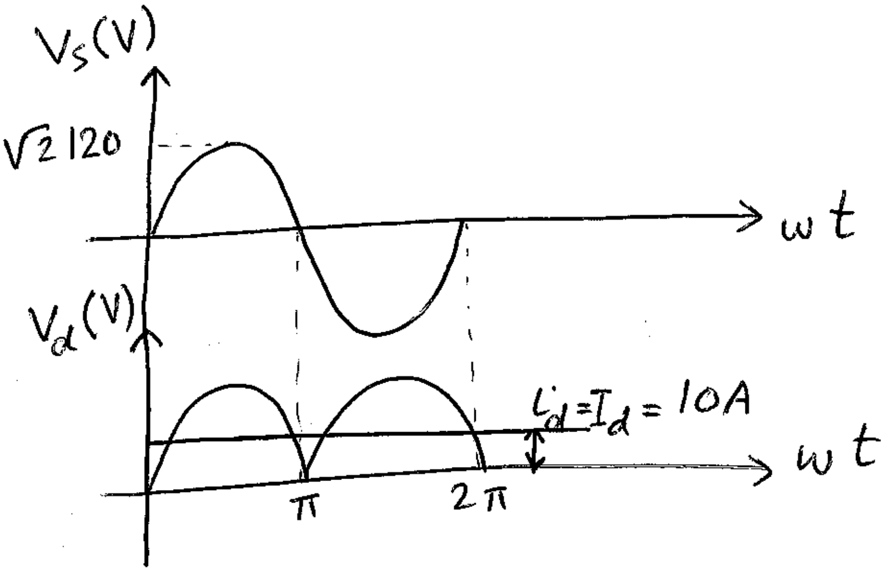


(a)



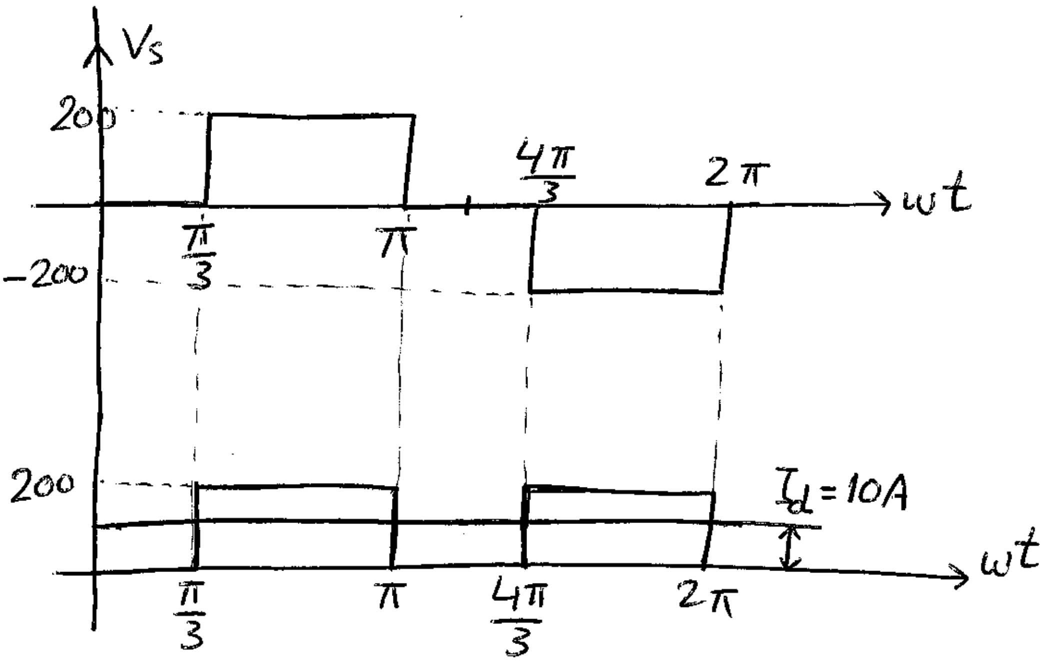
$$P = \frac{1}{T}\int_0^T v(t)\, i(t)\, dt$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} \sqrt{2}\,120 \sin(\omega t)\, d\omega t\,(10) + \int_{\pi}^{2\pi} \sqrt{2}\,120 \sin(\omega t)\, d(\omega t)\,(10)\right]$$

$$= \frac{(\sqrt{2}\,120)(10)}{2\pi}\left[\int_0^{\pi} \sin(\omega t)\, d(\omega t) + \int_{\pi}^{2\pi} \sin(\omega t)\, d(\omega t)\right]$$

$$= \left(\underbrace{0.9\, V_s}\right)(10) = (0.9)(120)(10) = 1080\ \text{W}$$

(b)



$$P = \frac{1}{T}\int_0^T v(t)\, i(t)\, dt$$

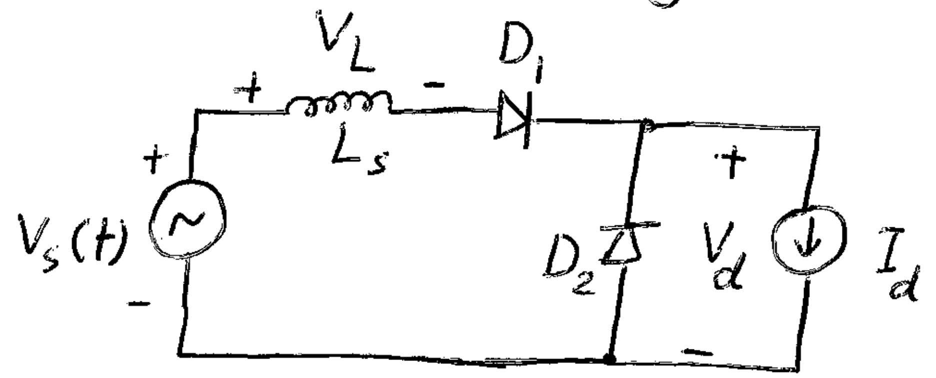
$$= \frac{1}{2\pi}\left[\int_{\pi/3}^{\pi}(200)(10)\,d(\omega t) + \int_{\frac{4\pi}{3}}^{2\pi}(200)(10)\,d(\omega t)\right]$$

$$= \frac{1}{2\pi}\left[(2000)\left(\pi - \frac{\pi}{3}\right) + (2000)\left(2\pi - \frac{4\pi}{3}\right)\right]$$

$$= \frac{1}{2\pi}\left[(2000)\,\frac{2\pi}{3} + (2000)\,\frac{2\pi}{3}\right] = \frac{2000}{2\pi}\,\frac{4\pi}{3}$$

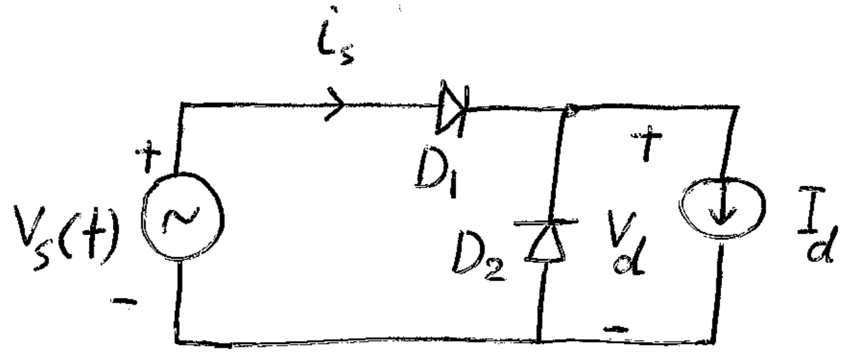
$$= \frac{8000}{6} = 1333.333 \text{ W}$$

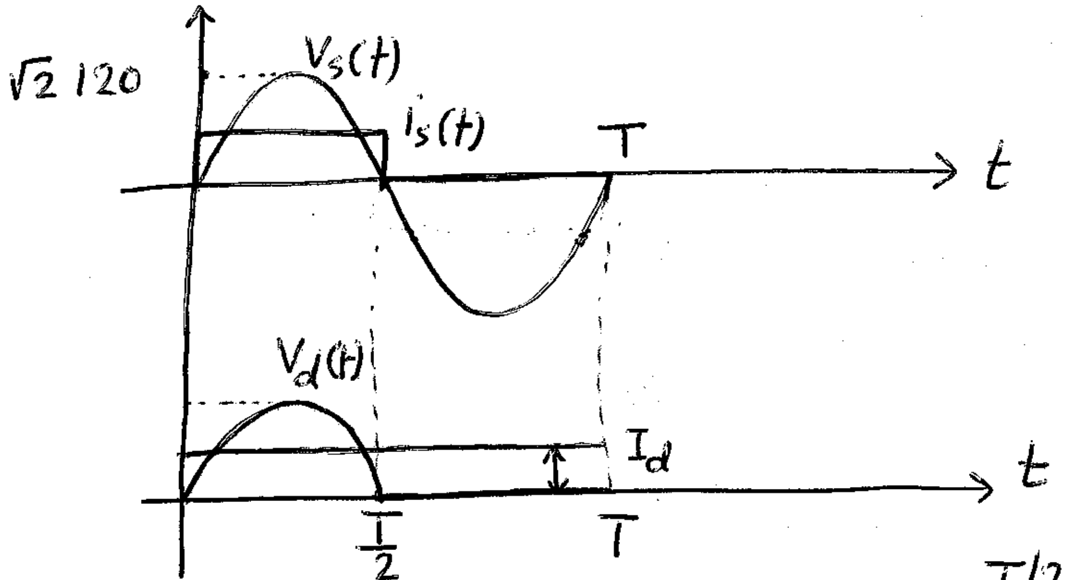
5-5 For the following circuit:



$I_d = 10A$

(a) $V_s = 120V$, $60Hz$, $L_s = 0$ $\quad V_d$? & P_d?





$$V_d = \frac{1}{T}\int_0^T \sqrt{2}\,120 \sin\omega t\, dt = \frac{1}{T}\int_0^{T/2} \sqrt{2}\,120 \sin\omega t\, dt$$

$$= \frac{\sqrt{2}\,120}{\omega T} \cos\omega t \Big|_{T/2}^{0} = \frac{(\sqrt{2})(120)}{\pi}$$

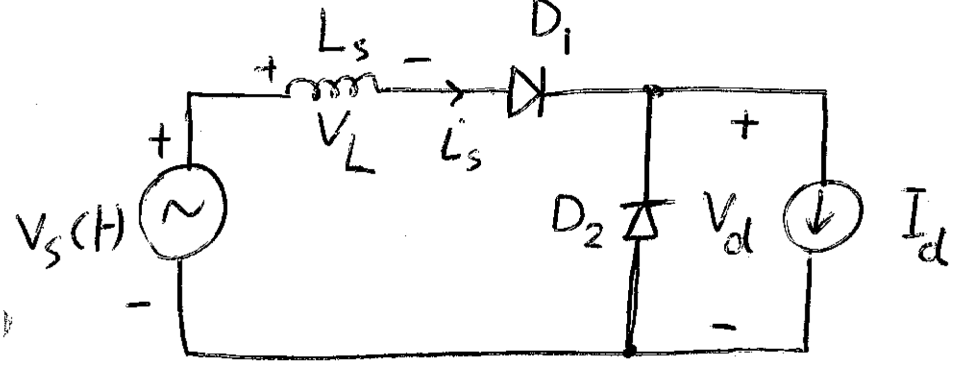
$$P_d = \frac{1}{T}\int_0^T v_d(t)\, i_d(t)\,dt$$

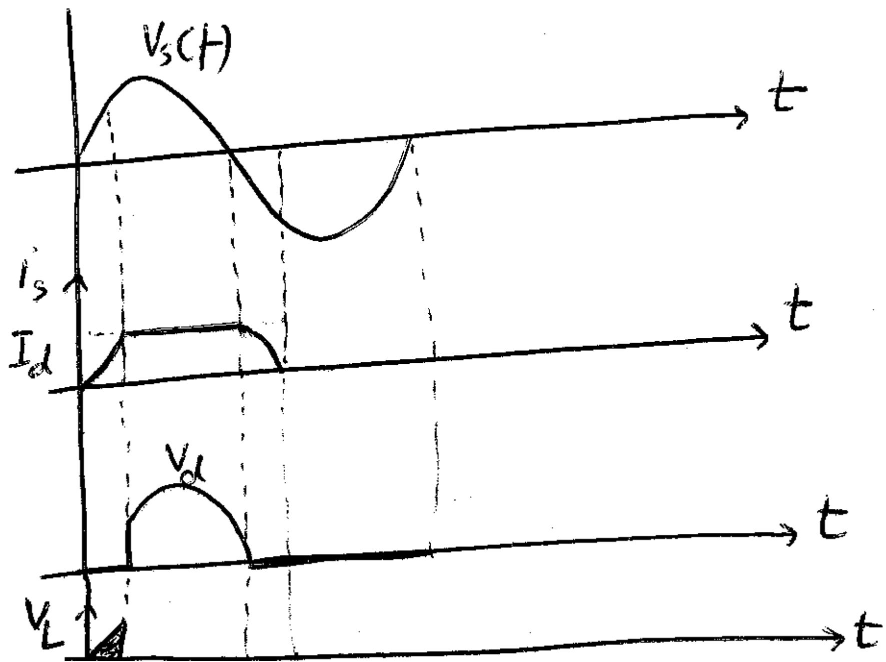
$$= \frac{1}{T}\int_0^{T/2} \sqrt{2}\,120 \sin\omega t \; I_d\,dt + \frac{1}{T}\int_{T/2}^{T} (0)\, I_d\,dt$$

$$= \frac{\sqrt{2}\,120}{\omega T}\, I_d \cos\omega t \Big|_{T/2}^{0} = \frac{\sqrt{2}\,120}{\frac{2\pi}{T}\,T}\, I_d \left[\cos 0 - \cos\frac{2\pi}{T}\frac{T}{2}\right]$$

$$= \frac{\sqrt{2}\,120}{2\pi}\, I_d \left[1+1\right] = \frac{\sqrt{2}\,120}{\pi}\, I_d$$

(b) $V_s = 120V$, $60Hz$, $L_s = 5mH$ $\quad u, V_d$ & P_d





$$\cos u = 1 - \frac{\omega L_s I_d}{\sqrt{2} V_s}$$

$$\cos u = 1 - \frac{(377)(5\times10^{-3})(10)}{(\sqrt{2})(120)}$$

$$= 1 - \frac{18.85}{169.2} = 1 - 0.1114 = 0.8886$$

$$u = \cos^{-1}(0.8886)$$

$$V_d = \frac{1}{2\pi}\int_u^{\pi} \sqrt{2} V_s \sin\omega t \, d\omega t$$

$$V_d = 0.45 V_s - \frac{\omega L_s}{2\pi} I_d$$

$$= 0.45 \quad (120) - \frac{(377)(5\times10^{-3})(10)}{2\pi}$$

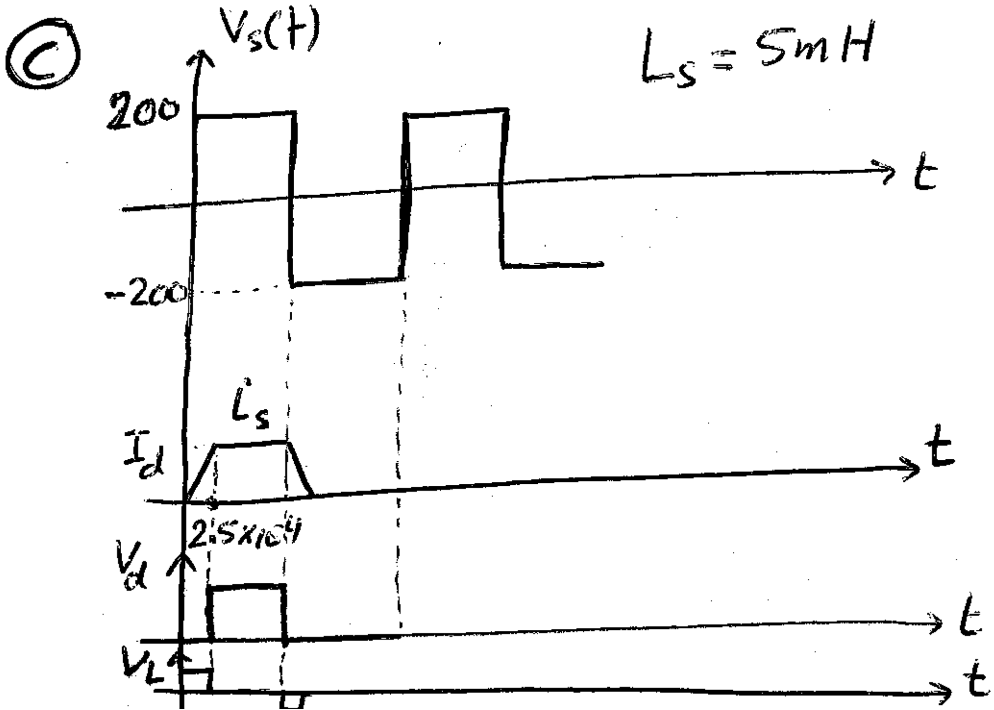
$$= 54 - \frac{18.85}{2\pi} = 54 - 3 = 51\,V$$

$$P_d = V_d I_d = (51)(10) = 510\,W$$

$$V_L = L_s \frac{di_s}{dt}$$

ⓒ

$L_s = 5mH$



$$T = 2$$

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{377}$$

$$\frac{T}{2} = \frac{\pi}{377} =$$

$$= 8.32\times10^{-3}$$

91

$$V_{do} = \frac{1}{T}\int_0^{T/2} 200\,dt = \frac{1}{T}\int_0^{8.32\times10^{-3}} 200\,dt$$

average value if $L_s = 0$

$$= \frac{1}{16.64\times10^{-3}}\, 200 \Big]_0^{8.32\times10^{-3}}$$

$$= \frac{(200)(8.32\times10^{-3})}{16.64\times10^{-3}} = 100\,V$$

$$V_L = \frac{1}{T}\int_0^{u} 200\,dt = \frac{1}{2\pi}\int_0^{u} 200\,d\omega t = \frac{1}{2\pi}\,200u$$

$$= \frac{100u}{\pi}$$

average value of the inductor voltage during commutation

at the end of the commutation

during Commutation

$$200 = L\frac{di_s}{dt}$$

$$\frac{200}{L} = \frac{di_s}{dt}$$

$$i_s = \frac{200}{L}t$$

$$I_d = 10 = \frac{200}{L}t_0 \Rightarrow t_0 = \frac{10L}{200} = 2.5\times10^{-4}$$

$$\uparrow 5\times10^{-3}$$

$$\left.\begin{aligned} 8.32\times10^{-3} &\longrightarrow \frac{T}{2} \\ 2.5\times10^{-4} &\longrightarrow X \end{aligned}\right\} \Rightarrow$$

$$X = \frac{2.5\times10^{-4} * \frac{T}{2}}{8.32\times10^{-3}} = 0.03\,\frac{T}{2} = (0.03)\,\frac{2\pi}{2}$$

$$X = 5.4^\circ$$

$$V_L = \frac{(100)(5.4)\frac{\pi}{180}}{\pi} = \frac{(100)(5.4)\pi}{180\pi}$$

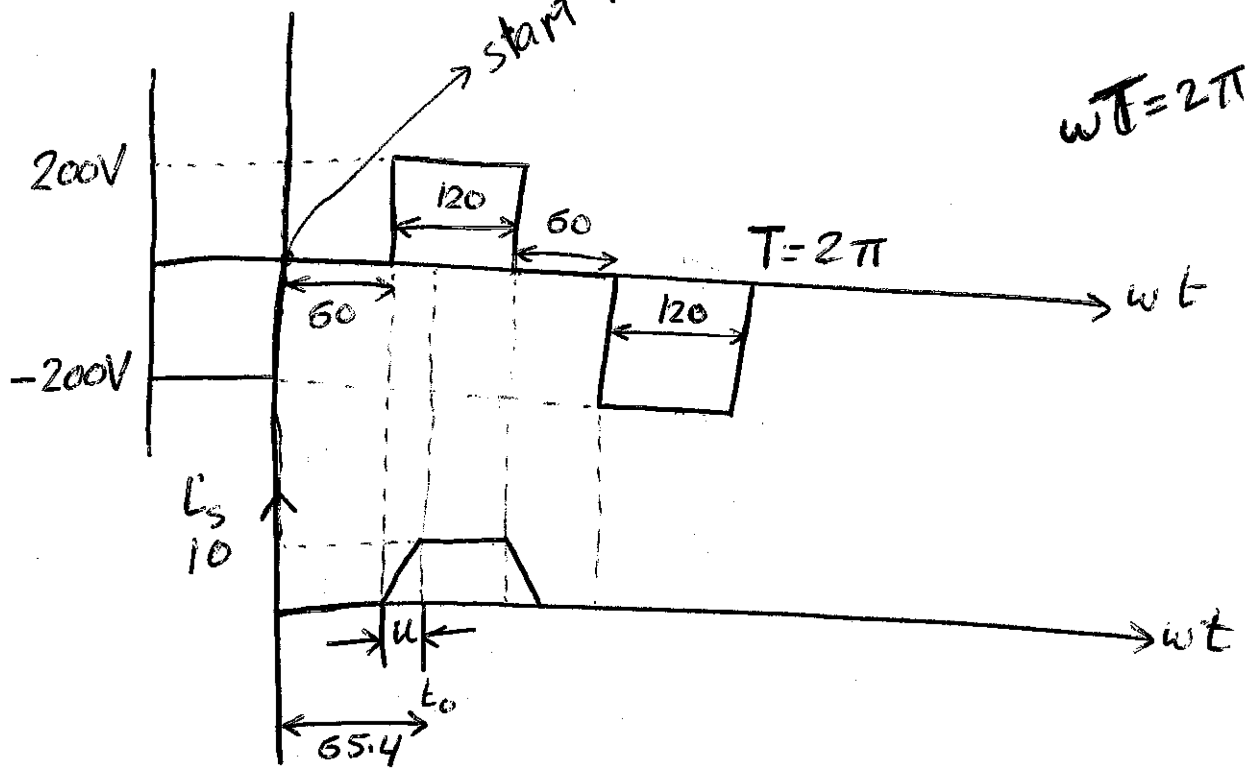
$$= 3\,V$$

(92)

$V_d = 100 - 3 = 97\,V$

$P_d = 97 * 10 = 970\,W$

(d)



$$200 = L\frac{di_s}{dt} \Rightarrow i_s(t) = \frac{200}{L}t \quad \text{during commutation}$$

at the end of the commutation

$$10 = \frac{200\,t_o}{L} \Rightarrow t_o = \frac{10L}{200} = \frac{(10)(5\times10^{-3})}{200} = 2.5\times10^{-4}\,s$$

$$T = 16.64\times10^{-3}\,s \Rightarrow 2\pi$$

$$2.5\times10^{-4}\,s \Rightarrow X$$

$$X = \frac{(2.5\times10^{-4})\,2\pi}{16.64\times10^{-3}} = 0.03\,\pi = 5.4^\circ$$

$V_L = 200\,V$ during the commutation interval

$$V_d = \frac{1}{2\pi}\int_{\pi/3}^{\pi} 200\, d\omega t = \frac{1}{2\pi}(200)\left[\pi - \frac{\pi}{3}\right]$$

↓ the average value with $L_s = 0$

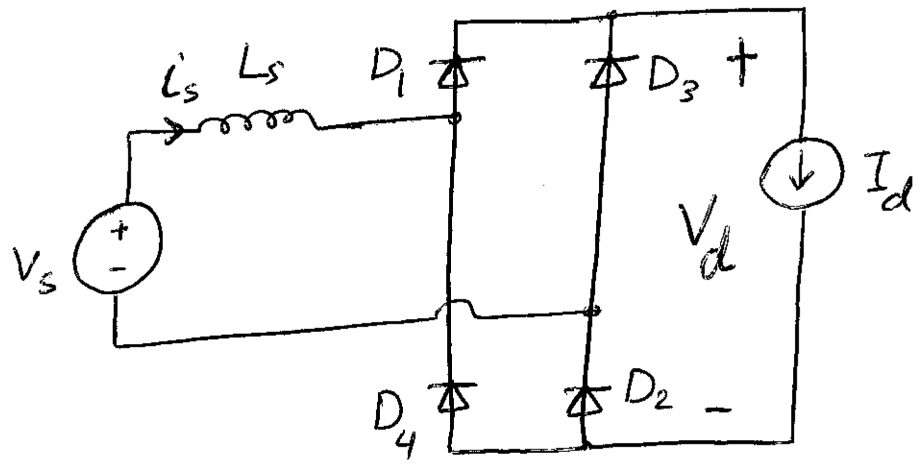
$$= \frac{1}{2\pi}(200)\,\frac{2\pi}{3} = \frac{200}{3}\,V$$

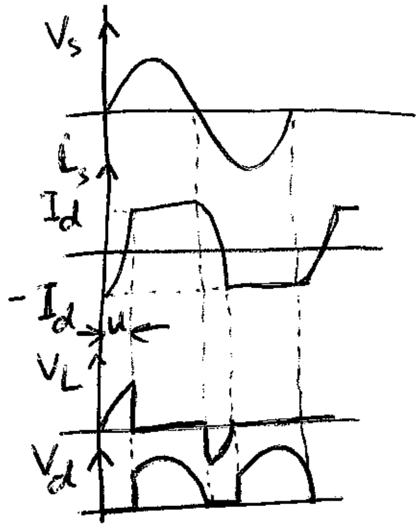
$$V_{do} = \frac{200}{3} - \frac{1}{2\cancel{\pi}}(200)\left(0.03\cancel{\pi}\right)$$

$$= \frac{200}{3} - 3 = 63.67\,V$$

$$P_d = V_{do}\, I_d = (63.67)(10) = 636.7\ W$$

problem 5-8 PP. 115





$V_s = 120V$, $L_s = 1mH$, $I_d = 10A$

u, V_d & P_d ?

$$\cos u = 1 - \frac{2\omega L_s}{\sqrt{2}V_s} I_d$$

$$= 1 - \frac{(2)(377)(1\times10^{-3})}{(\sqrt{2})(120)} 10 = 1 - \frac{7.54}{169.2}$$

$$= 0.955$$

$$u = \cos^{-1}(0.955) =$$

$$V_d = V_{do} - \frac{\text{area } A_u}{\pi} = 0.9V_s - \frac{2\omega L_s I_d}{\pi}$$

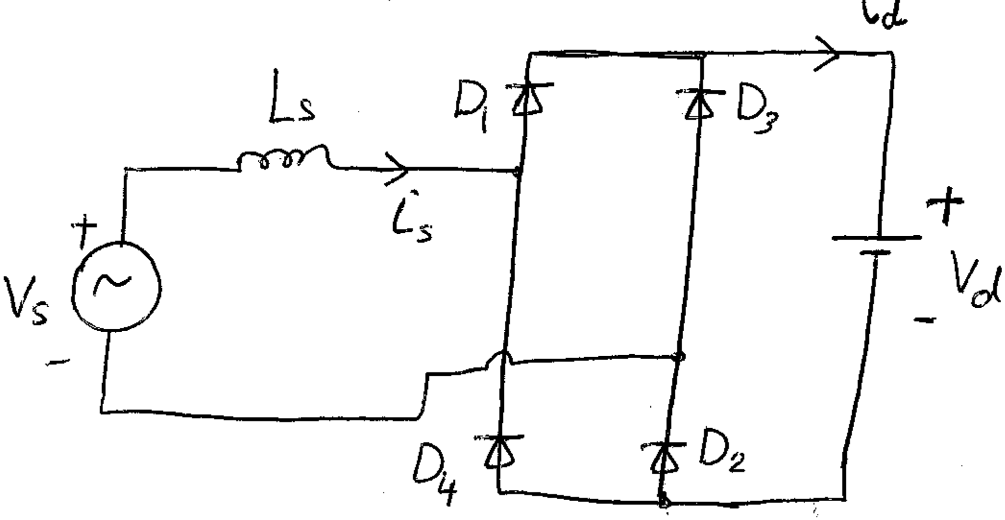
$$= (0.9)(120) - \frac{(2)(377)(1\times10^{-3})(10)}{\pi} =$$

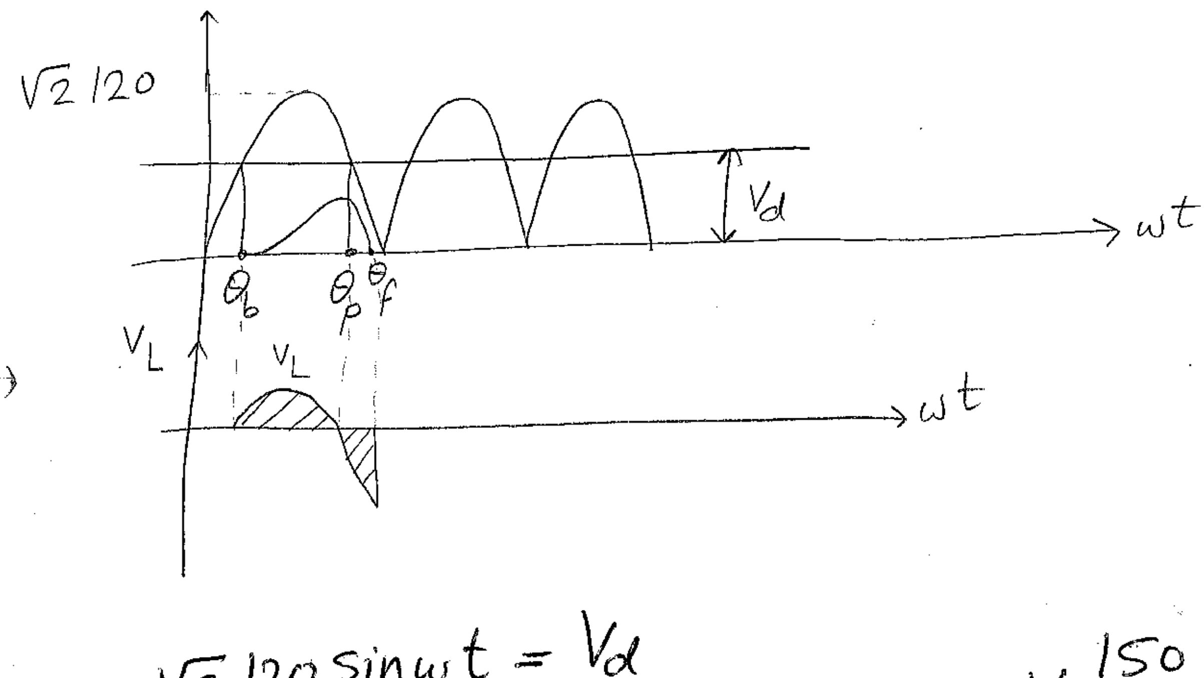
$$P_d = V_d I_d =$$

problem 5-11 PP. 115

$V_s = 120V$, $60 Hz$, $L_s = 1 mH$ $V_d = 150V$

i_d, θ_b, θ_f, $I_{d,peak}$, I_d



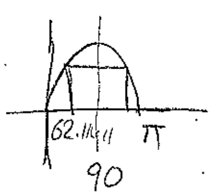


$\sqrt{2}\,120 \sin\omega t = V_d$

$\sqrt{2}\,120 \sin\theta_b = 150 \Rightarrow \theta_b = \sin^{-1}\left(\frac{150}{\sqrt{2}\,120}\right)$

$\theta_b = 1.0841 \text{ rad} = 62.1144°$

$\theta_p = 117.8856° = 2.0575 \text{ rad}$



90 − 62.1144

$$V_L = L_s \frac{d i_d}{dt} = \sqrt{2}\, V_s \sin(\omega t) - V_d$$

$$\omega L_s \int_{\theta_b}^{\theta} d i_d = \int_{\theta_b}^{\theta} (\sqrt{2} V_s \sin \omega t - V_d)\, d(\omega t)$$

$$\omega L_s \Big[i_d(\theta) - \underbrace{\cancel{i_d(\theta_b)}}_{\text{zero}} \Big] = \int_{\theta_b}^{\theta} (\sqrt{2} V_s \sin \omega t - V_d)\, d(\omega t)$$

$$\omega L_s \; i_d(\theta) = \int_{\theta_b}^{\theta} (\sqrt{2} V_s \sin \omega t - V_d)\, d(\omega t)$$

$$\rightarrow i_d(\theta) = \frac{1}{\omega L_s} \int_{\theta_b}^{\theta} (\sqrt{2} V_s \sin \omega t - V_d)\, d(\omega t)$$

$$i_d(\theta) = \frac{1}{\omega L_s} \left[\sqrt{2}\, V_s \cos \omega t \Big|_{\theta}^{\theta_b} - V_d\, \omega t \Big|_{\theta_b}^{\theta} \right]$$

$$= \frac{1}{\omega L_s} \left[\sqrt{2}\, V_s \left[\cos \theta_b - \cos \theta \right] - V_d \theta + V_d \theta_b \right]$$

$$i_d(\theta) = \frac{\sqrt{2} V_s}{\omega L_s} (\cos \theta_b - \cos \theta) - \frac{1}{\omega L_s} V_d \theta + \frac{1}{\omega L_s} V_d \theta_b$$

$$i_d(\theta_f) = 0 = \frac{\sqrt{2} V_s}{\omega L_s} (\cos \theta_b - \cos \theta_f) - \frac{1}{\omega L_s} V_d \theta_f + \frac{1}{\omega L_s} V_d \theta_b$$

$$\frac{\sqrt{2}V_s}{\omega L_s}\cos\theta_b - \frac{\sqrt{2}V_s}{\omega L_s}\cos\theta_f - \frac{1}{\omega L_s}V_d\theta_f$$

$$+\frac{1}{\omega L_s}V_d\theta_b = 0$$

$$\frac{(\sqrt{2})(120)}{(377)(1\times10^{-3})}\cos(62.1144) - \frac{\sqrt{2}\ (120)}{(377)(1\times10^{-3})}\cos\theta_f -$$

$$\frac{1}{(377)(1\times10^{-3})}(150)\theta_f + \frac{1}{(377)(1\times10^{-3})}(150)\ (1.0841) = 0$$

$$\rightarrow \theta_f =$$

$$I_d = \frac{\int_{\theta_b}^{\theta_f} i_d(\theta)\,d\theta}{\pi}$$

$$I_d = \frac{\int_{\theta_b}^{\theta_f}\left(\frac{\sqrt{2}V_s}{\omega L_s}(\cos\theta_b - \cos\theta) - \frac{1}{\omega L_s}V_d\theta + \frac{1}{\omega L_s}V_d\theta_b\right)d\theta}{\pi}$$

problem 5-23 PP. 116

$$I_a = \frac{1}{2\pi}\int_0^{2\pi/3} I_d \, d\theta = \frac{1}{2\pi} I_d \frac{2\pi}{3} = \frac{I_d}{3} \text{ (average)}$$

$$I_a = \sqrt{\frac{1}{2\pi}\int_0^{2\pi/3} I_d^2 \, d\theta} = \sqrt{\frac{1}{2\pi} I_d^2 \frac{2\pi}{3}} = \frac{I_d}{\sqrt{3}}$$

problem 5-24 PP. 116

~~V_{an} V_{cn} = K ωt ⇒ increasing linearly~~

ⓐ $$L_s \frac{di_u}{dt} = \sqrt{2} V_{LL} \omega t$$

$$\omega L_s \int_0^{I_d} di_u = \int_0^{u} \sqrt{2} V_{LL} \omega t \, d\omega t = \frac{\sqrt{2} V_{LL} u^2}{2}$$

$$\omega L_s I_d = \frac{\sqrt{2} V_{LL} u^2}{2}$$

$$\frac{\sqrt{2} V_{LL}}{2} u^2 = \omega L_s I_d \Rightarrow u = \left(\frac{2\omega L_s I_d}{\sqrt{2} V_{LL}}\right)^{1/2}$$

$$u = \left(\frac{(2)(377)(2\times10^{-3})(10)}{\sqrt{2}\ (208)} \right)^{1/2}$$

$$= 0.2264 \text{ rad} = \quad 12.9728^{\circ}$$

$$\cos u = 1 - \frac{2\omega L_s I_d}{\sqrt{2}\, V_{LL}}$$

$$= 1 - \frac{(2)(377)(2\times10^{-3})(10)}{(\sqrt{2})(208)}$$

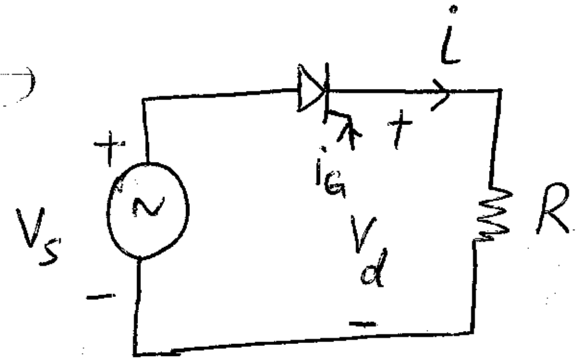
$$= 0.3216 = 18.4256^{\circ}$$

"Chapter 6"

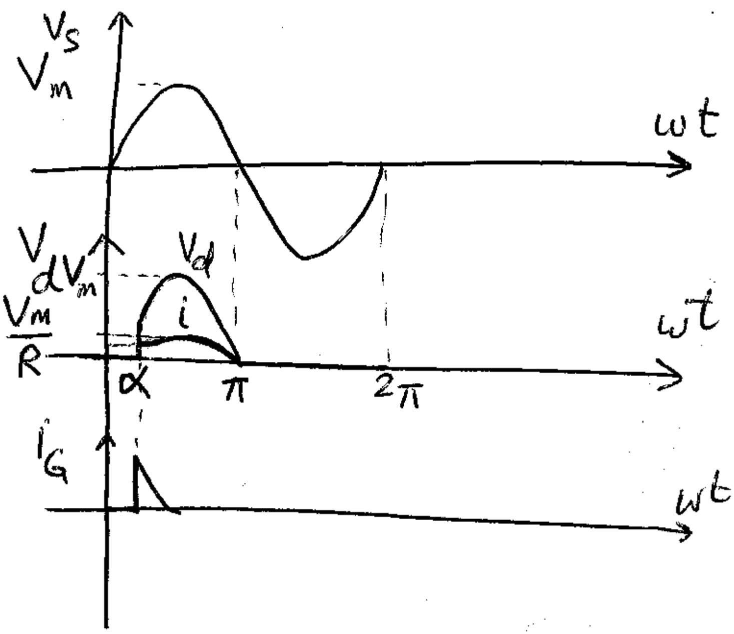
Line-Frequency Phase-Controlled Rectifiers and Inverters: Line-Frequency ac → Controlled dc

Controlled dc → Thyristor is used

Basic Thyristor Circuits



half_wave controlled rectifier with pure resistive load



$$V_d = \frac{1}{2\pi}\int_{\alpha}^{\pi} \sqrt{2}V_s \sin\omega t \, d\omega t$$

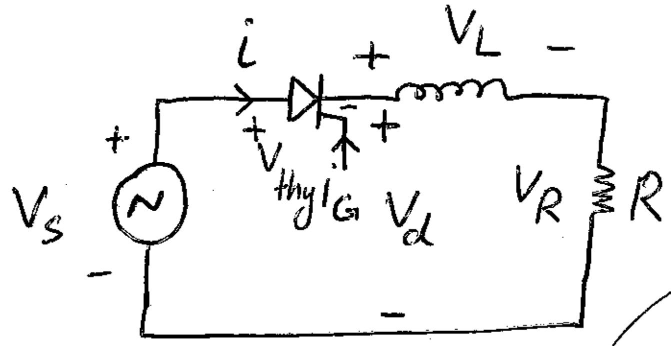
α → firing angle

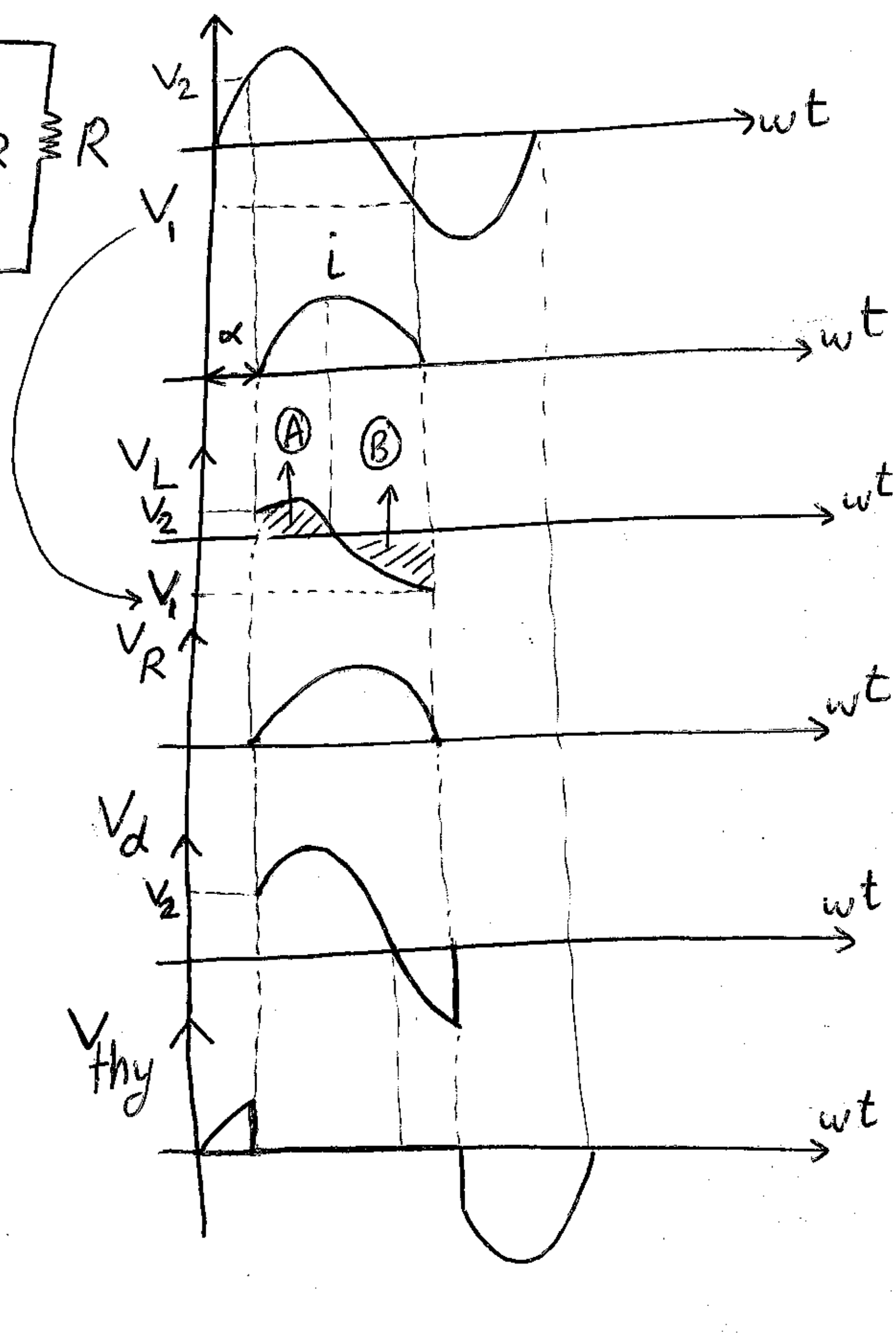
$$V_d = \frac{1}{2\pi}\int_{\alpha}^{\pi} \sqrt{2}\, V_s \sin(\omega t)\, d(\omega t)$$

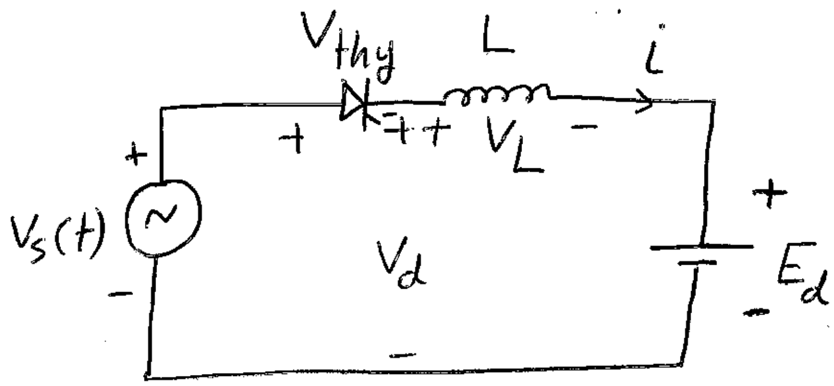
$$= \frac{\sqrt{2}}{2\pi} V_s \cos\omega t \Big]_{\pi}^{\alpha} = \frac{\sqrt{2} V_s}{2\pi}(\cos\alpha - \cos\pi)$$

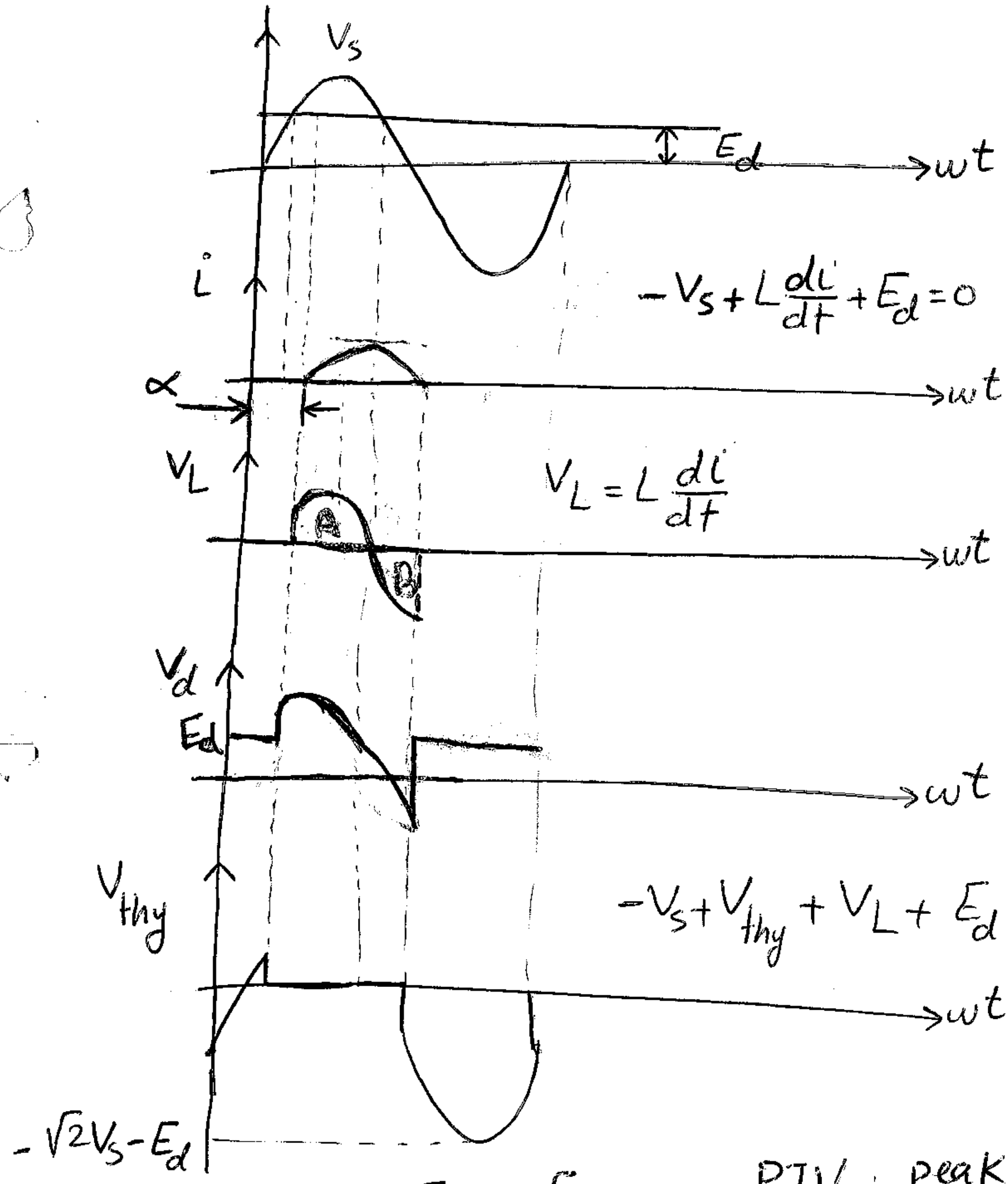
$$= \frac{\sqrt{2}\, V_s}{2\pi}(\cos\alpha + 1) = \frac{\sqrt{2}\, V_s}{2\pi}(1 + \cos\alpha)$$

α : firing angle or triggering angle









$$-V_s + L\frac{di}{dt} + E_d = 0$$

$$V_L = L\frac{di}{dt}$$

$$-V_s + V_{thy} + V_L + E_d = 0$$

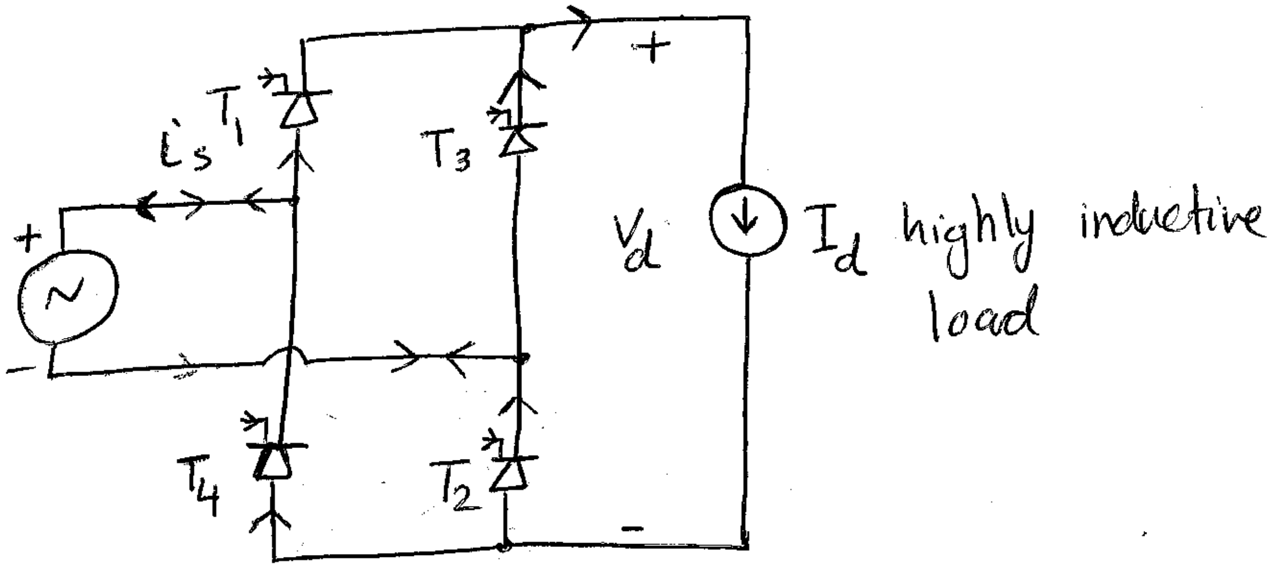
$$PIV = \sqrt{2}V_s + E_d$$

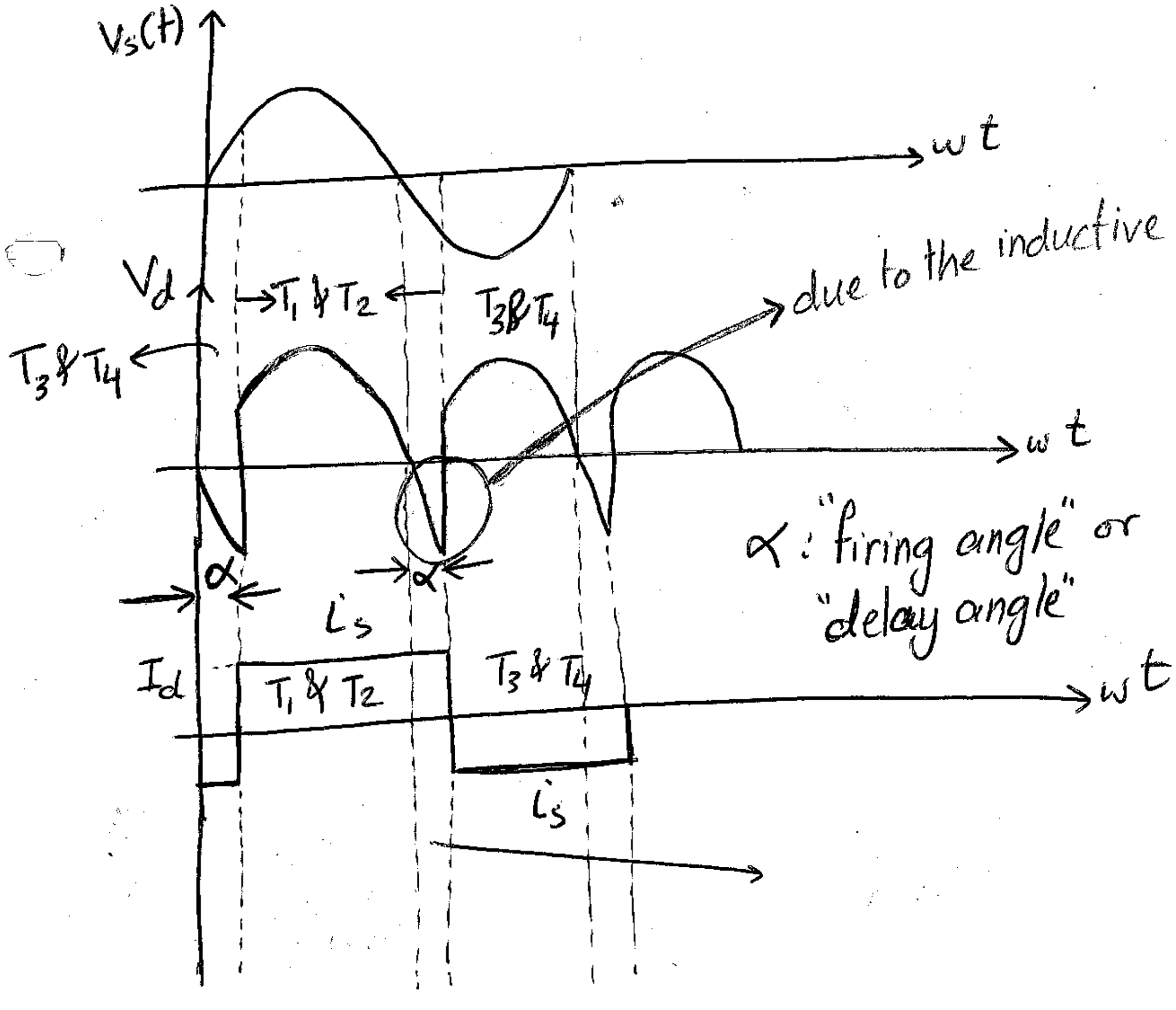
PIV : peak inverse voltage

The maximum voltage across the device when it's off.

Single-phase Converters

Idealized Circuit with $L_s = 0$ & $i_d(t) = I_d$





$$V_{d\alpha} = \frac{2}{2\pi} \int_{\alpha}^{\alpha+\pi} \sqrt{2} V_s \sin\omega t \, d(\omega t)$$

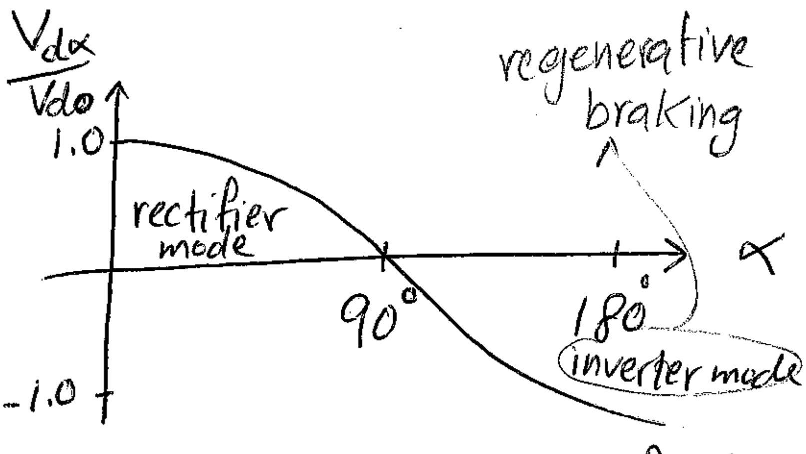
$$= \frac{1}{\pi} \sqrt{2} V_s \cos\omega t \Big]_{\alpha+\pi}^{\alpha} = \frac{\sqrt{2} V_s}{\pi} \left(\cos\alpha - \cos(\alpha+\pi)\right)$$

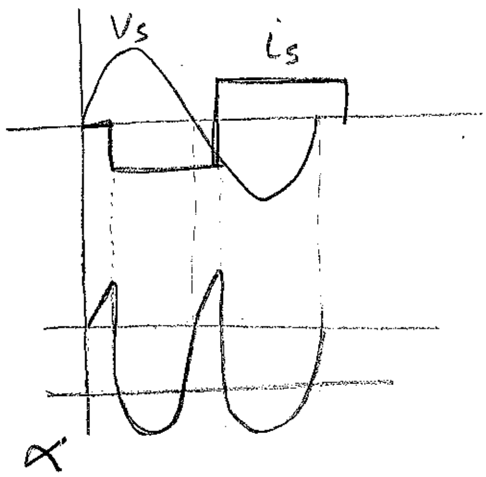
$$= \frac{\sqrt{2} V_s}{\pi} \left(\cos\alpha - (\cos\alpha \cos\pi - \sin\alpha \sin\pi)\right)$$

$$= \frac{\sqrt{2} V_s}{\pi} \left(\cos\alpha - (-\cos\alpha)\right) = \frac{2\sqrt{2} V_s}{\pi} \cos\alpha$$

$$= \underbrace{0.9 V_s}_{V_{do}} \cos\alpha$$

with $\alpha = 0$, $V_{do} = 0.9 V_s$ (as before)

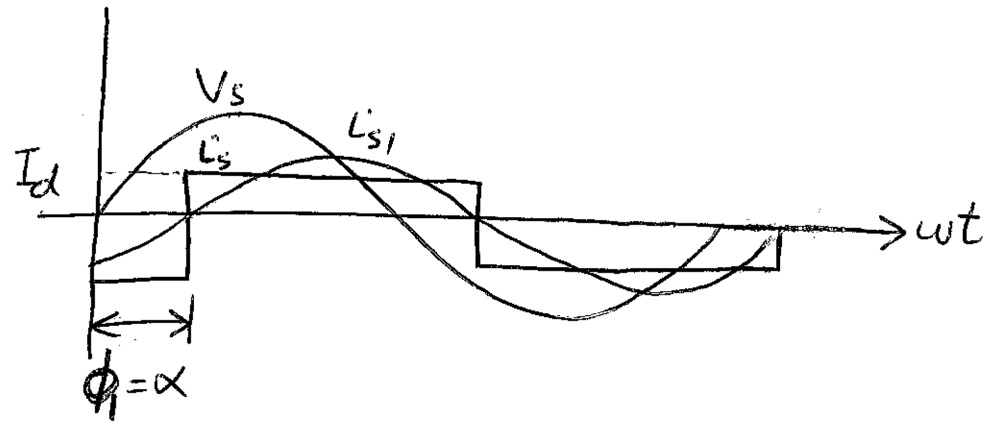




Normlized V_d as function of α

$$P = \frac{1}{T} \int_0^T p(t) dt = \frac{1}{T} \int_0^T V_d i_d dt = \underbrace{0.9 V_s \cos\alpha \, I_d}_{\text{average power}}$$

* For V_s & i_s ⇒ find THD, PF, DPF.



By means of Fourier analysis :

$$i_s(\omega t) = \sqrt{2}\, I_{s_1} \sin(\omega t - \alpha) + \sqrt{2}\, I_{s_3} \sin[3(\omega t - \alpha)]$$

$$+ \sqrt{3}\, I_{s_5} \sin[5(\omega t - \alpha)] + \cdots$$

$$I_{s_1} = \frac{2}{\pi}\sqrt{2}\, I_d = 0.9\, I_d$$

$$I_{sh} = \frac{I_{s1}}{h} \quad \text{for odd } h.$$

$$I_s = I_d \quad (\text{rms value of } i_s)$$

$$\%THD = 100 \times \frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s_1}} = 100 \times \frac{\sqrt{I_d^2 - 0.9^2 I_d^2}}{0.9\, I_d}$$

$$= 48.43\%$$

$$DPF = \cos\phi_1 = \cos\alpha$$

$$PF = \frac{I_{s1}}{I_s} DPF = \frac{0.9 I_d}{I_d} \cos\alpha$$

$$= 0.9 \cos\alpha$$

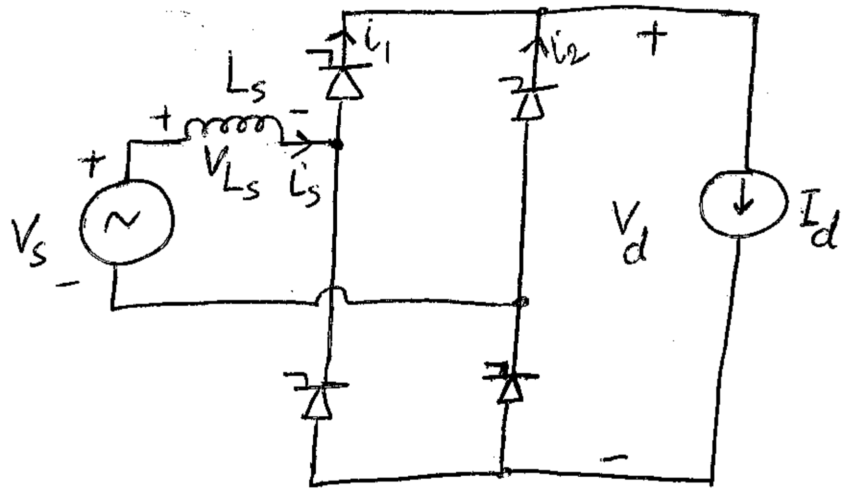
$$P = V_s I_{s_1} \cos\phi_1$$

$$P = 0.9 V_s I_d \cos\alpha$$

$$Q = V_s I_{s_1} \sin\phi_1 = 0.9 V_s I_d \sin\alpha$$

$$S = V_s I_{s_1} = \sqrt{P^2 + Q^2}$$

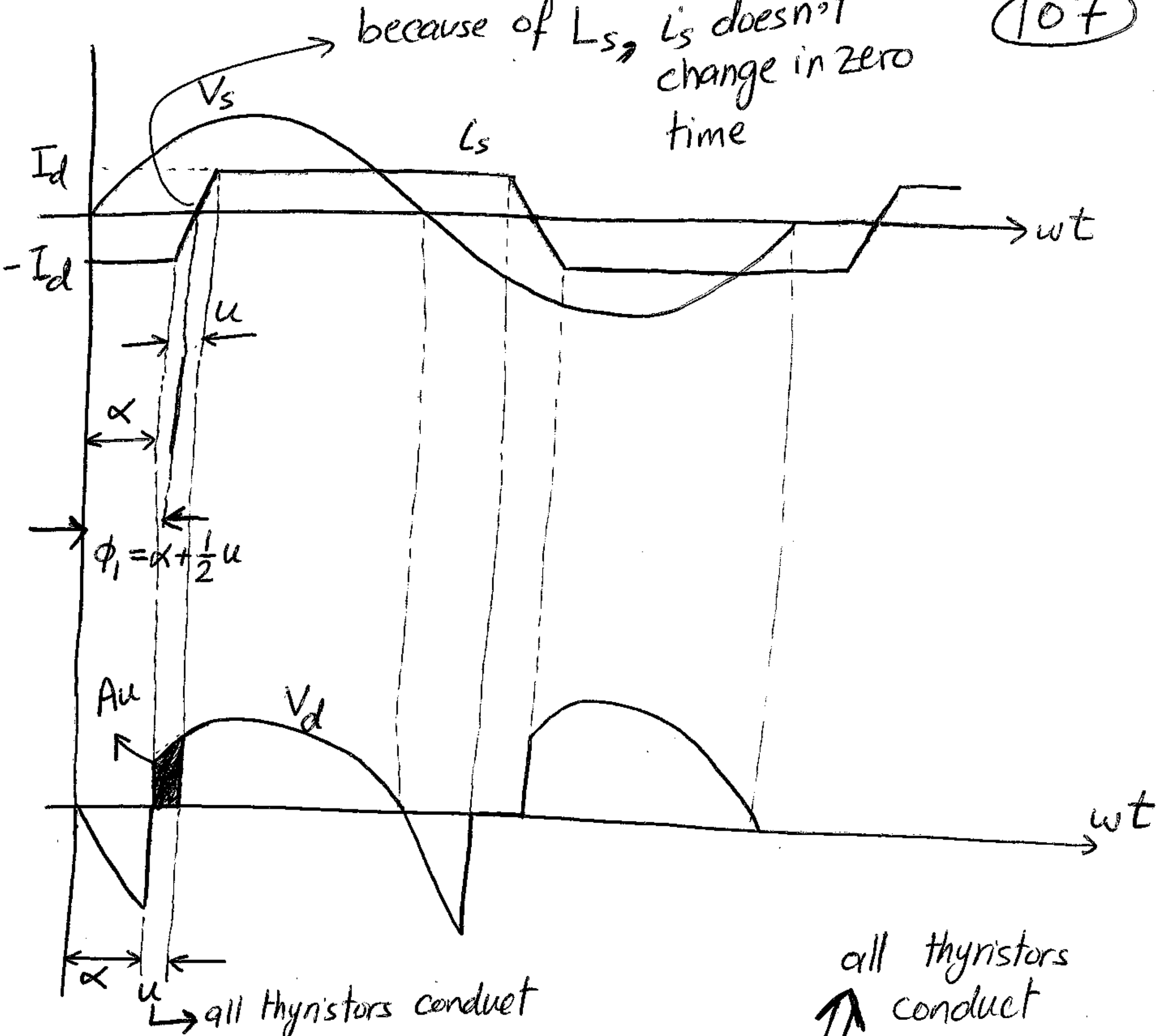
Effect of L_s



$i_1 + i_2 = I_d$

$i_1 = -i_2$

$\frac{di_1}{dt} = -\frac{di_2}{dt}$



* During the commutation process $\underline{V_d = 0}$ and so

$$V_s = L_s \frac{di_s}{dt}$$

$$\sqrt{2}\, V_s \sin\omega t = \omega L_s \frac{di_s}{d(\omega t)}$$

$$\int_{\alpha}^{\alpha+u} \sqrt{2}\, V_s \sin\omega t \, d(\omega t) = \omega L_s \int_{-I_d}^{I_d} di_s = 2\omega L_s I_d$$

$$\sqrt{2}\, V_s \cos\omega t \Big]_{\alpha+u}^{\alpha} = 2\omega L_s I_d$$

$$\sqrt{2}\, V_s \left[\cos(\alpha) - \cos(\alpha+u)\right] = 2\omega L_s I_d$$

$$\cos(\alpha+u) = \cos\alpha - \frac{2\omega L_s I_d}{\sqrt{2}\, V_s} \Rightarrow$$ u can be calculated

$$V_d = V_{d\alpha} - \frac{A_u}{\pi}$$ A_u for half cycle

$$= 0.9 V_s \cos\alpha - \frac{2\omega L_s I_d}{\pi}$$ for half cycle

* For THD, PF, DPF, I_s, I_{s1} they all can be calculated.

Homework $\Rightarrow$ Do them.

The average value of V_d can be calculated as:

$$V_d = \frac{2}{2\pi}\left[\int_{\alpha}^{\alpha+u} 0\, d\omega t + \int_{\alpha+u}^{\pi+\alpha} \sqrt{2}\, V_s \sin\omega t\, d(\omega t)\right]$$

$$= \frac{1}{\pi}\int_{\alpha+u}^{\pi+\alpha} \sqrt{2}\, V_s \sin\omega t\, d(\omega t)$$

$$V_d = \frac{1}{\pi}\sqrt{2}V_s \cos\omega t \Big]_{\pi+\alpha}^{\alpha+u}$$

$$= \frac{\sqrt{2}V_s}{\pi}\left[\cos(\alpha+u) - \cos(\pi+\alpha)\right]$$

$$= \frac{\sqrt{2}V_s}{\pi}\left[\cos\alpha - \frac{2\omega L_s I_d}{\sqrt{2}V_s} - \cos\pi\cos\alpha + \underbrace{\sin\pi\sin\alpha}_{\text{zero}}\right]$$

$$= \frac{\sqrt{2}V_s}{\pi}\left[\cos\alpha - \frac{2\omega L_s I_d}{\sqrt{2}V_s} + \cos\alpha + 0\right]$$

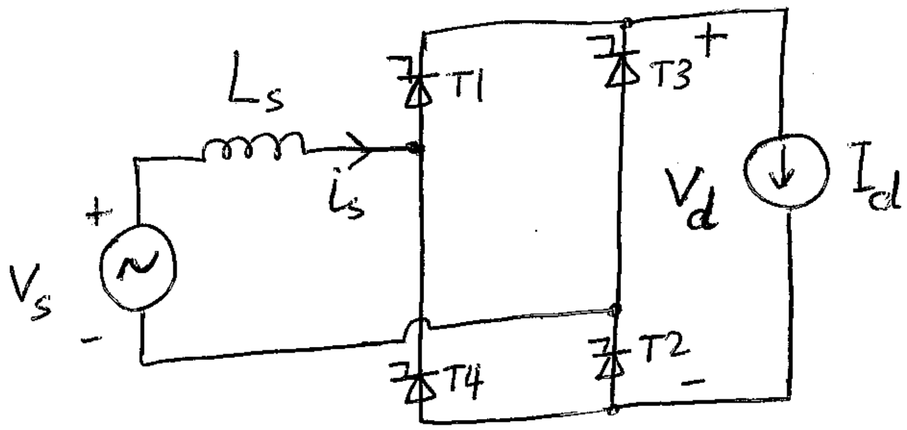
$$= \frac{\sqrt{2}V_s}{\pi}\left[2\cos\alpha - \frac{2\omega L_s I_d}{\sqrt{2}V_s}\right]$$

$$= \frac{2\sqrt{2}V_s}{\pi}\cos\alpha - \frac{1}{\pi}2\omega L_s I_d$$

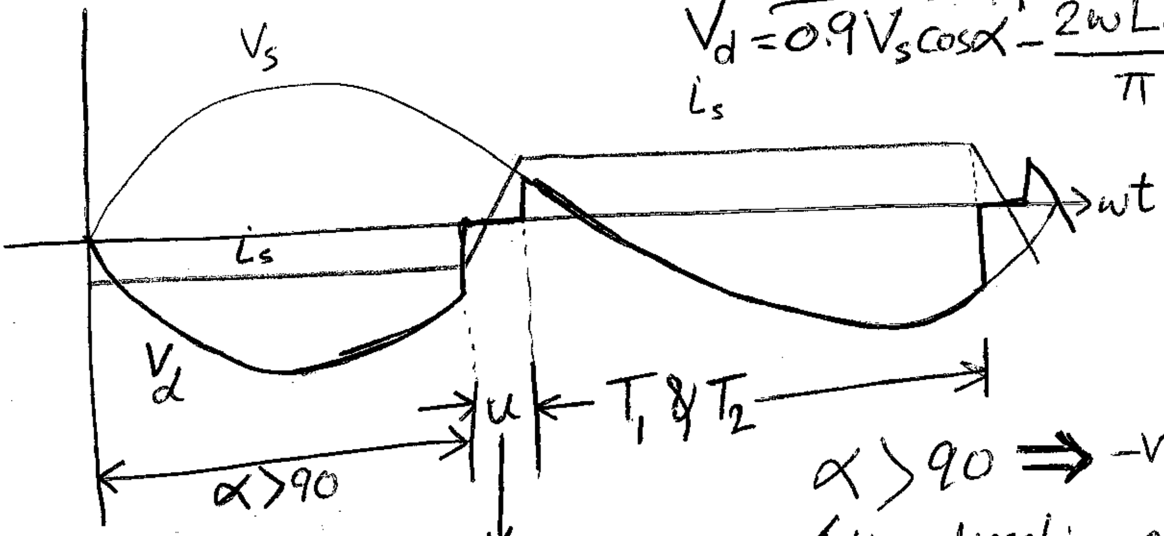
$$V_d = 0.9V_s\cos\alpha - \underbrace{\frac{2\omega L_s I_d}{\pi}}_{\text{due to } L_s}$$

Inverter Mode of Operation

* Thyristor converters can operate in an inverter mode when V_d has a negative value as shown below in the figures if $90° < \alpha < 180°$:



$$V_d = \overbrace{0.9 V_s \cos\alpha}^{V_{do}} - \overbrace{\frac{2\omega L_s I_d}{\pi}}^{\text{due to } L_s}$$

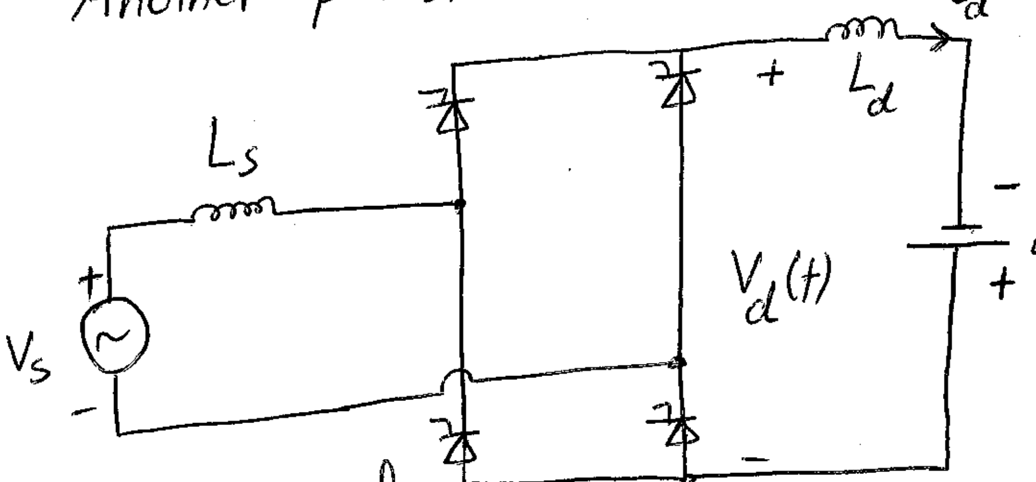


T_3 & T_4

commutation interval (T_1, T_2, T_3 & T_4)

$\alpha > 90 \Rightarrow -ve\ P_d$ (the direction of power flow is reversed)

Another possible circuit is:



① battery
② photovoltaic source
③ dc voltage produced by wind-electric system

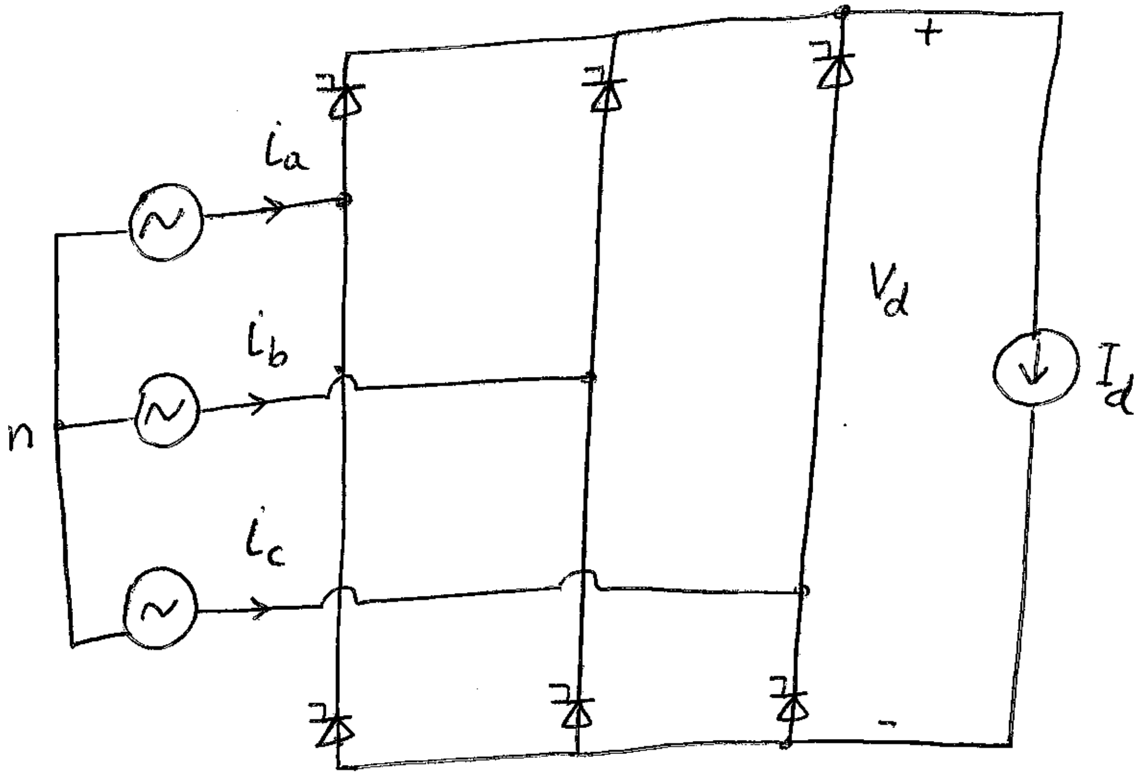
average value of $V_d(t)$

$$V_d = V_{do} \cos\alpha - \frac{2}{\pi}\omega L_s I_d = E_d \underline{\underline{< 0}}$$

inverter mode of operation

Three-phase Converters

Idealized Circuit with $L_s = 0$ and $i_d(t) = I_d$



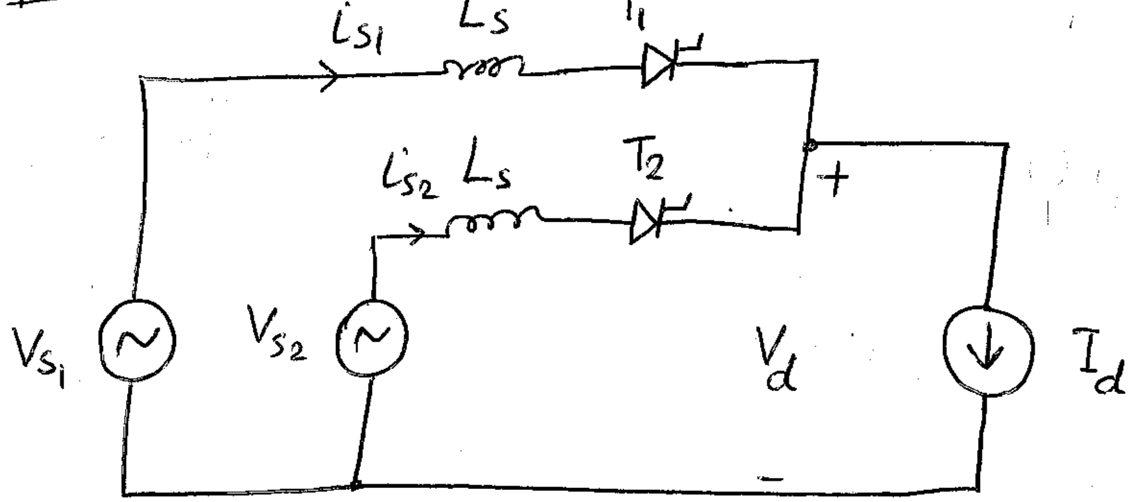
$$V_{do} = \frac{3\sqrt{2}}{\pi} V_{LL} = 1.35\, V_{LL}$$

$$V_{d\alpha} = 1.35\, V_{LL} \cos\alpha = V_{do} \cos\alpha$$

$$P = V_d I_d = 1.35\, V_{LL} I_d \cos\alpha$$

صوّر صفحة 139 للطلاب واشرحها
في المحاضرة .

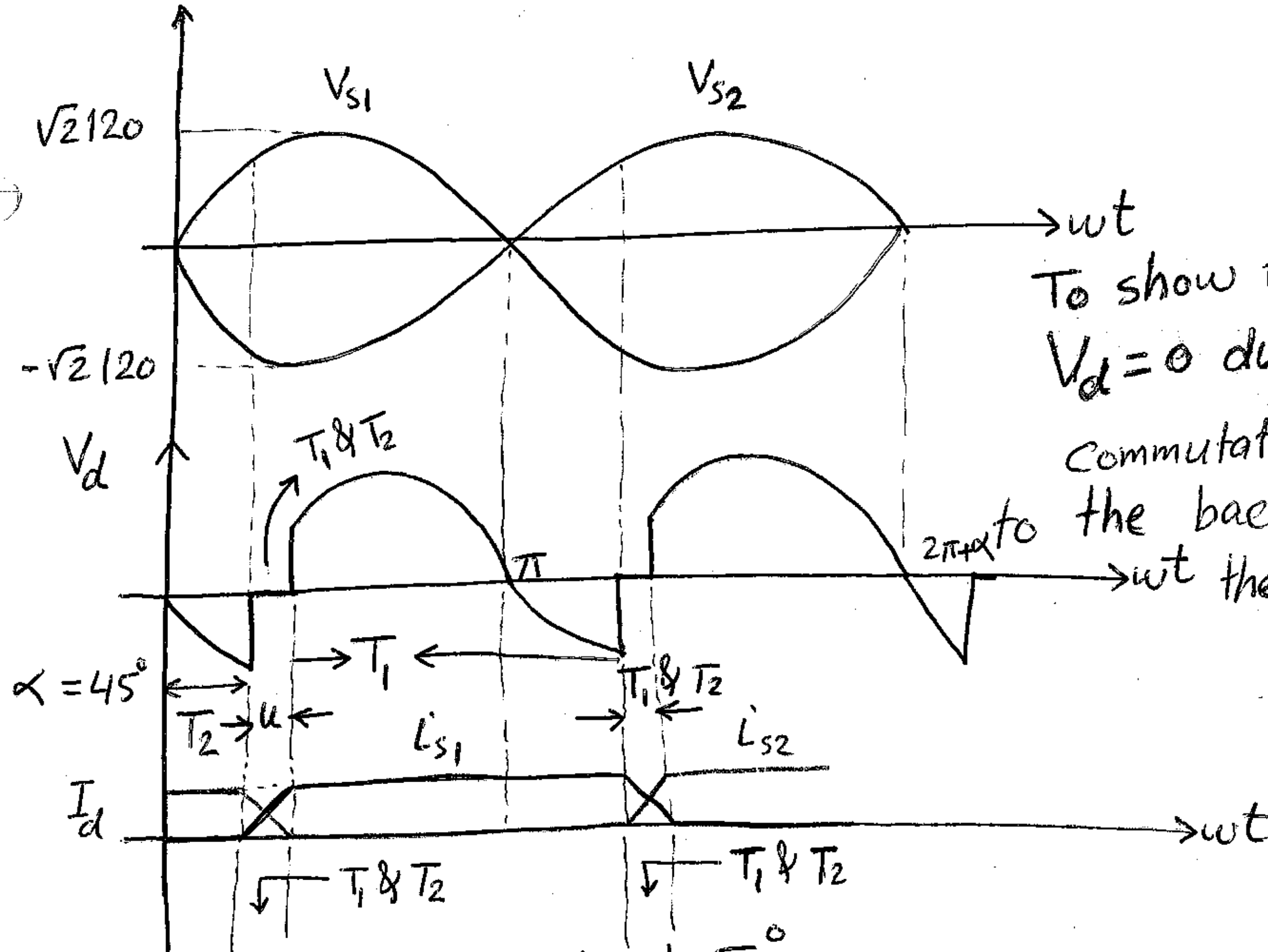
problem 6-1 PP. 154



$V_{s_1} = V_{s_2} = 120V$ (rms) (180° out of phase)

60Hz, $L_s = 5mH$, $I_d = 10A$

For $\alpha = 45°$ & $135°$ draw the waveforms of V_{s_1}, i_{s1} & V_d. Calculate V_d & u.



To show that $V_d = 0$ during commutation, go to the back of the page →

$\alpha = 45°$

$$V_d = \frac{1}{2\pi}\left[\int_{\alpha+u}^{\pi+\alpha} \sqrt{2}\, V_{s_1} \sin\omega t \, d(\omega t) + \int_{\pi+\alpha+u}^{2\pi+\alpha} \sqrt{2}\, V_{s_2} \sin\omega t \, d(\omega t)\right]$$

$$= \frac{2}{2\pi}\int_{\alpha+u}^{\pi+\alpha} \sqrt{2}\, V_{s_1} \sin\omega t \, d\omega t = \frac{1}{\pi}\int_{\alpha+u}^{\pi+\alpha} \sqrt{2}\, V_{s_1} \sin\omega t \, d\omega t$$

$$= \left.\frac{\sqrt{2}\, V_{s1}}{\pi} \cos\omega t\right]_{\pi+\alpha}^{\alpha+u} = \frac{\sqrt{2}\, V_{s1}}{\pi}\left[\underbrace{\cos(\alpha+u)}_{\text{can be obtained from the commutation}} - \cos(\pi+\alpha)\right]$$

During commutation: $(V_d = 0)$

$$V_L = L_s \frac{di_{s_1}}{dt} = \sqrt{2}\, V_{s_1} \sin\omega t$$

$$\omega L_s \frac{di_{s1}}{d\omega t} = \sqrt{2}\, V_{s_1} \sin\omega t$$

$$\omega L_s \, di_{s_1} = \sqrt{2}\, V_{s_1} \sin\omega t \, d(\omega t)$$

$$\omega L_s \int_0^{I_d} di_{s_1} = \int_{\alpha}^{\alpha+u} \sqrt{2}\, V_{s_1} \sin\omega t \, d(\omega t)$$

$$\omega L_s I_d = \left.\sqrt{2}\, V_{s_1} \cos\omega t\right]_{\alpha+u}^{\alpha}$$

$$\omega L_s I_d = \sqrt{2} V_{s_1} \left[\cos\alpha - \cos(\alpha+u) \right]$$

$$\frac{\omega L_s I_d}{\sqrt{2} V_{s_1}} = \cos\alpha - \cos(\alpha+u)$$

$$\cos(\alpha+u) = \cos\alpha - \frac{\omega L_s I_d}{\sqrt{2} V_{s_1}}$$

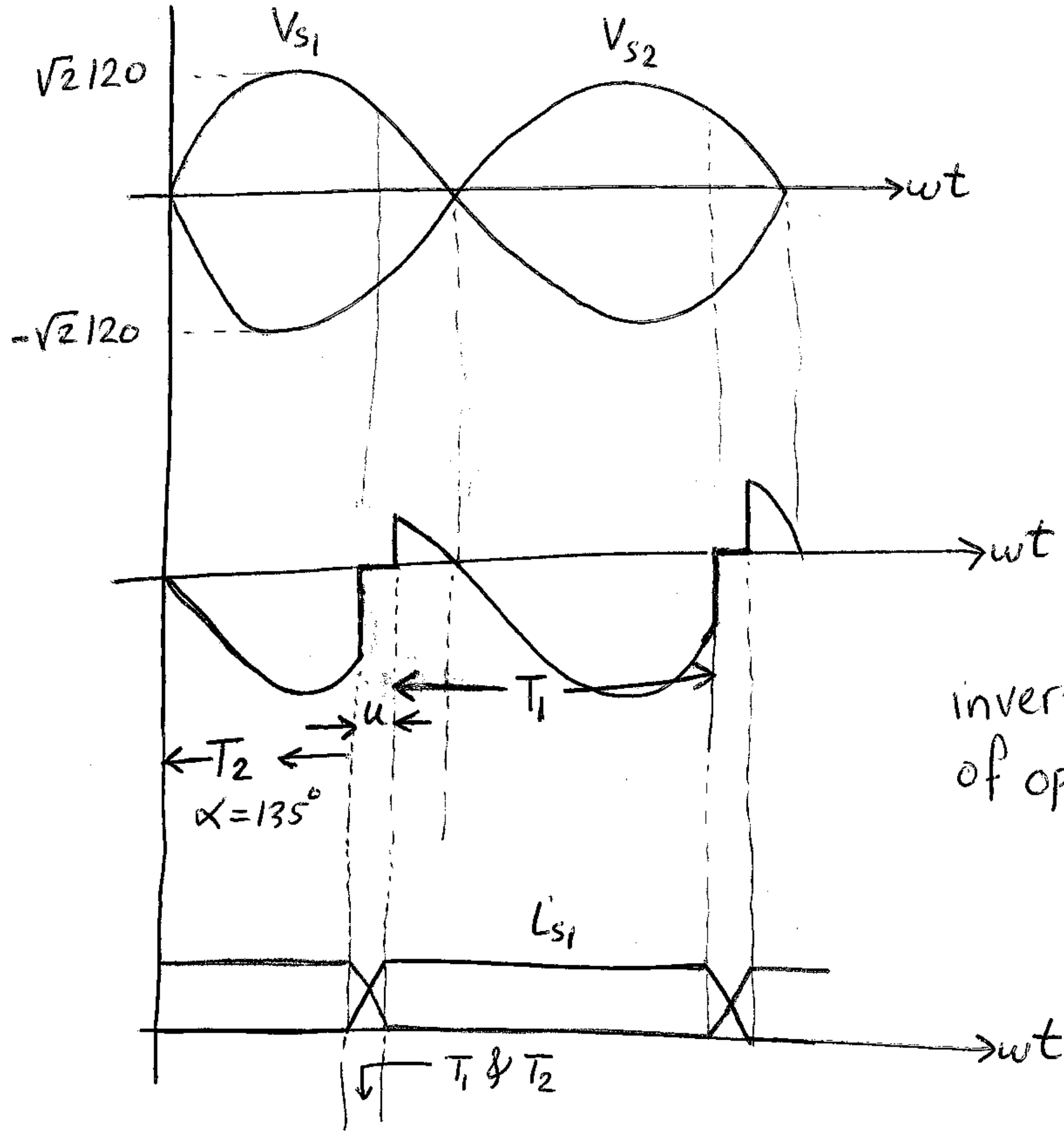
$$V_d = \frac{\sqrt{2} V_{s1}}{\pi} \left[\cos(\alpha+u) - \cos(\pi+\alpha) \right]$$

$$= \frac{\sqrt{2} V_{s1}}{\pi} \left[\cos\alpha - \frac{\omega L_s I_d}{\sqrt{2} V_{s_1}} - \underbrace{\cos\pi\cos\alpha}_{-\cos\alpha} + \underbrace{\sin\pi\sin\alpha}_{\text{Zero}} \right]$$

$$V_d = 2\frac{\sqrt{2} V_{s1}}{\pi} \cos\alpha - \frac{\omega L_s I_d}{\pi}$$

$$= 2\,\frac{\sqrt{2}(120)}{\pi} \cos 45^\circ - \frac{(377)(5\times10^{-3})(10)}{\pi}$$

For $\alpha = 135°$



inverter mode of operation.

During Commutation:

$$V_L = L_s \frac{di_{s1}}{dt} = \sqrt{2}\, V_{s_1} \sin\omega t$$

$$\omega L_s \frac{di_{s1}}{d(\omega t)} = \sqrt{2}\, V_{s_1} \sin\omega t$$

$$\cos(\alpha + u) = \cos\alpha - \frac{\omega L_s I_d}{\sqrt{2} V_{s_1}}$$

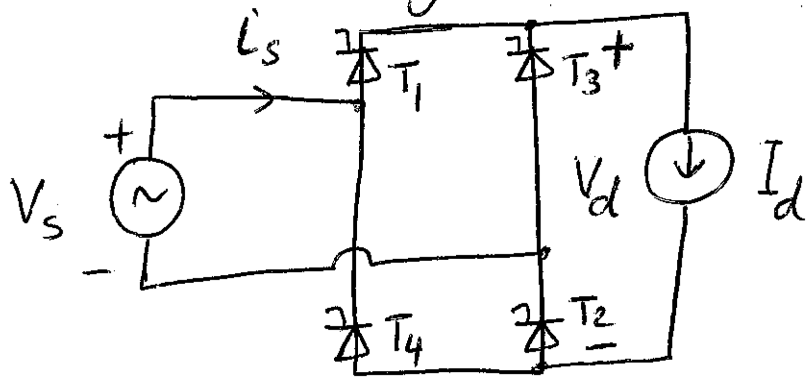
$$V_d = 2\,\frac{\sqrt{2}\,V_{s1}}{\pi}\cos\alpha - \frac{\omega L_s I_d}{\pi}$$

$$= 2\,\frac{\sqrt{2}\,(120)}{\pi}\cos(135^\circ) - \frac{(377)(5\times10^{-3})(10)}{\pi}$$

$$=$$

problem 6-3   pp. 155

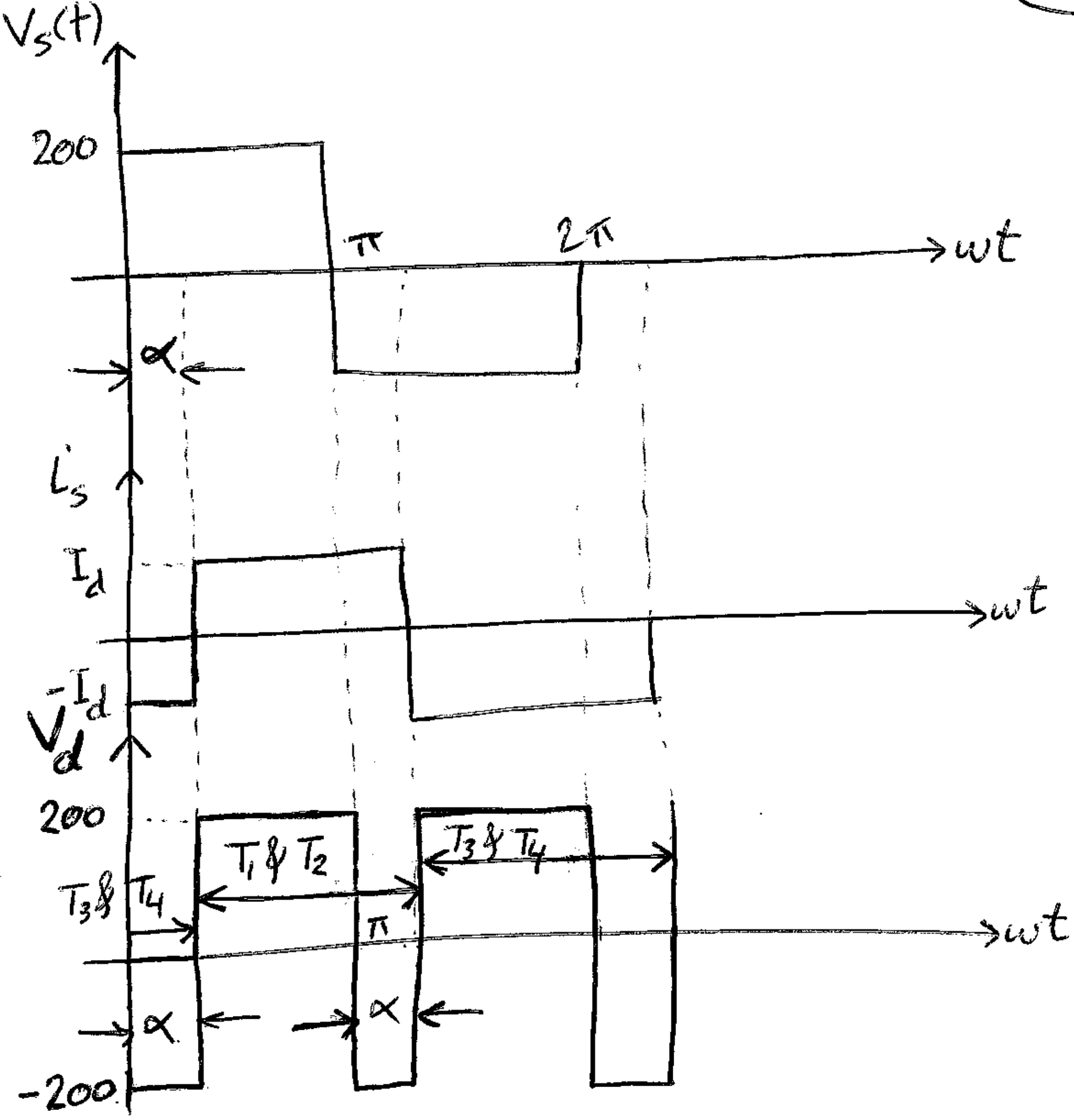
For the following circuit



V_s is a square waveform with an amplitude of 200V

$f = 60\,Hz$, $I_d = 10A$ V_d?

Draw V_d. Calculate V_d for $\alpha = 45^\circ$ & 135°



$$V_d = \frac{1}{2\pi}\left[\int_0^{\alpha} -\hat{V}_d\, d\omega t + \int_{\alpha}^{\pi} \hat{V}_d\, d\omega t + \int_{\pi}^{\pi+\alpha} -\hat{V}_d\, d\omega t + \int_{\pi+\alpha}^{2\pi} \hat{V}_d\, d\omega t\right]$$

$$= \frac{1}{2\pi}\left[-\hat{V}_d\alpha + \hat{V}_d\pi - \hat{V}_d\alpha - \hat{V}_d(\pi+\alpha) + \hat{V}_d\pi \right.$$

$$\left. + \hat{V}_d 2\pi - \hat{V}_d(\pi+\alpha)\right]$$

$$V_d = \frac{1}{2\pi}\Big[-\hat{V}_d\alpha + \hat{V}_d\pi - \hat{V}_d\alpha - \hat{V}_d\pi - \hat{V}_d\alpha +$$
$$\hat{V}_d\pi + \hat{V}_d 2\pi - \hat{V}_d\pi - \hat{V}_d\alpha\Big]$$

$$= \frac{1}{2\pi}\Big[-4\hat{V}_d\alpha + \hat{V}_d 2\pi\Big] = \frac{-2\hat{V}_d\alpha}{\pi} + \hat{V}_d$$

$$\boxed{V_d = \frac{-2}{\pi}\hat{V}_d\alpha + \hat{V}_d}$$

For $\alpha = 45^\circ = \frac{\pi}{4}$

$$V_d = \frac{-2}{\pi}\hat{V}_d\frac{\pi}{4} + \hat{V}_d = -0.5\hat{V}_d + \hat{V}_d = 0.5\hat{V}_d$$

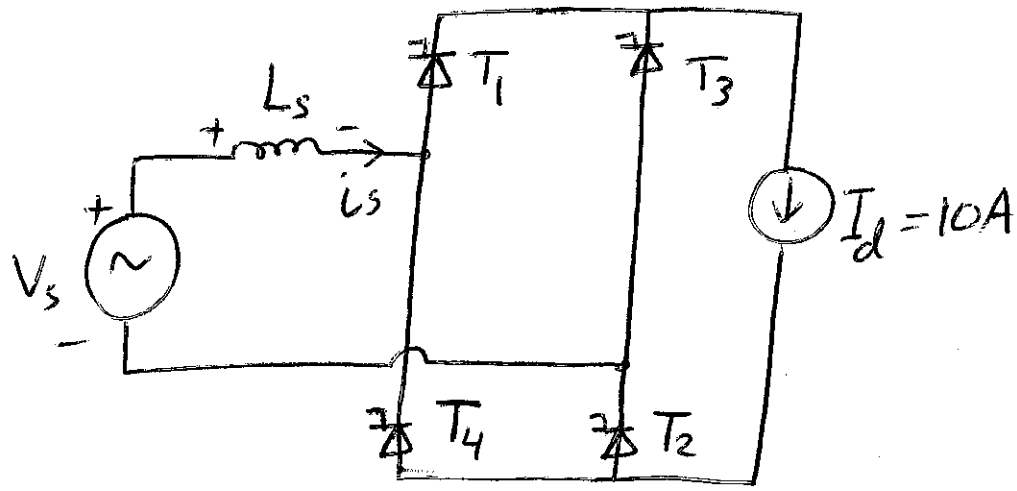
For $\alpha = 135^\circ = \frac{3}{4}\pi$

$$V_d = \frac{-2}{\pi}\hat{V}_d\frac{3}{4}\pi + \hat{V}_d = \frac{-6}{4}\hat{V}_d + \hat{V}_d = \frac{-3}{2}\hat{V}_d + \hat{V}_d$$
$$= -0.5\,V_d$$

OR: (Directly from the waveform)

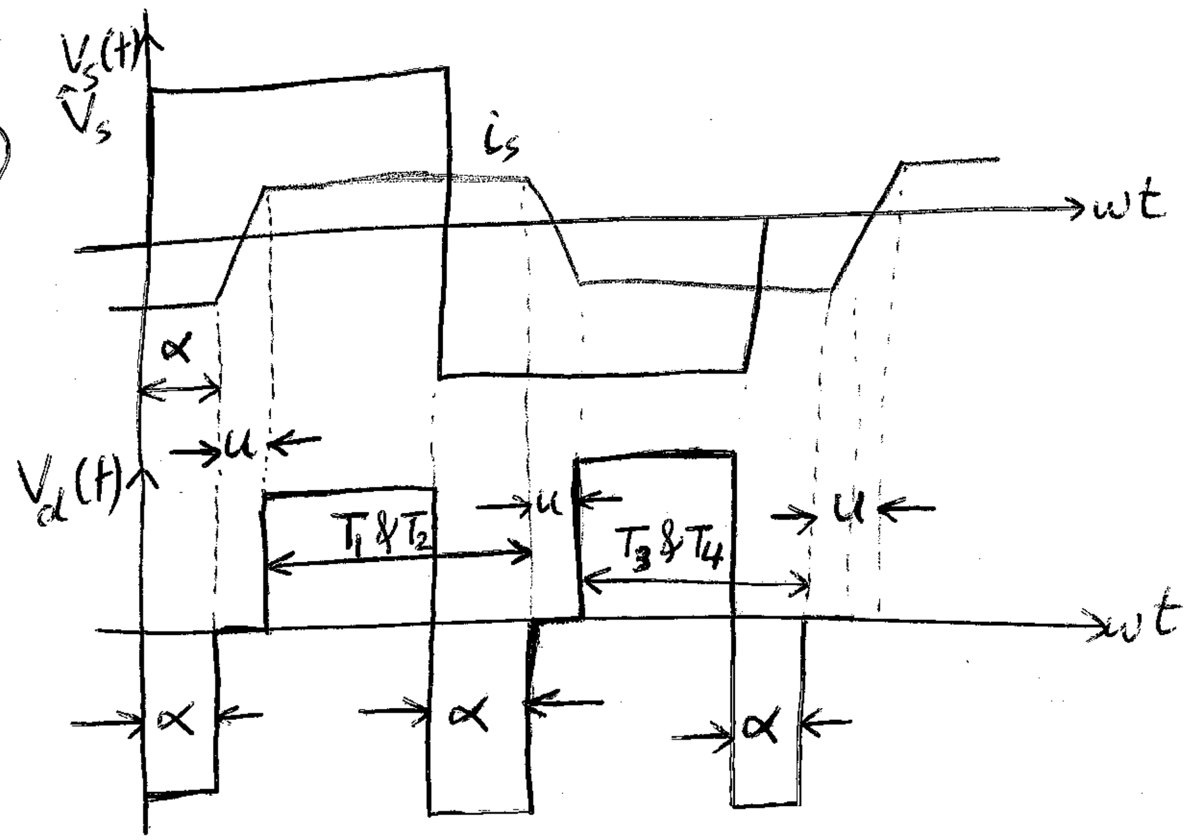
$$V_d = \frac{1}{\pi}\Big[-\hat{V}_d\alpha + \hat{V}_d(\pi - \alpha)\Big] = \frac{1}{\pi}\Big[-2\hat{V}_d\alpha + \hat{V}_d\pi\Big]$$
$$= \frac{-2}{\pi}\hat{V}_d\alpha + \hat{V}_d$$

problem 6-4 PP. 155



V_s is a square wave with amplitude of 200V and frequency of 60Hz. $L_s = 3mH$ & $I_d = 10A$

(a)



$$L_s \frac{di_s}{dt} = V_s(t)$$

$$\omega L_s \frac{di_s}{d(\omega t)} = \hat{V}_s$$

$$\int_{-I_d}^{I_d} \omega L_s \, di_s = \int_{\alpha}^{\alpha+u} \hat{V}_s \, d(\omega t)$$

$$\omega L_s 2 I_d = \hat{V}_s(\alpha+u) - \hat{V}_s\alpha$$

$$2\omega L_s I_d = \hat{V}_s u$$

$$u = \frac{2\omega L_s I_d}{\hat{V}_s}$$

$$V_d = \frac{1}{\pi}\left[\int_0^{\alpha} -\hat{V}_s d\omega t + \int_{\alpha+u}^{\pi} \hat{V}_s d\omega t\right]$$

$$= \frac{1}{\pi}\left[-\hat{V}_s\alpha + \hat{V}_s\pi - \hat{V}_s(\alpha+u)\right]$$

$$= \frac{1}{\pi}\left[-\hat{V}_s\alpha + \hat{V}_s\pi - \hat{V}_s\alpha - \hat{V}_s u\right]$$

$$V_d = \frac{1}{\pi}\left[-2\hat{V}_s\alpha + \hat{V}_s\pi - \hat{V}_s u\right]$$

$$= \frac{-2}{\pi}\hat{V}_s\alpha + \hat{V}_s - \frac{u}{\pi}\hat{V}_s$$

$$= \frac{-2}{\pi}\hat{V}_s\alpha + \hat{V}_s - \frac{1}{\pi}\hat{V}_s \frac{2\omega L_s I_d}{\hat{V}_s}$$

$$V_d = \frac{-2}{\pi}\hat{V}_s\alpha + \hat{V}_s - \frac{1}{\pi}2\omega L_s I_d$$

(b) $\underline{\alpha = 35^\circ}$

$$u = \frac{(2)(377)(3\times10^{-3})(10)}{200} =$$

$$V_d = \frac{-2}{\pi}(200)\left(35\frac{\pi}{180}\right) + 200 - \frac{1}{\pi}(2)(377)(3\times10^{-3})(10)$$

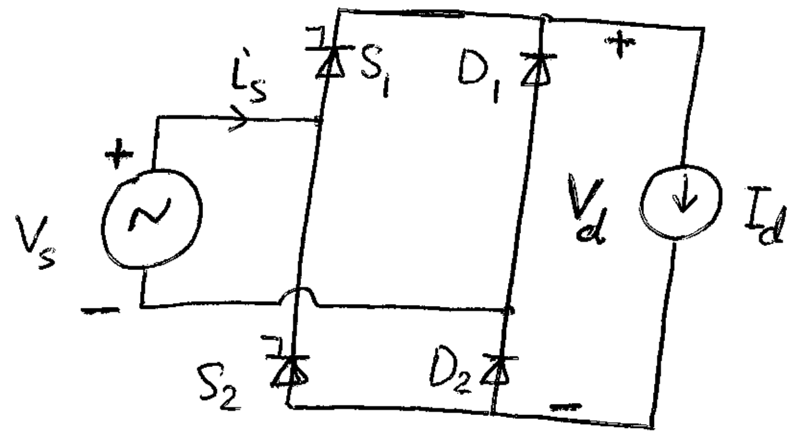
$\underline{\alpha = 135^\circ}$

$u =$

$$V_d = \frac{-2}{\pi}(200)\left(135\frac{\pi}{180}\right) + 200 - \frac{1}{\pi}(2)(377)(3\times10^{-3})(10)$$

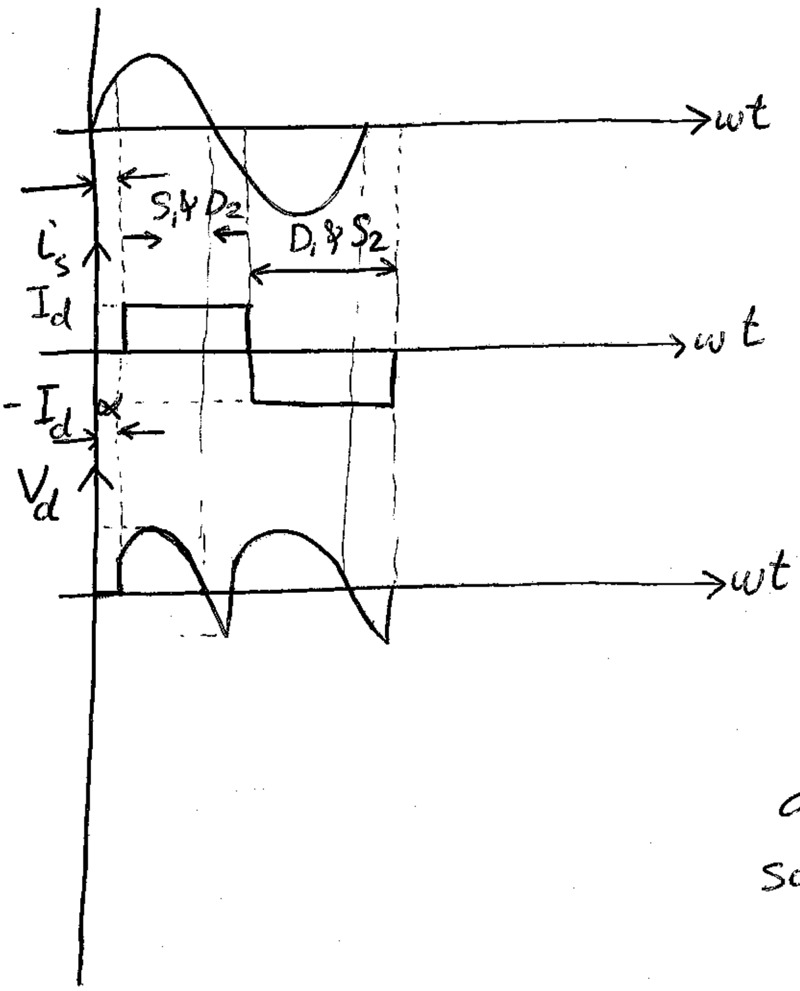
problem 5-5 PP. 155

For the following circuit:



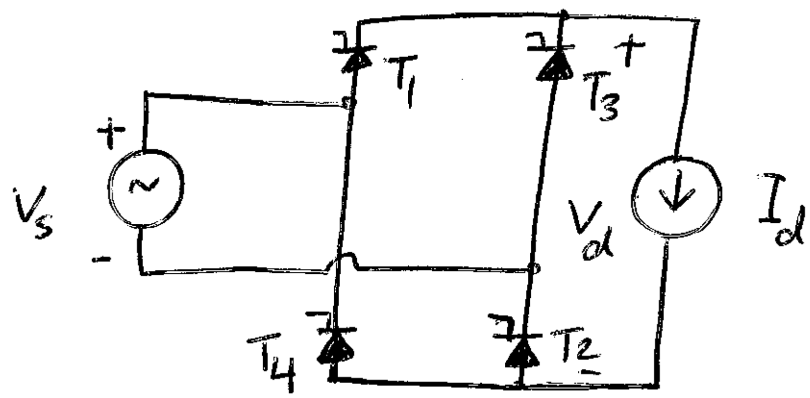
single-phase half-controlled converter (rectifier)

For $\alpha = 45°, 90°$ and $135°$ draw V_s, i_s, V_d, DPF, PF, THD

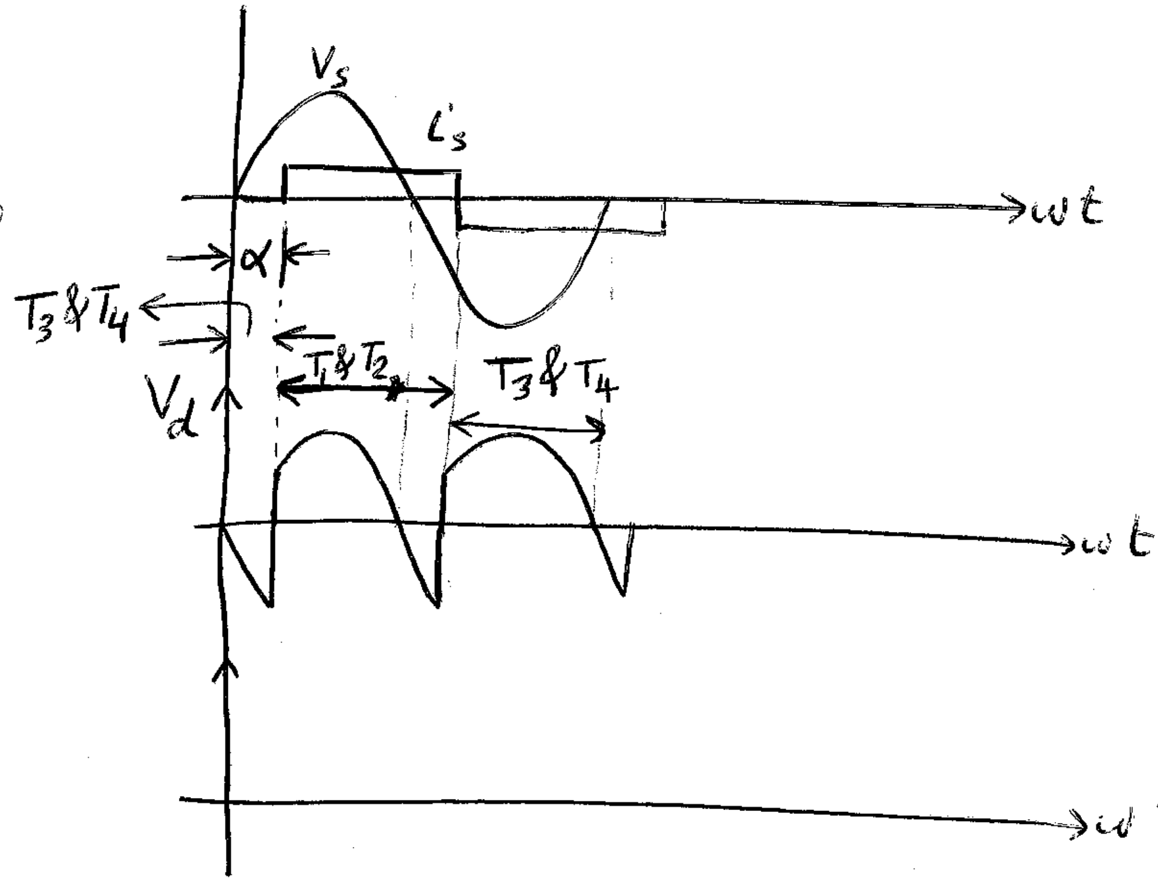


and so on and so forth

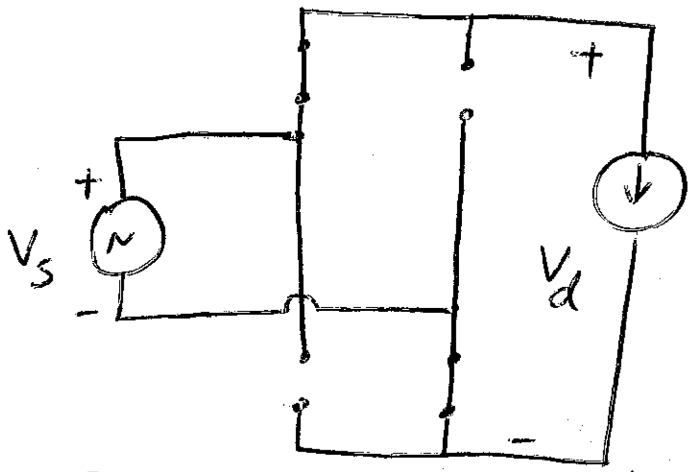
problem 6-6 pp. 155



$V_s = \sqrt{2} V_s \sin\omega t$



when T_1 & T_2 are on , T_3 & T_4 are off



PIV = $\sqrt{2} V_s$ ⟶ peak inverse voltage

* The average value of the current through each thyristor is zero.

* The rms value of the current through each thyristor is I_d.

How to Solve Differential Equations @ in MATLAB

* First Order Equation:

$$\dot{y} = A \sin 10t - By$$

$$A = 10, \quad B = 5$$

```
function yprime = RL(t,x,FLAG,A,B)
yprime = [A*sin(10*t)-B*y];
```

m-file called RL.m

```
>> A=10;
>> B=5;
>> Tspan = [0   5];
>> yo = [1];
>> options = [];
>> [t1,x1] = ode45('RL', Tspan,yo,options,A,B)
>> plot(t1,x1);
```

* Second Order Equation: (b)

$$\frac{d^2x}{dt^2} - \mu(1-x^2)\frac{dx}{dt} - x = 0$$

In state space representation:

$$y_1 = x$$

$$y_2 = \dot{y}_1 = \dot{x}$$

$$\boxed{\dot{y}_1 = y_2}$$

$$\boxed{\dot{y}_2 = \mu(1-y_1^2)y_2 - y_1}$$

m-file example2.m

```
function yprime = example2(t, y, FLAG, mu)
  yprime = [y(2);
           mu*(1-(y(1))^2)*y(2) - y(1)];
```

```
>> mu = 2;
>> Tspan = [0 30];
>> yo = [1;0];
>> [t,y] = ode45('example2', Tspan, yo, options, mu);
>> plot(t, y(:,1));    >> plot(t, y(:,2));    >> plot(y(:,1), y(:,2))
```

C

$$
\left.\begin{aligned}
A\dot{x}_1 + B\dot{x}_2 &= \sin x_1 + e^{x_2} \\
C\dot{x}_1 + D\dot{x}_2 &= 3x_1 + 5x_2
\end{aligned}\right\}
\quad
\begin{aligned}
A &= 2 \\ B &= 5 \\ C &= 1 \\ D &= 6
\end{aligned}
$$

```
>> syms x1 x2
>> A = 2;
>> B = 5;
>> C = 1;
>> D = 6;
>> AA=[A B; C D];
>> F1= sin x1 + exp(x2);
>> F2= 3*x1 + 5*x2;
>> F = inv(A)*[F1; F2];
>> F =
```

$$\begin{bmatrix} A \end{bmatrix}^{-1} \begin{bmatrix} F1 \\ F2 \end{bmatrix} =$$

(d)

$$x = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 5 \\ 6 \\ 7 \end{bmatrix} \qquad y = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 6 \\ 3 \\ 2 \end{bmatrix}$$

```
>> x = [0;1;3;5;6;7];
>> y = [2;3;4;6;3;2];
>> A = polyfit(x,y,3)
>> plot(x, y, x, A(1)*x.^3 + A(2)*x.^2 + A(3)*x + A(4))
```

⇒ coefficients of the polynomial representing y vs. x.

H.W

Full bridge uncontrolled rectifier feeding battary

$V_s(t) = \sqrt{2}\,110 \sin \underline{\underline{wt}}$, $w = 377$ rad/s

$L_s = 10mH$. ideal diodes!

dc-dc Switch-Mode Converters

* ## Applications

① dc motordrive applications.
② regulated power supplies.

* Switch-mode dc-dc converters are used to convert the unregulated dc input into a controlled dc output at a desired voltage level.

* The following dc-dc converters will be discussed:

1- step-down (buck) converter.
2- Step-up (boost) converter.

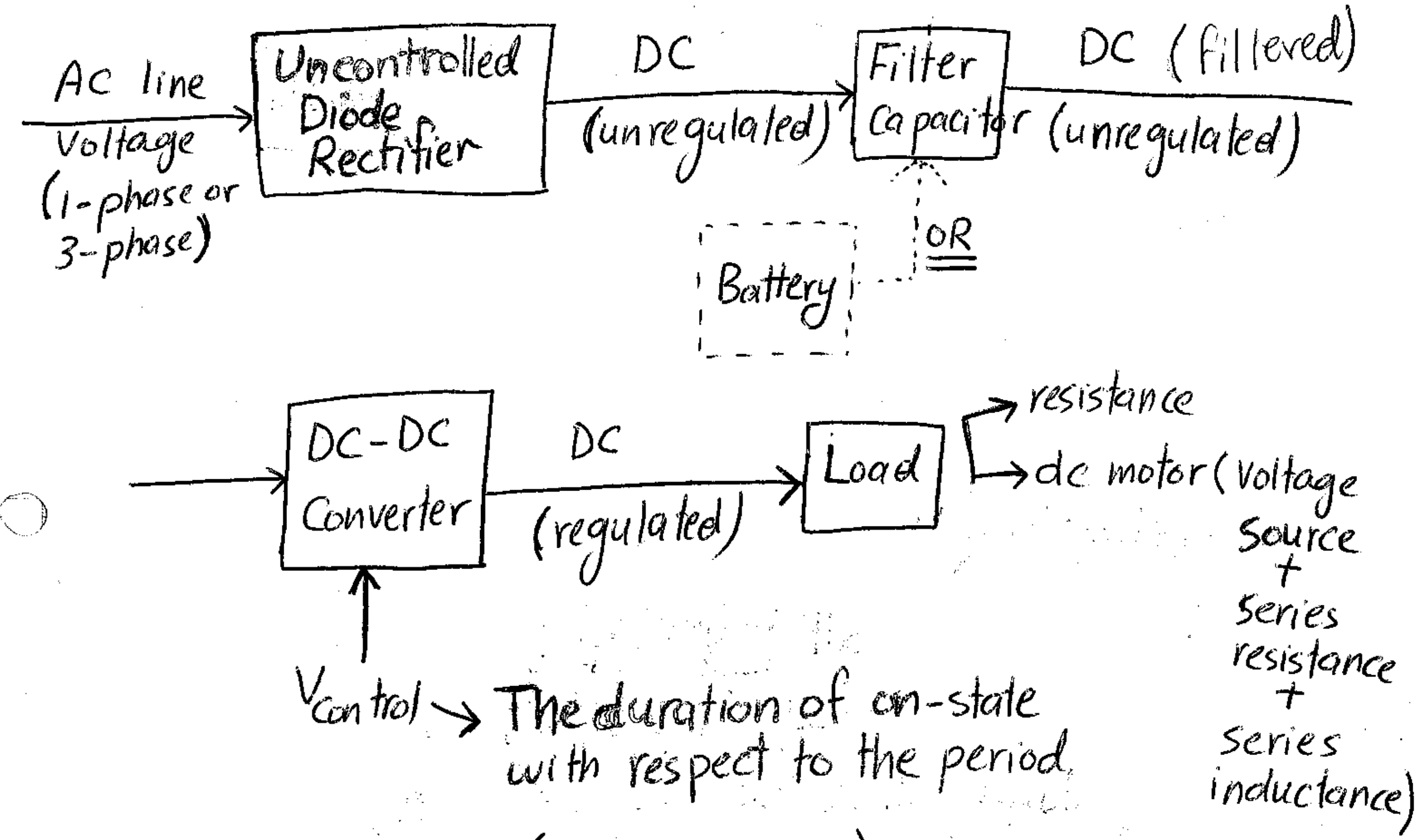
(1 and 2: Basic converters)

3- Step-down/step-up (buck-boost) converter.
4- Cûk converter → step up and stepdown.

(3 and 4: combination of both 1&2.)

5- Full bridge converter → derived from the step-down

* The dc-dc converter system is:

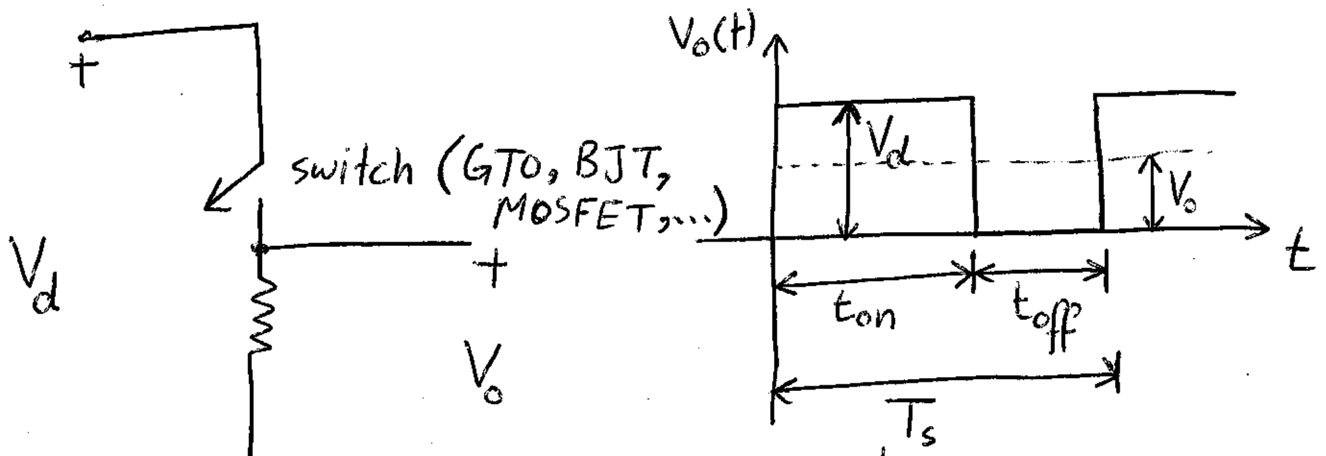


(duty cycle, D)
(ratio)

* In this chapter:
  - (a) the converters are analyzed in steady state.
  - (b) the switches are treated as ideal.
  - (c) the losses in the inductive and the capacitive elements are neglected.

Control of dc-dc converters

The concept of the switch mode converter is:

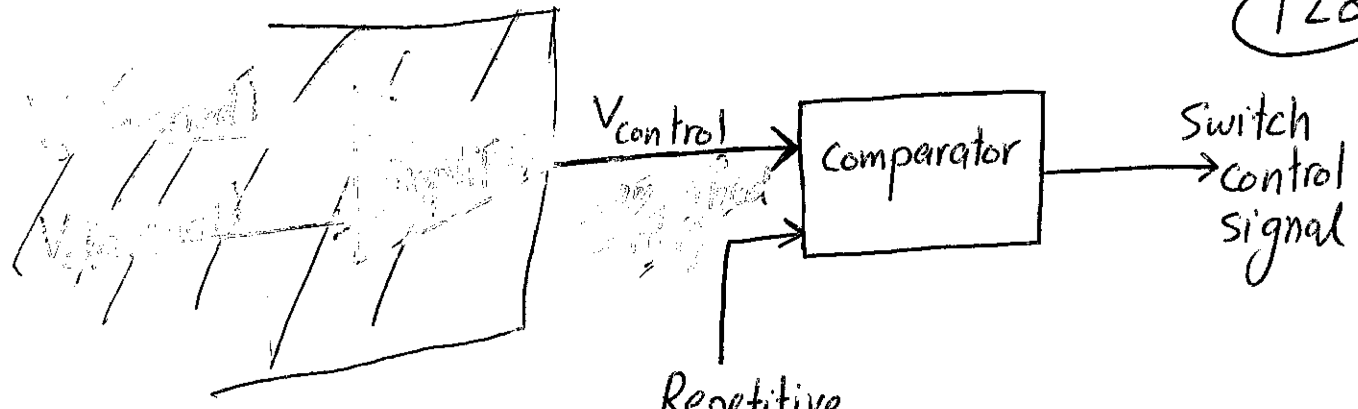


$$V_o = \frac{1}{T_s}\int_0^{t_{on}} V_d\,dt = \frac{t_{on}}{T_s} V_d = D V_d$$

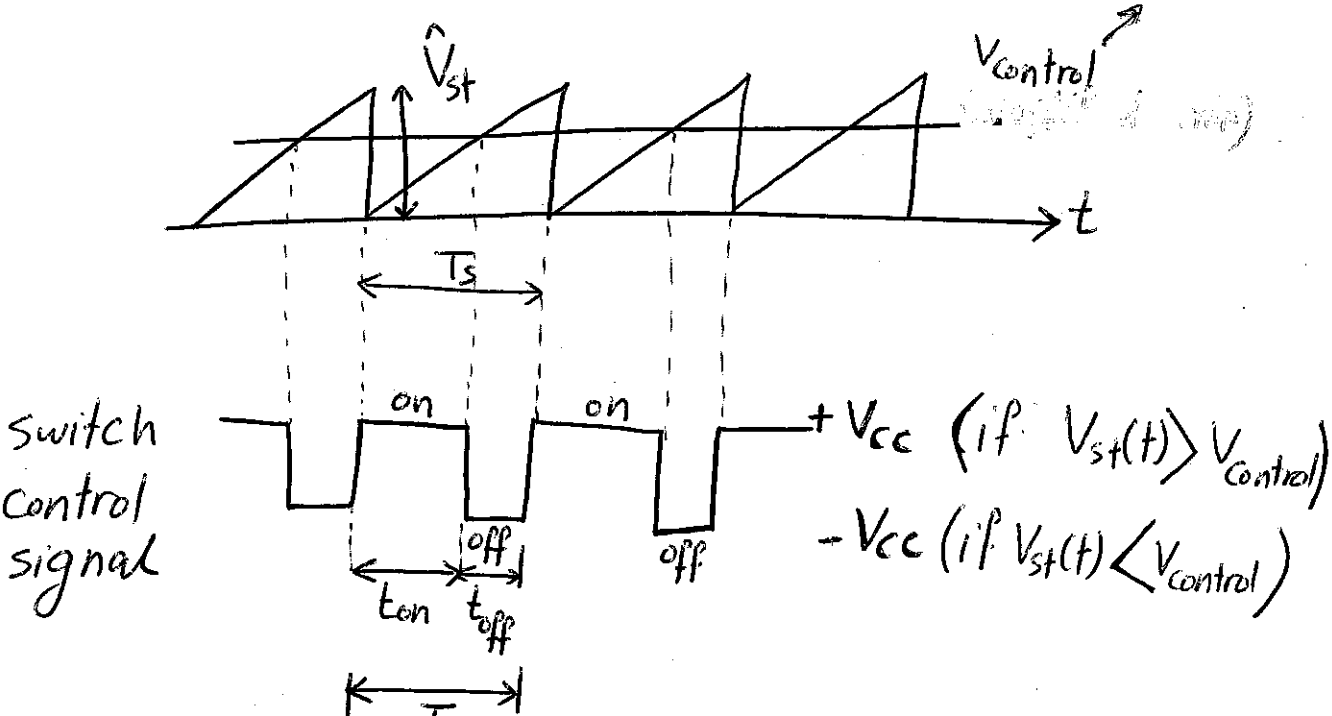
* There are two methods of control:

1. Varying t_{on} only. ⇒ will be discussed in this chapter.
2. varying t_{on} and T_s.

* One of the methods of controlling the output voltage is by switching at a constant frequency $\frac{1}{T_s}$ (constant switching time T_s) and adjusting the on duration of the switch to control the average output voltage. In this method, the duty ratio D is varied. This method is called Pulse Width Modulation (PWM)



(a)



$f_s = 1/T_s$ (b)

pulse width modulator (PWM) (a) block diagram

(b) comparator signals

$$\text{duty ratio} \leftarrow D = \frac{t_{on}}{T_s} = \frac{V_{control}}{\hat{V}_{st}}$$

* dc-dc converters have two distinct modes of operation:
  (1) continuous current conduction.
  (2) discontinuous current conduction.

Step-Down (Buck) Converter

* It is a step-down converter, which produces a lower average output voltage than the dc input voltage.

$$V_o < V_d .$$

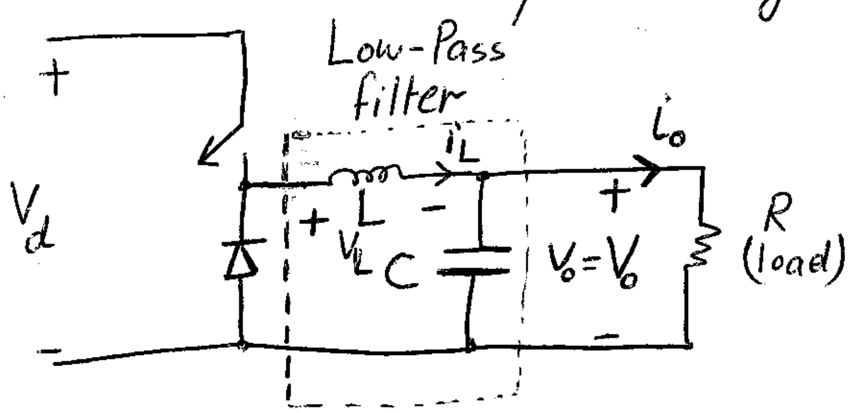
$$V_o = D V_d = \frac{t_{on}}{T_s} V_d$$

$$V_o = \frac{V_d}{\hat{V}_{st}} V_{control} = k \, V_{control}$$

D

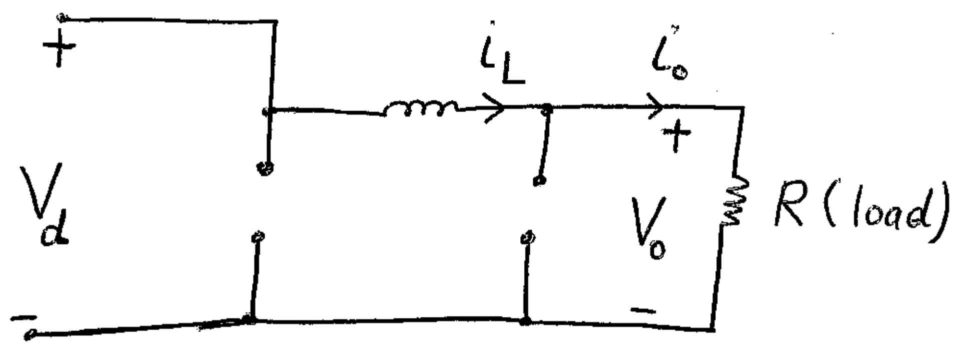
* The circuit of step-down dc-dc converter is:

→ Passes only the low frequencies

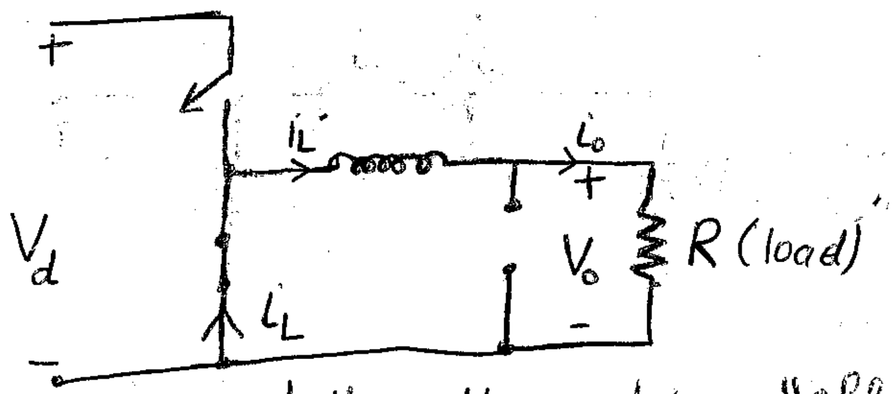


* C is very large such that $v_o(t) \approx V_o$.

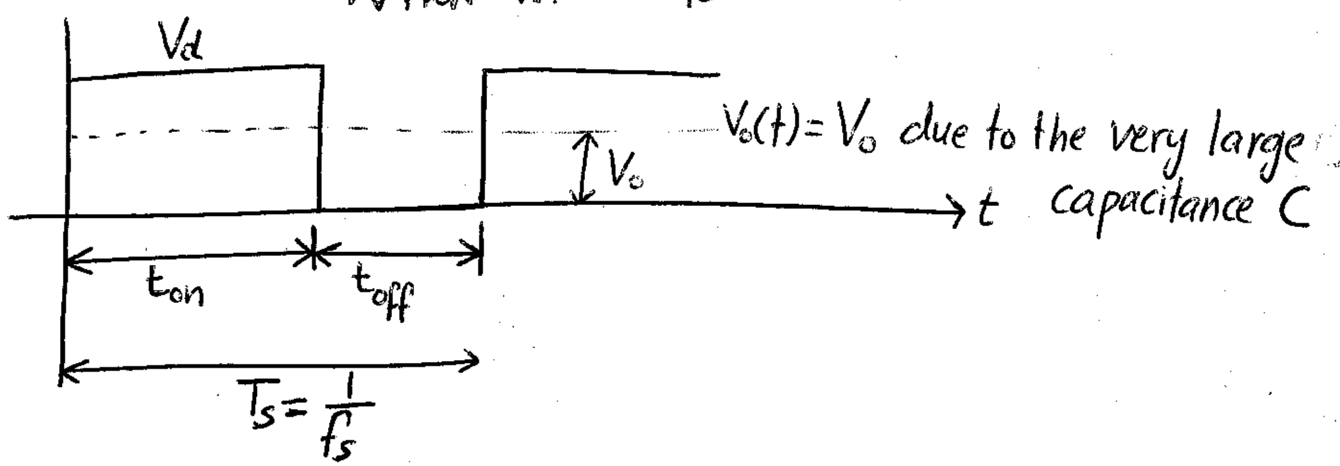
* When the switch is on, the diode becomes reverse biased and the input provides energy to the load as well as to the inductor. During the interval when the switch is off, the inductor current flows through the diode, transferring some of its stored energy to the load.



When the switch is "on"



When the switch is "off"

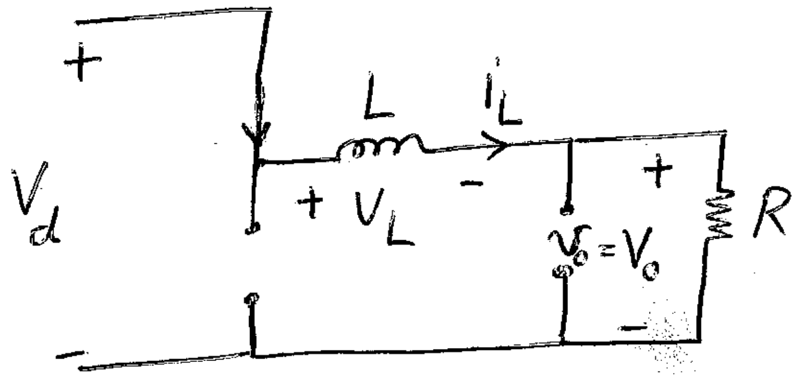


* The average inductor current I_L equals the average load current I_o since the average capacitor current I_C in steady state equals zero

$$i_c = C\frac{dV_o(t)}{dt} = C\frac{dV_o}{dt} = 0 \Rightarrow I_c = 0.$$

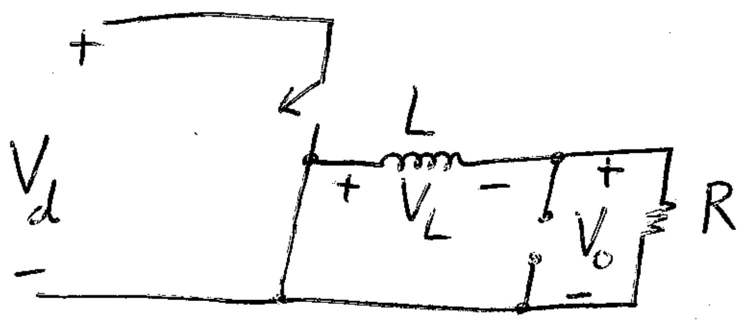
Continuous-Conduction Mode

When the switch is on, we have the following circuit:



$$-V_d + V_L + V_o = 0 \Rightarrow \boxed{V_L = V_d - V_o}$$

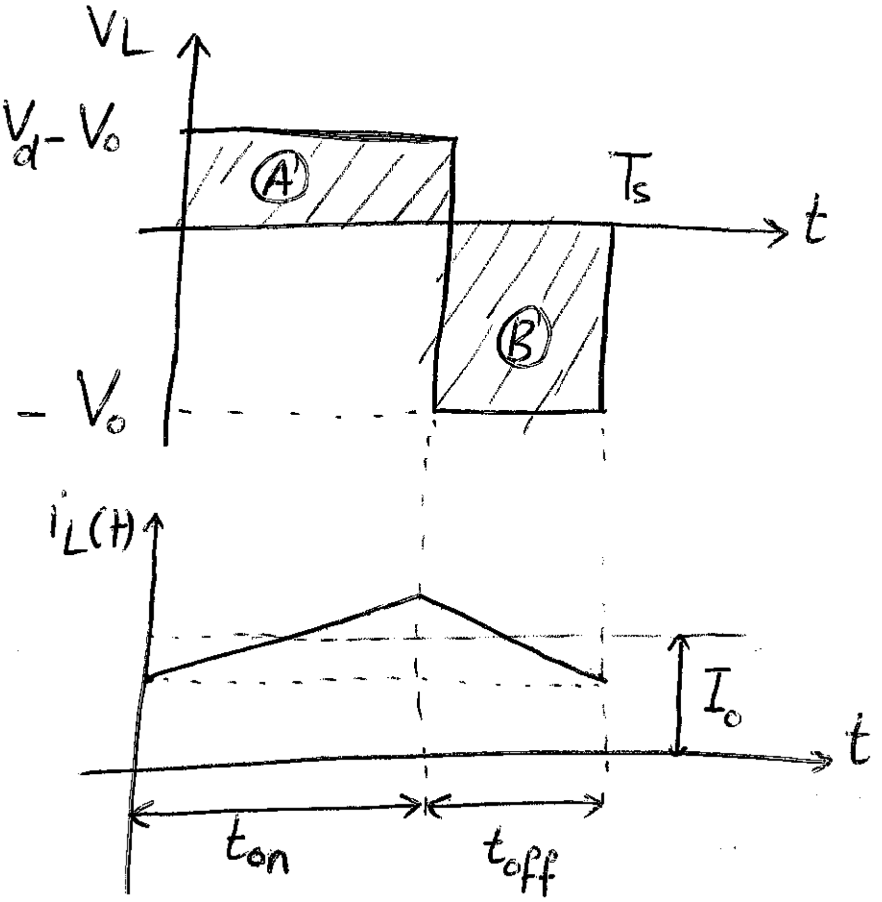
when the switch is off, we have the following circuit



$$+V_L + V_o = 0$$

$$\boxed{V_L = -V_o}$$

* The waveforms of V_L & i_L will be:



$$A = B$$

$$V_L = L\frac{di_L}{dt}$$

$$i_L = \frac{1}{L}\int V_L dt$$

$$= \frac{1}{L}\int (V_d - V_o)dt$$

$$i_L = \frac{1}{L}(V_d - V_o)t$$

$$T_s = t_{on} + t_{off}$$

$$\int_0^{T_s} V_L dt = \int_0^{t_{on}} V_L dt + \int_{t_{on}}^{T_s} V_L dt = 0$$ $\Rightarrow$ average value of V_L

$$\int_0^{t_{on}} (V_d - V_o)dt + \int_{t_{on}}^{T_s} -V_o\, dt = 0$$

$$(V_d - V_o)\, t_{on} = V_o(T_s - t_{on})$$

$$V_d\, t_{on} - \cancel{V_o t_{on}} = V_o T_s - \cancel{V_o t_{on}}$$

$$V_d\, t_{on} = V_o T_s$$

$$\frac{V_o}{V_d} = \frac{t_{on}}{T_s} = D \text{ (duty ratio)}$$

* Therefore, the output voltage ($V_o = D V_d$) varies linearly with the duty ratio of the switch (doesn't depend on the circuit parameters).

* $P_d = P_o$ (neglecting power losses in all circuit elements), (assuming lossless inductance, capacitance and switch)

input power ← P_d; P_o → output power

$$V_d I_d = V_o I_o$$

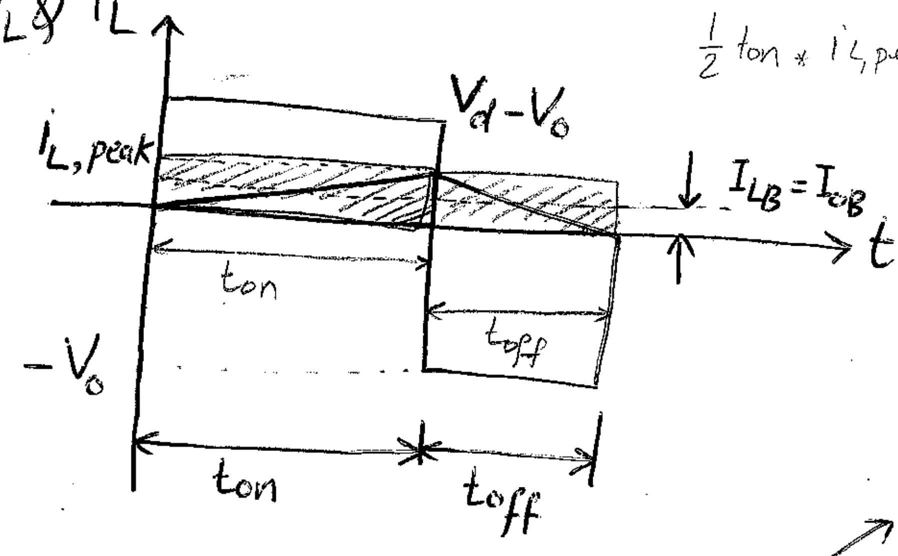
$$\frac{V_d}{V_o} = \frac{I_o}{I_d} = \frac{1}{D} \Rightarrow$$ *dc transformer

* the turns ratio can be continuously controlled in the range of 0–1 by controlling the duty ratio of the switch.

Boundary Between Continuous and Discontinuous Conduction

The following figure shows the waveforms of V_L & i_L at the boundary between the continuous and the discontinuous mode. The inductor current i_L goes to zero at the end of the off period.

$\frac{1}{2} t_{on} * i_{L,peak} + \frac{1}{2} t_{off}\, i_{L,peak} \left(\frac{t_{on} + t_{off}}{T_s}\right)$



$D = \frac{t_{on}}{T_s}$

$$I_{LB} = \frac{1}{T_s}\left[\frac{1}{2} t_{on}\, i_{L,peak} + \frac{1}{2} t_{off}\, i_{L,peak}\right]$$

$i_{L,peak}$

$$I_{LB} = \frac{1}{2} i_{L,peak} = \frac{t_{on}}{2L}(V_d - V_o) = \frac{D T_s}{2L}(V_d - V_o) = I_{oB}$$

$i_{L,peak}$:

$\Downarrow$ show this:

$$i_L = \frac{1}{L}\int (V_d - V_o)\,dt = \frac{1}{L}(V_d - V_o)t + K$$

$$i_L(0) = 0 = \frac{1}{L}(V_d - V_o)(0) + K \Rightarrow K = 0$$

$$i_L(t) = \frac{1}{L}(V_d - V_o)t \qquad 0 < t < t_{on}$$

$$i_L(t)_{peak} = \frac{1}{L}(V_d - V_o)\,t_{on} \quad \text{at} \quad t = t_{on}$$

$$\boxed{i_{L,peak} = \frac{1}{L}(V_d - V_o)\, t_{on}}$$

$$I_{LB} = \frac{1}{2}\, i_{L,peak} = \frac{1}{2}\,\frac{1}{L}(V_d - V_o)\, t_{on}$$

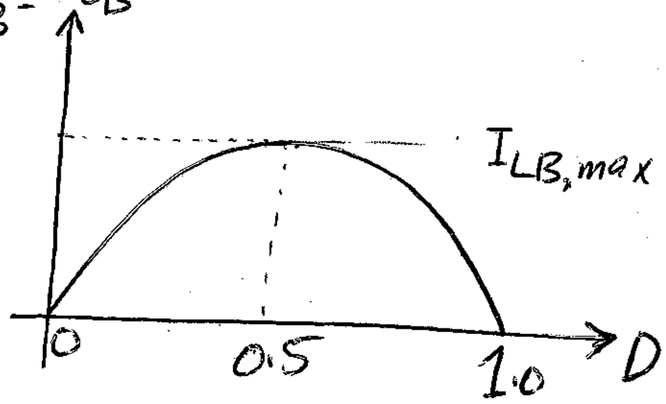
$$I_{LB} = \frac{t_{on}}{2L}(V_d - V_o)$$

but $D = \dfrac{t_{on}}{T_s}$

$$I_{LB} = \frac{D\,T_s}{2L}(V_d - V_o)$$

* I_{LB} as function of D is:

$$I_{LB} = \frac{D\,T_s}{2L}(V_d - D V_d) = \frac{D\,T_s V_d}{2L} - \frac{D^2 T_s V_d}{2L}$$



OR

$$I_{oB} = I_{LB} = \frac{1}{t_{on}}\int_0^{t_{on}} \left(\frac{V_d - V_o}{L}\, t\right) dt = \frac{1}{t_{on}}\,\frac{V_d - V_o}{L}\,\frac{t^2}{2}\Bigg]_0^{t_{on}}$$

$$= \frac{1}{t_{on}}\,\frac{V_d - V_o}{L}\,\frac{t_{on}^2}{2} = \frac{t_{on}}{2}\,\frac{V_d - V_o}{L} = \frac{D\,T_s}{2L}(V_d - V_o)$$

Discontinuous-Conduction Mode

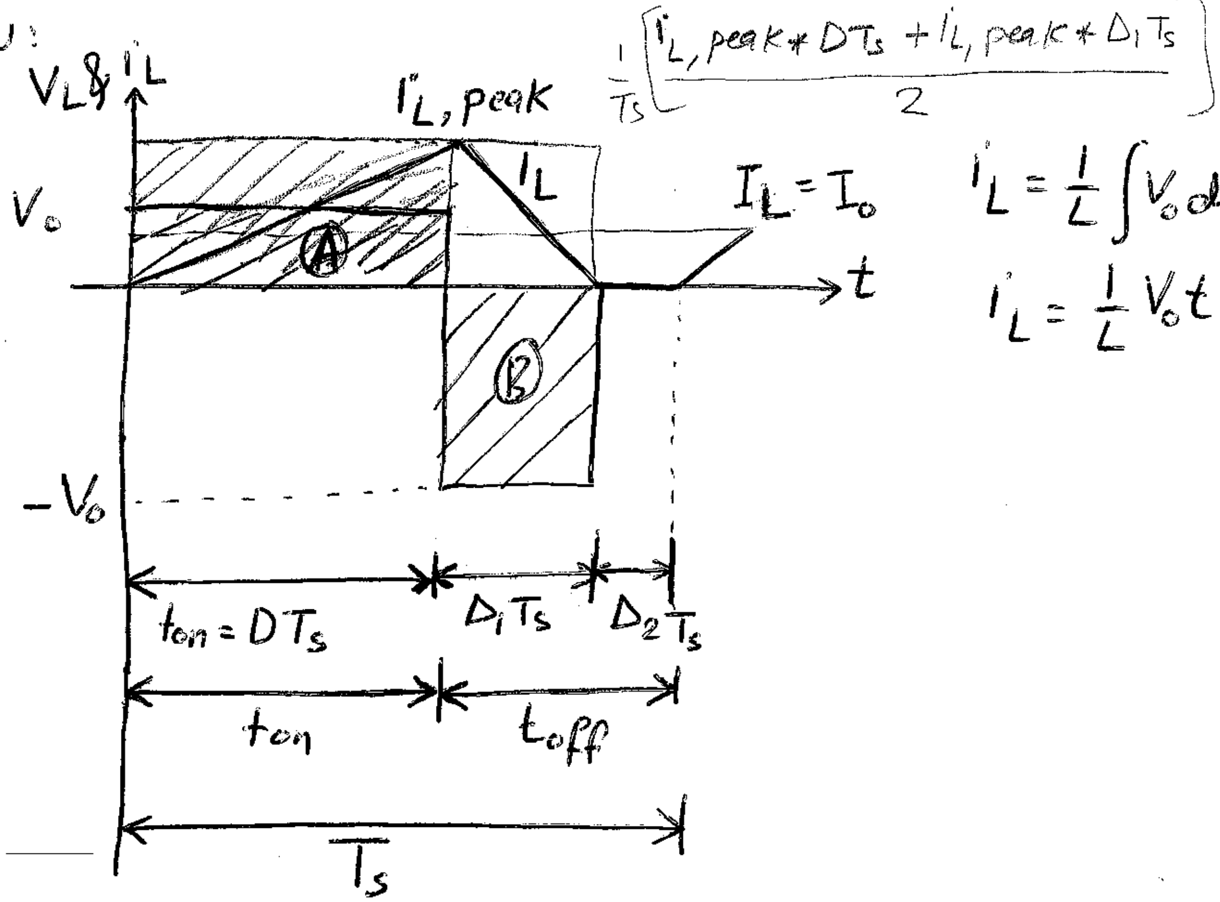
depending on the application of these converters, either the input voltage V_d or the output voltage V_o remains constant during the converter operation.

1- Discontinuous-Conduction Mode with Constant V_d

* To control the speed of a dc motor, V_d should remain constant and V_o should vary by adjusting the duty cycle (ratio) D.

* Let us assume that initially the converter is operating at the edge of the continuous conduction for given values of T_s, L, V_d and D. If these parameters are kept constant and the output power is decreased (load resistance increases), then the average load current will decrease. This

dictates a higher value of V_o than before and results in a discontinuous inductor current as shown below:



$$i_L = \frac{1}{L}\int V_o\, dt$$

$$i_L = \frac{1}{L} V_o t$$

* During the interval $\Delta_2 T_s$ where the inductor current is zero, the power to the load resistance is supplied by the filter capacitor alone. V_L during this interval equals zero.

Again:

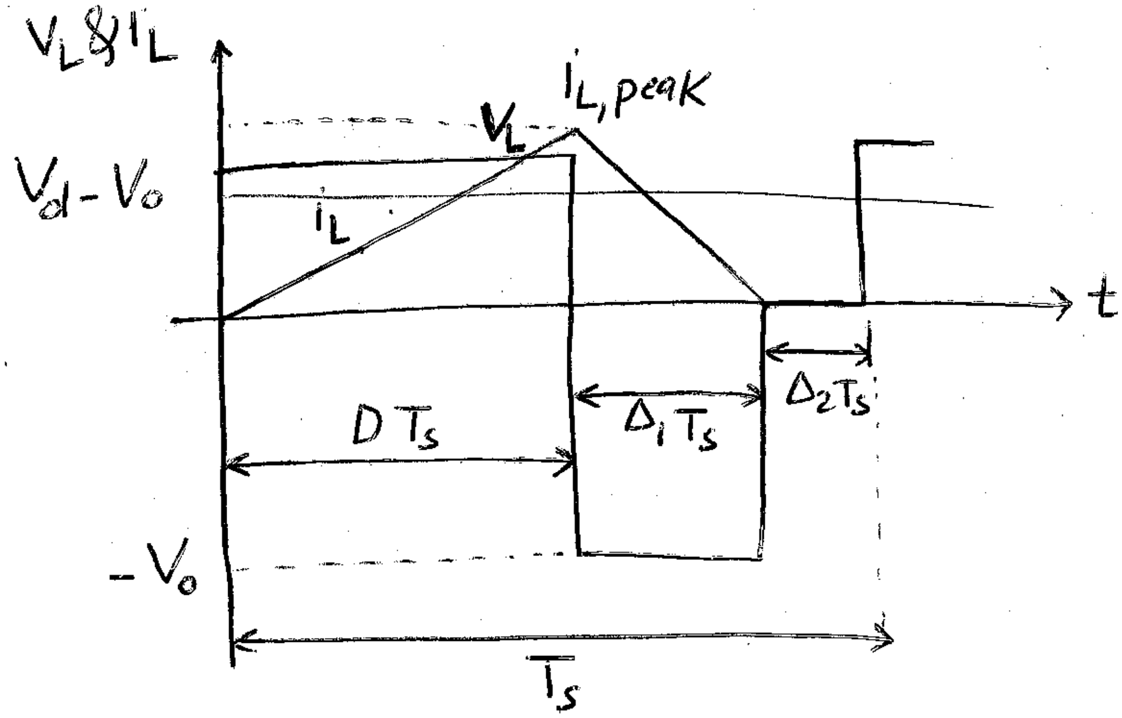
$$A = B$$

$$(V_d - V_o) DT_s = V_o \Delta_1 T_s$$

$$V_d DT_s - V_o DT_s = V_o \Delta_1 T_s$$

$$V_d D T_s = V_o \Delta_1 T_s + V_o D T_s$$

$$\frac{V_o}{V_d} = \frac{D}{D + \Delta_1}$$



$$I_L = \frac{1}{T_s}\left[\frac{1}{2} i_{L,peak} * D T_s + \frac{1}{2} i_{L,peak} * \Delta_1 T_s\right]$$

$$I_L = i_{L,peak}\left(\frac{D + \Delta_1}{2}\right)$$

but $\quad i_{L,peak} = \frac{V_d - V_o}{L} D T_s$

and $\quad \underbrace{(V_d - V_o) D T_s} = V_o \Delta_1 T_s$

$$\therefore \quad i_{L,peak} = \frac{V_o}{L} \Delta_1 T_s$$

$$I_L = \left(\frac{D+\Delta_1}{2}\right) i_{L,peak}$$

$$= \left(\frac{D+\Delta_1}{2}\right) \frac{V_o}{L} \Delta_1 T_s$$

$$= \left(\frac{D+\Delta_1}{2}\right) \frac{1}{L} \Delta_1 T_s \frac{D V_d}{D+\Delta_1}$$

constant

$$I_L = \frac{V_d T_s}{2L} D \Delta_1$$

$$\Delta_1 = \frac{2L I_L}{V_d T_s D}$$

fluctuated

$$\therefore \frac{V_o}{V_d} = \frac{D}{D+\Delta_1} = \frac{D}{D + \frac{2L I_L}{V_d T_s D}}$$

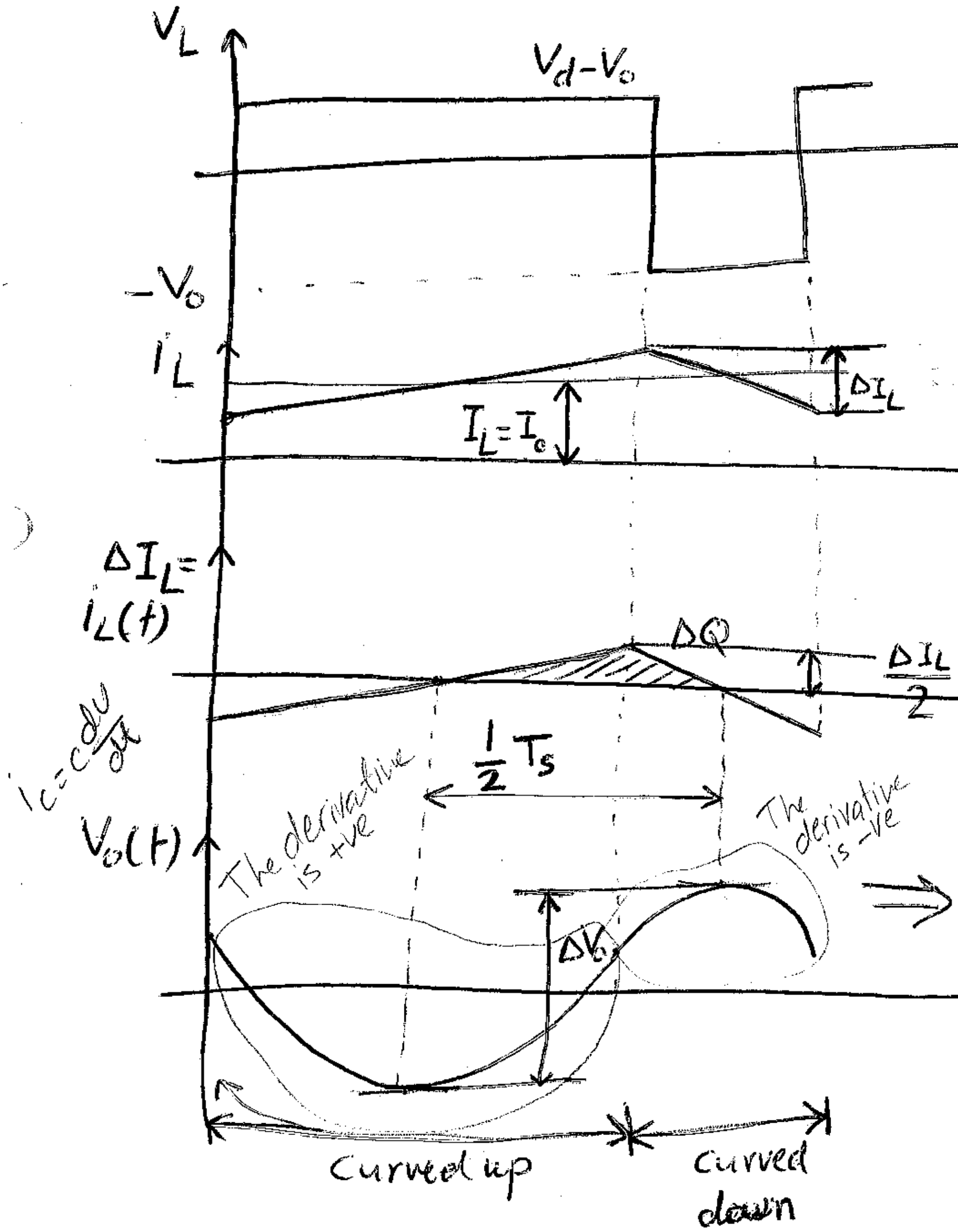
constant

Discontinuous - Conduction Mode with Constant V_o

* In applications such as regulated dc power supplies V_d may fluctuate but V_o is kept constant by adjusting the duty ratio D.

Output Voltage Ripple

In the previous analysis, the output capacitor is assumed to be so large as to yield $V_o(t) = V_o$. However, the ripple in the output voltage with a practical value of capacitance can be calculated by considering the waveforms:



through the capacitor

through the resistor

current ripple

$\underline{I_o} + \underbrace{\Delta I_L}_{i_L(t)\text{ ac component}}$

$i = \frac{dq}{dt} \Rightarrow q = \int i(t)\,dt$

$i_L(t) = I_o + i_{L_{ac}}(t)$

$\Rightarrow$ which is across the capacitor

$i_c = C\frac{dv}{dt}$

$V_o(t) = \frac{1}{C}\int \Delta I_L\,dt$

140''

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{1}{C}\,\frac{1}{2}\,\frac{T_s}{2}\,\frac{\Delta I_L}{2}$$

but $\quad \Delta I_L = \frac{V_o}{L}(1-D)\,T_s \Rightarrow$ Try to get it from the continuous mode of operation

$$\therefore \quad \Delta V_o = \frac{1}{8}\,\frac{T_s}{C}\,\frac{V_o}{L}(1-D)\,T_s$$

$$\frac{\Delta V_o}{V_o} = \frac{1}{8}\,\frac{T_s^2(1-D)}{LC}$$

$\Rightarrow$ The voltage ripple is independent of the output power.

<u>OR</u>

$$\frac{\Delta V_o}{V_o} = \frac{\pi^2}{2}(1-D)\left(\frac{f_c}{f_s}\right)^2$$

where $\quad f_c = \frac{1}{2\pi\sqrt{LC}}$ corner frequency of the filter (resonant frequency)

$\Rightarrow$ go back to page 140 to "Step-up Converter"

For discontinuous mode of operation:

$$\text{constant} \leftarrow V_o \qquad \frac{V_o}{V_d} = \frac{D}{D+\Delta_1}$$
$$\text{fluctuated} \leftarrow V_d$$

$$I_o = \frac{V_o T_s}{2L}(D+\Delta_1)\Delta_1$$

From which we can get Δ_1.

$\Longrightarrow$ go to the output voltage ripple next page

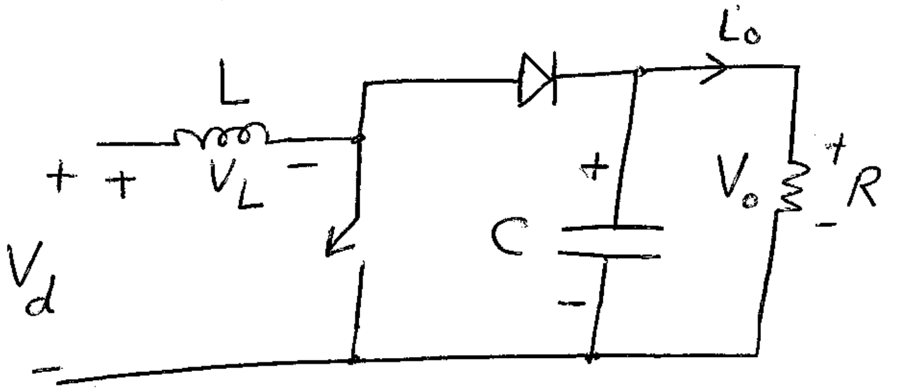
Step-Up (Boost) Converter

Applications:

1. regulated dc power supply.
2. Control of dc motors.

The step-up converter (boost converter) is:

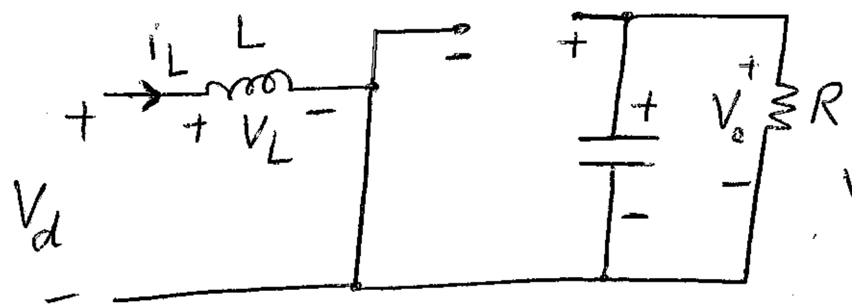
$\Rightarrow$



$-V_L + V_D + V_o = 0$

$V_D = V_L - V_o$

* When the switch is on, the diode is reversed biased and so we have the following circuit:



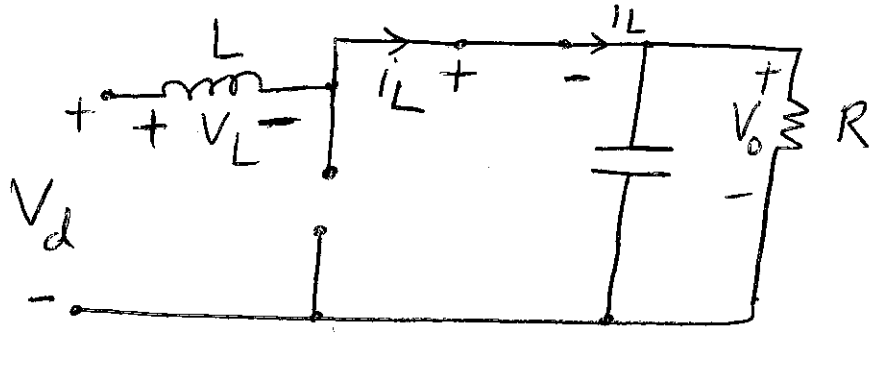
$\Rightarrow$ The circuit when the switch "on"

$V_L = V_d$

switch on and the diode is reversed biased

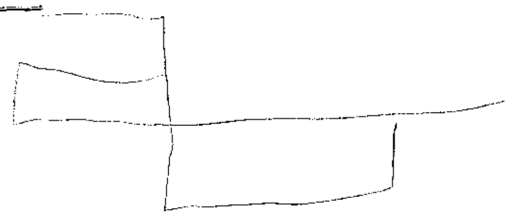
In this case the input supplies energy to the inductor.

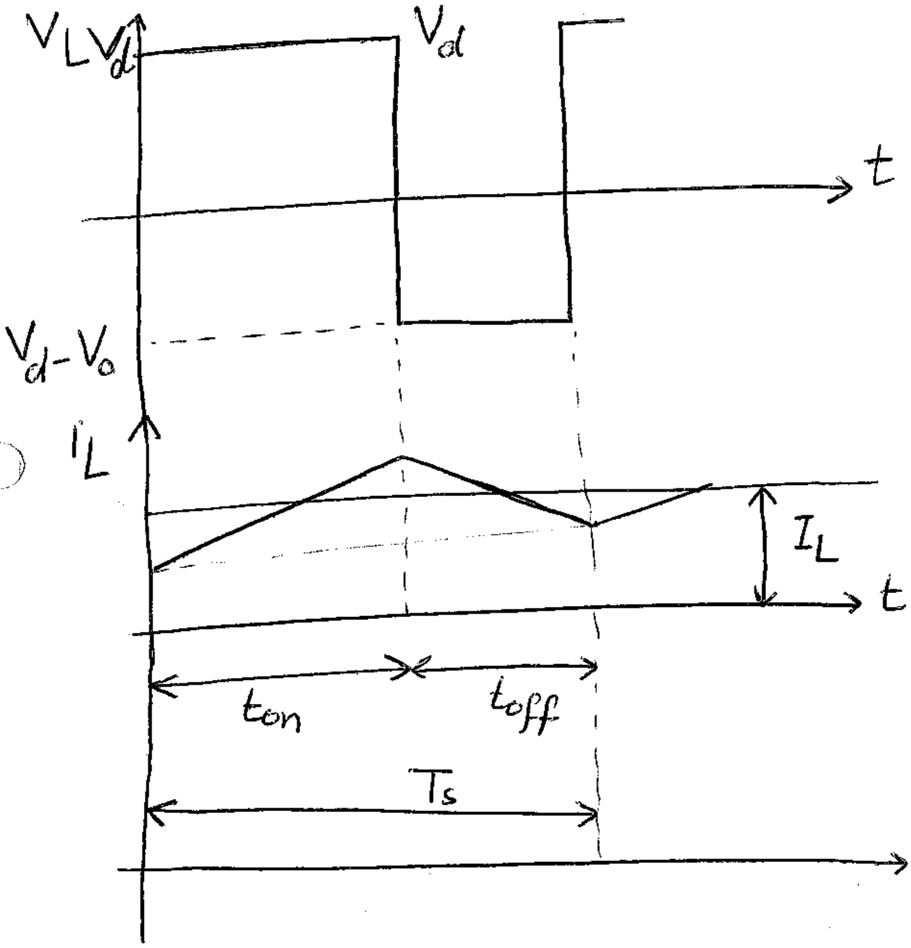
* When the switch is off, the output stage receives energy from the supply and from the inductor. The circuit is:



$\Rightarrow$ The circuit when the switch is off

Continuous-Conduction Mode





$$\int_0^{t_{on}} V_d\,dt + \int_{t_{on}}^{T_s} (V_d - V_o)\,dt = 0$$

The integral of the inductor voltage over one time period must be zero, therefore:

$$\int_0^{T_s} V_L\,dt = \int_0^{t_{on}} V_d\,t_{on} + \int_{t_{on}}^{T_s} (V_d - V_o)\,dt = 0$$

$$V_d\,t_{on} + (V_d - V_o)\,t_{off} = 0$$

$$V_d\,t_{on} = -(V_d - V_o)\,t_{off}$$

$$V_d\,t_{on} = (V_o - V_d)\,t_{off}$$

$$V_d\,t_{on} = V_o\,t_{off} - V_d\,t_{off}$$

$$V_d\,t_{on} + V_d\,t_{off} = V_o\,t_{off}$$

$$\int_0^{T_s} V_L\,dt = \int_0^{t_{on}} V_d\,dt + \int_{t_{on}}^{T_s} (V_d - V_o)\,dt$$

$$0 = V_d\,t_{on} + (V_d - V_o)\,t_{off}$$

$$V_d(t_{on}+t_{off}) = V_o t_{off}$$

$$\frac{V_o}{V_d} = \frac{t_{on}+t_{off}}{t_{off}} = \frac{T_s}{t_{off}} = \frac{T_s}{T_s - t_{on}} \qquad D = \frac{t_{on}}{T_s}$$

$$= \frac{T_s}{T_s - t_{on}} = \frac{T_s}{T_s - DT_s} = \frac{1}{1-D}$$

$$\boxed{\frac{V_o}{V_d} = \frac{1}{1-D}}$$

$$D = 0 \Rightarrow V_o = V_d$$

$$D = 0.5 \Rightarrow \frac{V_o}{V_d} = \frac{1}{1-0.5} = \frac{1}{0.5} = 2$$

$$\boxed{V_o = 2V_d}$$

$$D = 0.3 \Rightarrow \frac{V_o}{V_d} = \frac{1}{1-0.3} = \frac{1}{0.7}$$

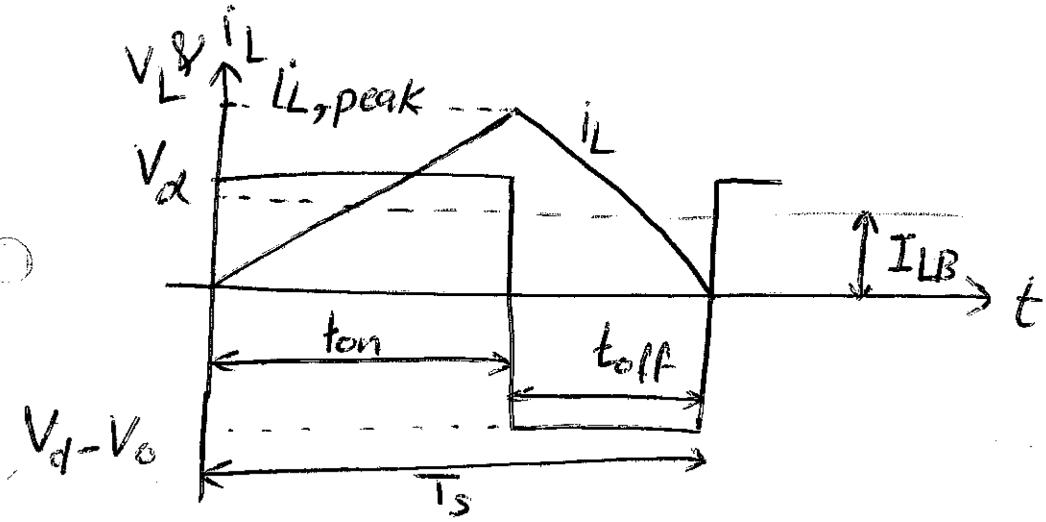
$$\boxed{V_o = \frac{1}{0.7}V_d = 1.428V_d}$$

$$D = 0.7 \Rightarrow \frac{V_o}{V_d} = \frac{1}{1-0.7} = \frac{1}{0.3}$$

$$\boxed{V_o = 3.333\, V_d}$$

Boundary Between Continuous and Discontinuous Conduction

The waveforms of this case are;



$$I_{LB} = \frac{1}{2} i_{L,peak} \Rightarrow \text{from the previous section}$$

$$= \frac{1}{2} \frac{V_d}{L} t_{on}$$

But $\frac{V_o}{V_d} = \frac{1}{1-D}$

$$V_d = V_o(1-D)$$

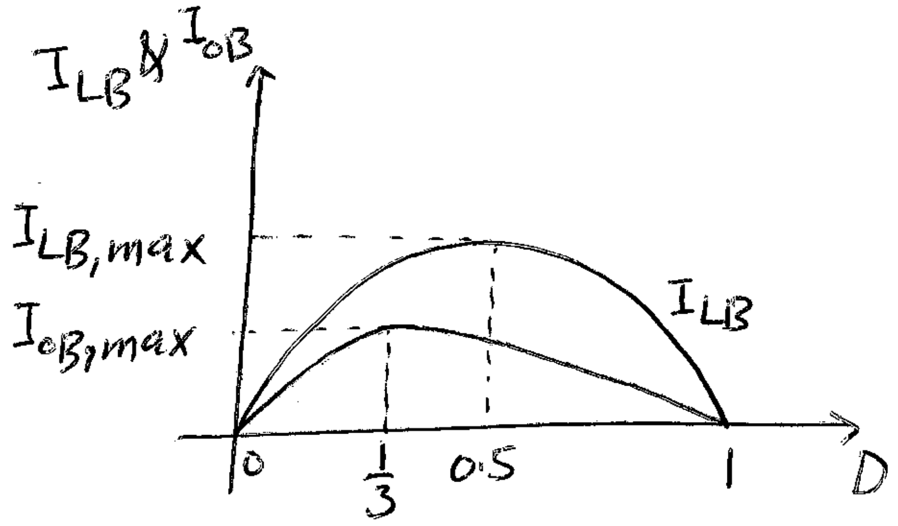
in Boost converter

$$I_{LB} = \frac{1}{2} \frac{1}{L} t_{on} V_o(1-D)$$

$$I_{LB} = \frac{1}{2L} D T_s V_o(1-D) = \frac{T_s V_o}{2L} D(1-D)$$

$$\left\{\begin{array}{l} I_d = I_L \\ I_o \neq I_L \\ \frac{I_o}{I_d} = 1-D \end{array}\right. \qquad I_{oB} = I_d(1-D) = \frac{T_s V_o}{2L} D(1-D)^2$$

* Lets draw I_{LB} & I_{oB} as function of D.



- I_{LB} is maximum when $D = 0.5$

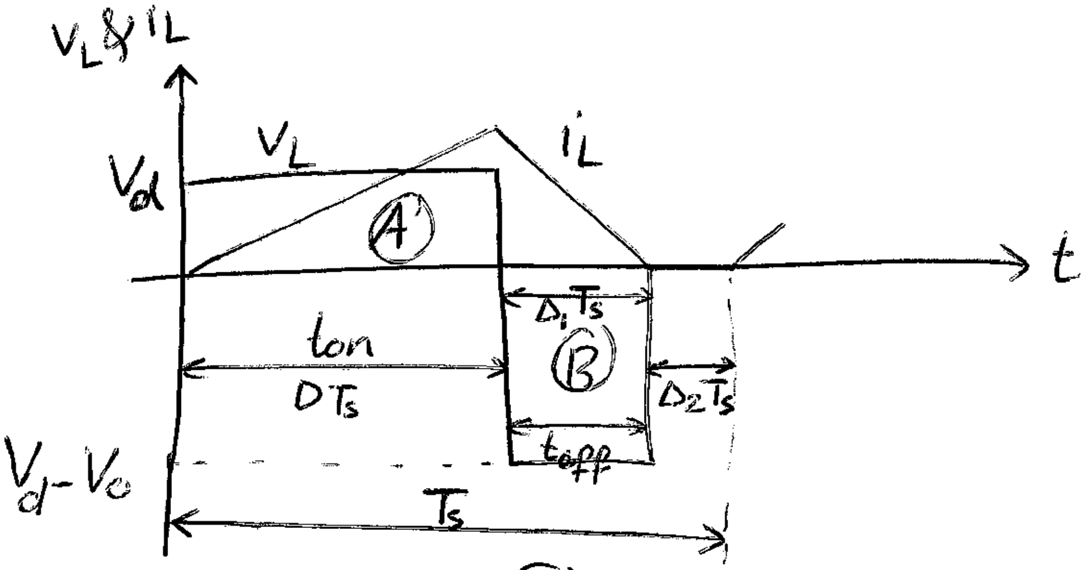
$$I_{LB,max} = \frac{T_s V_o}{8L}$$

- I_{oB} is maximum when $D = \frac{1}{3}$

$$I_{oB,max} = \frac{2}{27} \frac{T_s V_o}{L}$$

Discontinuous - Conduction Mode

Assume that as the output power decreases V_d and D remain constant. The waveforms for this mode are:



$$A + B = 0$$

$$V_d D T_s + (V_d - V_o)\Delta_1 T_s = 0$$

$$V_d D T_s + V_d \Delta_1 T_s = V_o \Delta_1 T_s$$

$$\frac{V_o}{V_d} = \frac{\Delta_1 + D}{\Delta_1}$$

$$\frac{I_o}{I_d} = \frac{\Delta_1}{\Delta_1 + D} \quad \text{since } P_d = P_o$$

$$I_d = I_L = i_{L,peak} \frac{D + \Delta_1}{2}$$ ⇒ see the previous section

$$= \frac{V_d}{2L} D T_s (D + \Delta_1)$$

$$I_o = I_d \frac{\Delta_1}{\Delta_1 + D} = \frac{V_d D T_s}{2L}(D + \Delta_1)\frac{\Delta_1}{\Delta_1 + D}$$

$$I_o = \frac{V_d D T_s}{2L} \Delta_1$$

Assuming a lossless circuit:

$$P_d = P_o$$

$$V_d I_d = V_o I_o$$

$$\frac{I_o}{I_d} = \frac{V_d}{V_o} = 1 - D$$

$\Longrightarrow$ go for the cases of boundary between continuous and discontinuous & for the case of discontinuous

Buck - Boost Converter

★ Application:

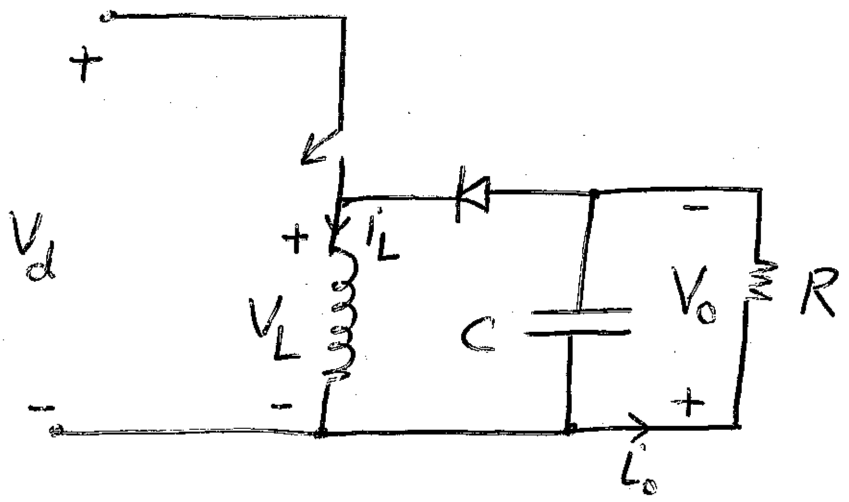
① regulated dc power supplies where a negative-polarity output may be desired with respect to the common terminal of the input voltage. and the output voltage can be either higher or lower than the input voltage

★ A buck-boost converter can be obtained by the cascade connection of the two basic converters: the step-down converter and the step-up converter. In this converter:

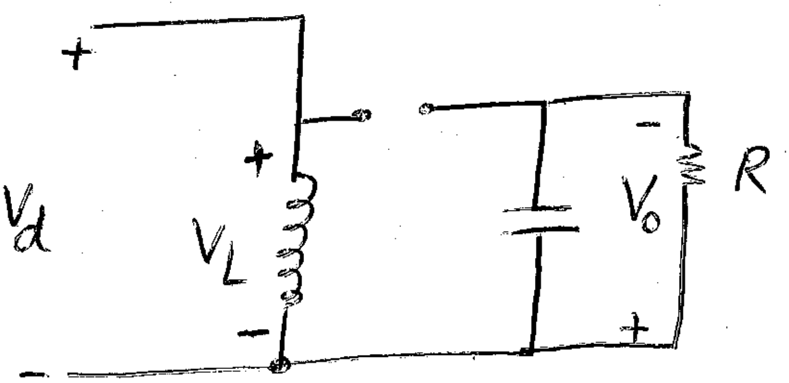
$$\frac{V_o}{V_d} = D \frac{1}{1-D}$$

Buck-Boost

* The topology of step-up and step-down converters is:



* When the switch is closed, the input provides energy to the inductor and the diode is reversed biased as shown in the following figure:



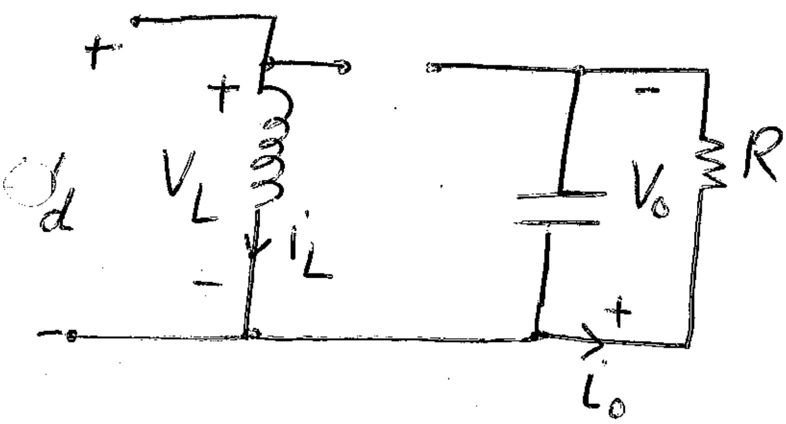
$V_d = V_L$

and the energy is stored in the inductor.

* When the switch is open, the energy stored in the inductor is transferred to the output. No energy is supplied by the input during this interval. The output capacitor is assumed to be very large which results in a constant output voltage $V_o(t) \approx V_o$.

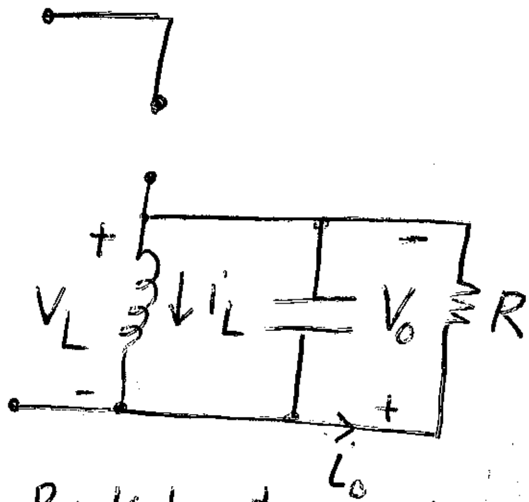
* Continuous- Conduction Mode

In this mode, the inductor current flows Continuously.



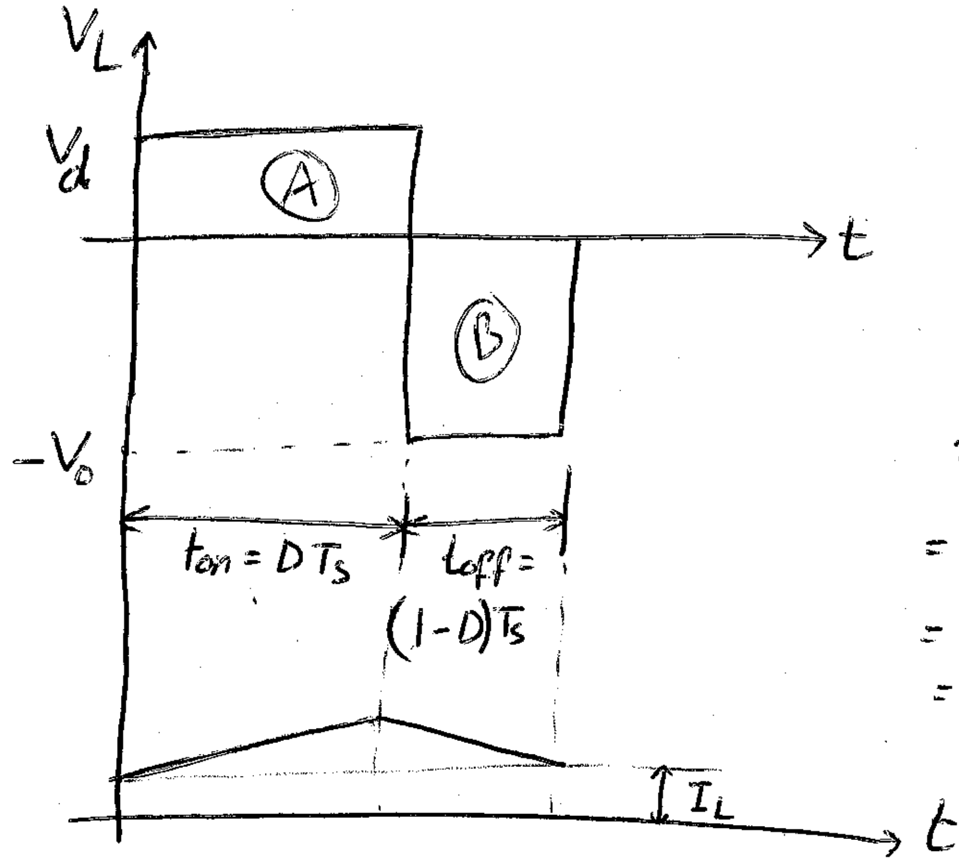
Buck- boost converter switch "on"

$V_L = V_d$



Buck-boost converter switch "off"

$V_L = -V_o$



$t_{on} + t_{off}$
$= DT_s + (1-D)T_s$
$= DT_s + T_s - DT_s$
$= T_s$

$$A + B = 0$$

$$V_d D T_s + (-V_o)(1-D)T_s = 0$$

$$V_d D T_s = V_o (1-D) T_s$$

$$\frac{V_o}{V_d} = \frac{D}{1-D}$$

$\frac{1}{T_s}\int_0^{T_s} v_L(t)\,dt = 0$

$\int_0^{t_{on}} V_d\,dt + \int_{t_{on}}^{T_s} -V_o\,dt = 0$

$V_d D T_s + (-V_o)(1-D)T_s = 0$

and $\frac{I_o}{I_d} = \frac{1-D}{D}$ assuming lossless capacitor and inductor ($P_d = P_o$)

So, if $D = 0.1$

$$\frac{V_o}{V_d} = \frac{0.1}{1-0.1} = \frac{0.1}{0.9} = \frac{1}{9} \Rightarrow \boxed{V_o = \frac{1}{9} V_d}$$ down

if $D = 0.5$

$$\frac{V_o}{V_d} = \frac{0.5}{1-0.5} = \frac{0.5}{0.5} = 1 \Rightarrow \boxed{V_o = V_d} \text{ unity}$$

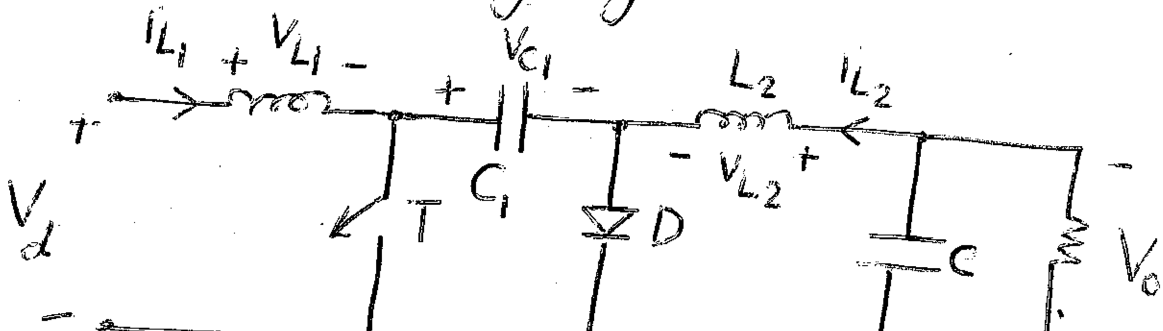
if $D = 0.9$

$$\frac{V_o}{V_d} = \frac{0.9}{1-0.9} = \frac{0.9}{0.1} = 9 \Rightarrow \boxed{V_o = 9V_d} \text{ up}$$

For $D < 0.5 \longrightarrow$ step down
For $D > 0.5 \longrightarrow$ step up

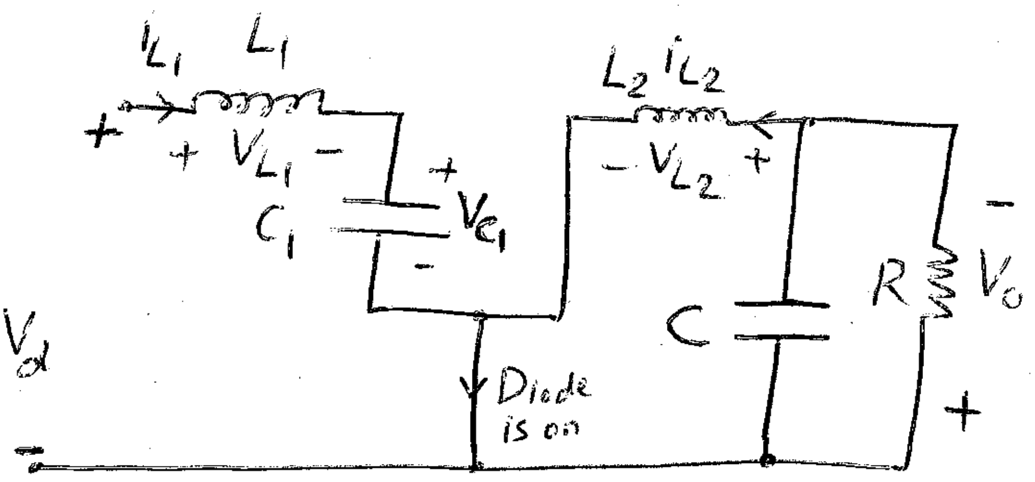
Cu'K dc-dc Converter

* Similar to the buck-boost converter, the cu'k Converter provides a negative-polarity regulated output voltage with respect to the common terminal of the input voltage. The cu'k converter is shown below in the following figure:



C_1: primary means of storing and transferring energy from the input to the output

* When the switch is off, i_{L_1} and i_{L_2} flow through the diode. The circuit is shown below:



when the switch is off

* C_1 is sufficiently large such that the variation in V_{C_1} from its average value V_{C_1} can be assumed to be negligibly small i.e. $V_{C_1} = V_{C_1}$, even though it stores and transfers energy from the input to the output.

* In steady state, V_{L_1} and V_{L_2} (the average inductor voltages) are zero. Therefore:

average voltage across the inductors

$$-V_d + V_{C_1} - V_o = 0 \quad (2) \qquad V_{L_1} \text{ \& } V_{L_2} = 0$$

$$\underline{V_{C_1} = V_d + V_o} \Rightarrow V_{C_1} \text{ is larger than } V_d \text{ and } V_o$$

$V_{C_1} > V_d$ & when $\boxed{\text{the switch is "off"}}$

$V_{C_1} > V_o$

* Assuming that C_1 is sufficiently large such that the variation in v_{C_1} from its average value V_{C_1} can be assumed to be negligibly small i.e. $v_{C_1}(t) \simeq V_{C_1}$.

* When the switch is "off," the inductor currents i_{L_1} and i_{L_2} flow through the diode. Capacitor C_1 is charged through the diode by energy from both the input and L_1. Current i_{L_1} decreases (3) because V_{C_1} is larger than V_d:

$$-V_d + L_1 \frac{d i_{L_1}}{dt} + V_{C_1} = 0$$

$$L_1 \frac{d i_{L_1}}{dt} = V_d - V_{C_1} < 0$$

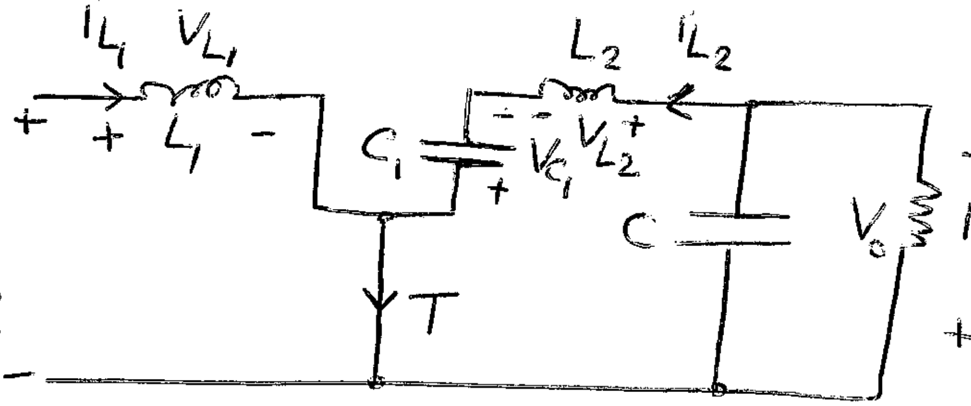
$$i_{L_1} = \frac{1}{L_1}\int (V_d - V_{C_1})\, dt \Rightarrow V_{C_1} > V_d$$

$\therefore i_{L_1}$ is decreasing

The switch is off

* During the state when the switch is "off"
Energy stored in L_2 feeds the output (load), therefore i_{L_2} also decreases.

* When the switch is "on", V_{C_1} reverse biases the diode. The inductor currents i_{L_1} & i_{L_2} flow through the switch as shown below;



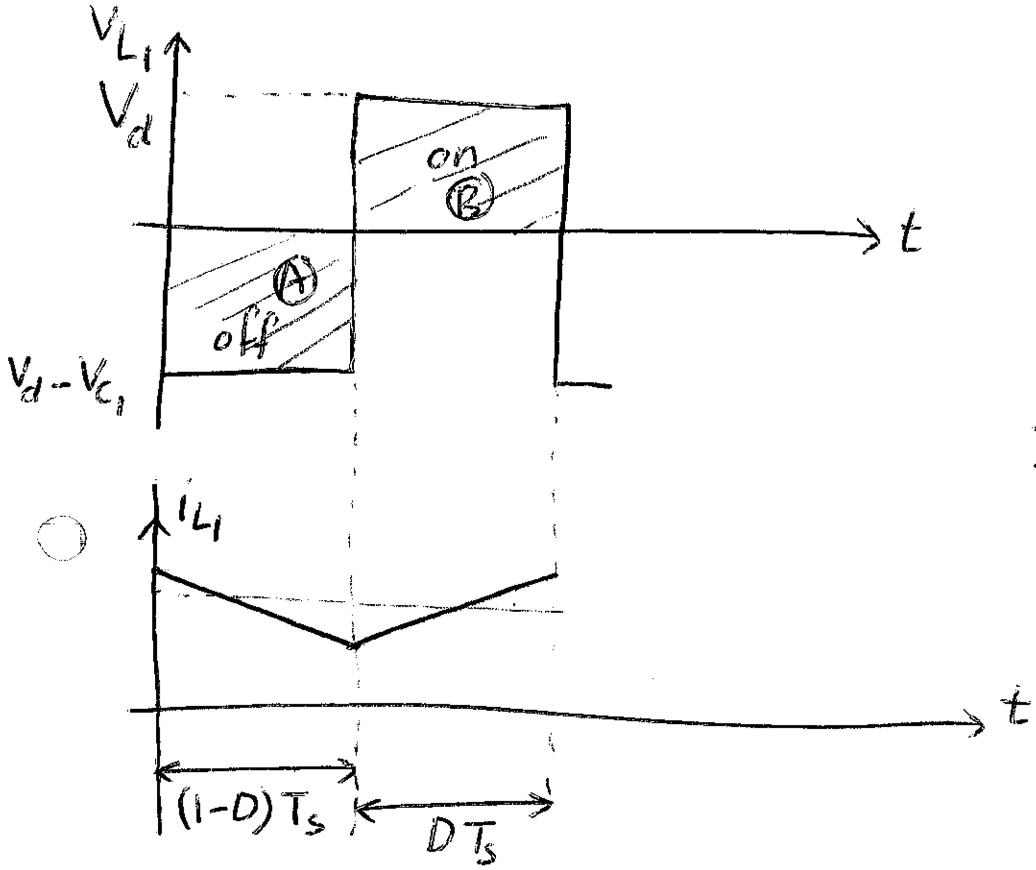
When the switch is on

* Since $V_{C_1} > V_o$, C_1 discharges through the switch, transferring energy to the output and L_2. Therefore i_{L_2} increases. The input feeds energy to L_1 causing i_{L_1} to increase.

i_{L_2} increasing

$$V_o + L_2 \frac{di_{L_2}}{dt} - V_{C_1} = 0 \Rightarrow L_2 \frac{di_{L_2}}{dt} = V_{C_1} - V_o > 0$$

* The waveforms of V_{L_1} & V_{L_2} and i_{L_1} & i_{L_2} are:



off

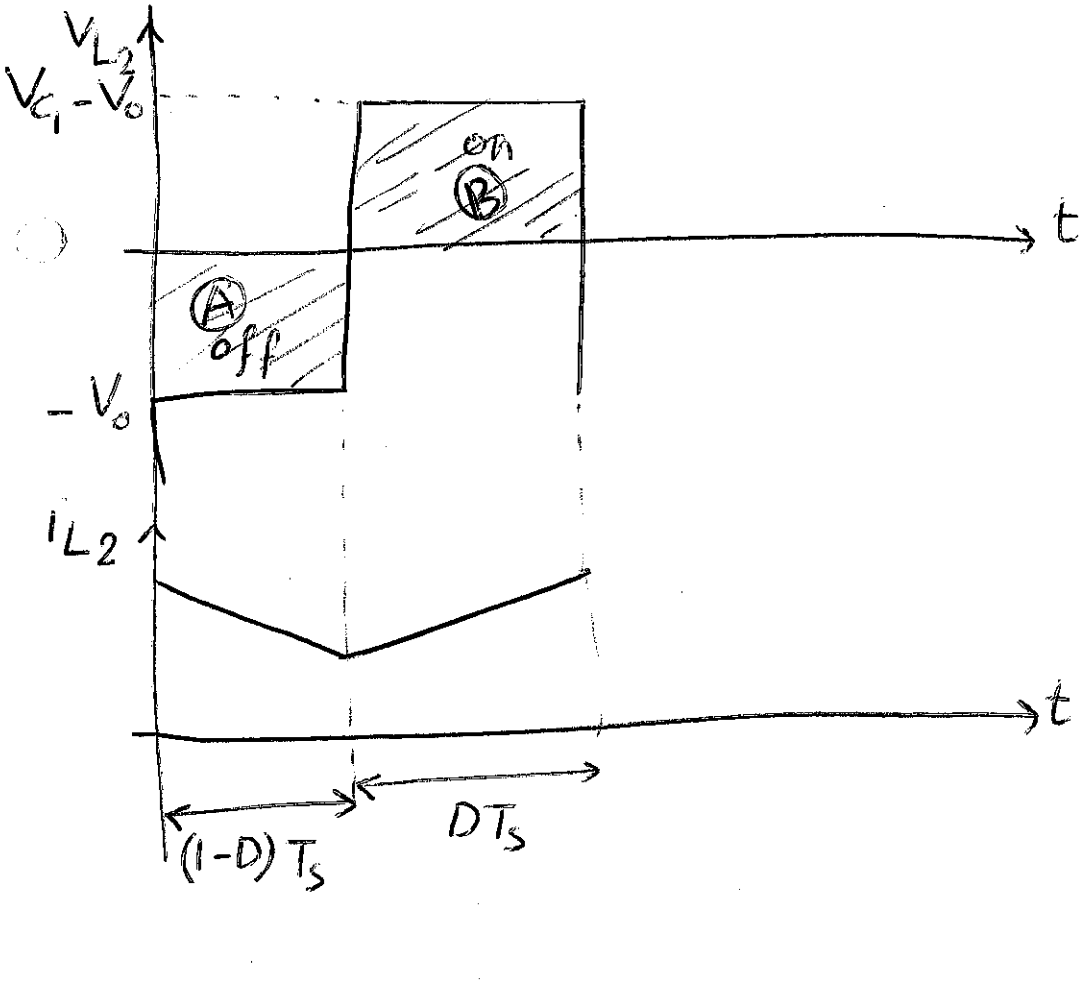
$$-V_d + V_{L_1} + V_{c_1} = 0$$

$$V_{L_1} = V_d - V_{c_1}$$

on

$$-V_d + V_{L_1} = 0$$

$$V_{L_1} = V_d$$



off

$$-V_{L_2} - V_o = 0$$

$$V_{L_2} = -V_o$$

on

$$+V_{c_1} - V_{L_2} - V_o = 0$$

$$V_{L_2} = V_{c_1} - V_o$$

* For L_1 :

$$(V_d - V_{c_1})\, t_{off} + V_d\, t_{on} = 0$$

$$\left.\begin{aligned} D = \frac{t_{on}}{T_s} \Longrightarrow \quad t_{on} = D T_s \\ t_{off} = T_s - t_{on} = T_s - D T_s = (1-D)\, T_s \end{aligned}\right\}$$

$$(V_d - V_{c_1})(1-D)\, T_s + V_d\, D T_s = 0$$

$$(V_d - V_{c_1})(1-D)\, T_s = -V_d\, D T_s$$

$$(V_{c_1} - V_d)(1-D)\, \cancel{T_s} = V_d\, D\, \cancel{T_s}$$

$$V_{c_1}(1-D) = V_d D + V_d (1-D)$$

$$V_{c_1}(1-D) = V_d$$

$$\therefore \quad \boxed{V_{c_1} = \frac{1}{1-D} V_d} \quad \text{--- (1)}$$

* For L_2

$$-V_o (1-D)\, T_s + (V_{c_1} - V_o)\, D T_s = 0$$

$$-V_o (1-D) = (V_o - V_{c_1})\, D$$

$$-V_o + \cancel{V_o D} = \cancel{V_o D} - V_{c_1} D$$

$$-V_o = -V_{c_1} D \Longrightarrow \boxed{V_{c_1} = \frac{1}{D} V_o} \quad \text{--- (2)}$$

From ① & ②

$$\frac{1}{1-D} V_d = \frac{1}{D} V_o$$

$$\frac{V_o}{V_d} = \frac{D}{1-D}$$

Assuming $P_d = P_o$ gives

$$\frac{I_o}{I_d} = \frac{1-D}{D}.$$

where $I_{L_1} = I_d$ & $I_{L_2} = I_o$.

Example 7-3 PP. 186

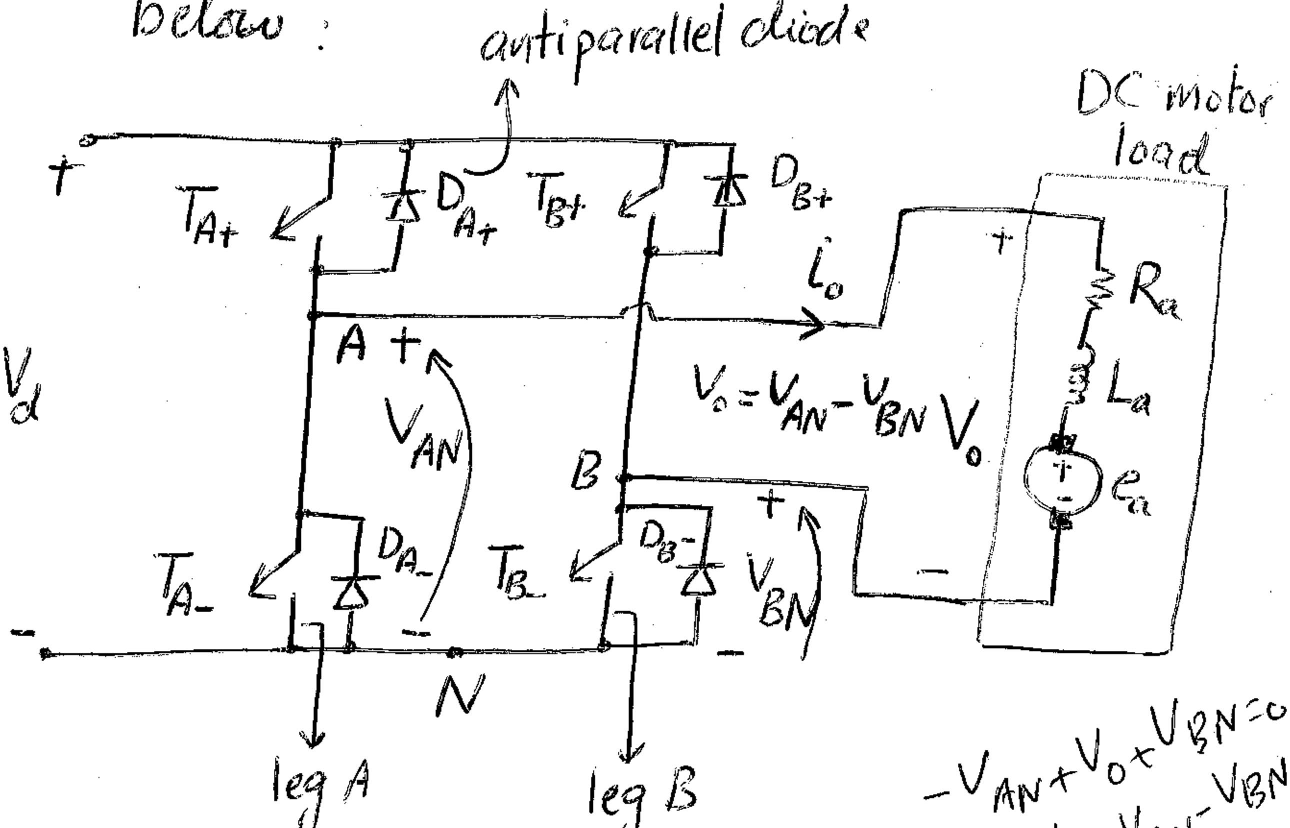
Do it!

Full-Bridge dc-dc Converter

Applications

1. dc motor drives
2. dc-to-ac (sine wave) conversion in single-phase uninterruptible ac power supplies
3. dc-to-ac (high intermediate frequency) conversion in switch-mode transformer isolated dc power supplies.

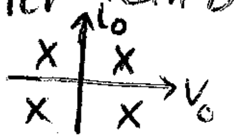
* The full-bridge converter circuit is shown below:



$-V_{AN} + V_o + V_{BN} = 0$

$V_o = V_{AN} - V_{BN}$

* The input for the full-bridge converter is a fixed-magnitude dc voltage V_d. The output is a dc voltage V_o which can be controlled in both <u>magnitude & polarity</u>. Similarly, the magnitude and the direction of the current i_o can be controlled. Therefore, the full-bridge converter can operate in all four quadrants of the i_o-V_o plane and so the power flow through the converter can be in either direction.

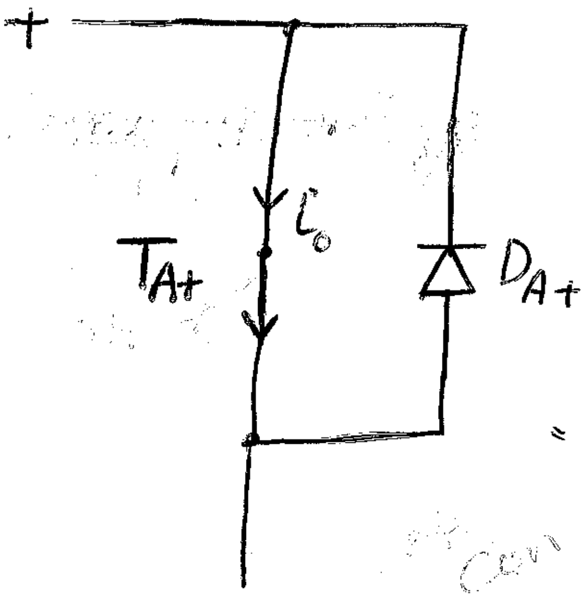


* The diodes are connected in antiparallel with the switches.

* <u>on-state of the switch</u>: the switch is on and may or may not conduct a current depending on the direction of the output current i_o.

* <u>Conducting state</u> of the switch: if the switch conducts a current.

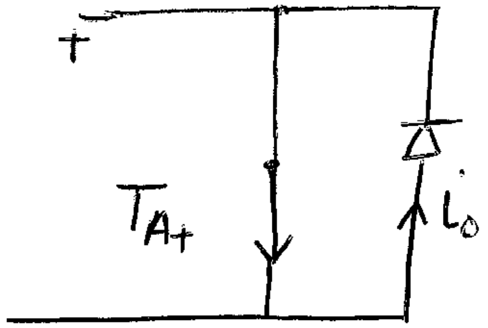
* The full-bridge converter consists of two legs, A & B. The two switches in each leg are switched in such a way that when one of them is in its off state, the other switch is on. Therefore, the two switches are never off simultaneously.



i_o is +ve

"conducting state"

The switch is on and conducting current (conducting state)



i_o is -ve

"on state"

The switch is on but doesn't conduct current (on state)

* The output voltage is dictated by the status of the switches. V_{AN} is dictated by the switch status as follows:

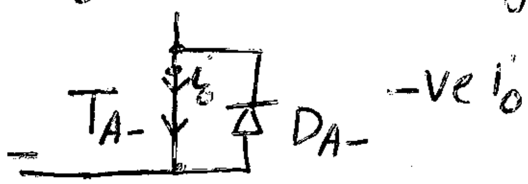
When
① T_{A+} is on, i_o will flow through T_{A+} if i_o is +ve.

When
② T_{A+} is on, i_o will flow through D_{A+} if i_o is −ve.

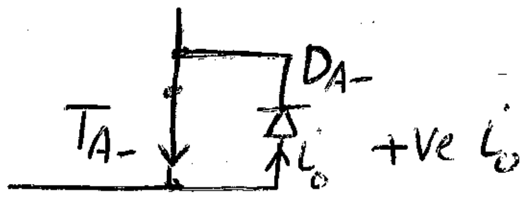
In both cases T_{A+} is on and

$$V_{AN} = V_d \quad (T_{A+} \text{ is on } \& \ T_{A-} \text{ is off})$$

Similarly,
① When T_{A-} is on, i_o will flow through T_{A-} if i_o is −ve.



② When T_{A-} is on, i_o will flow through D_{A-} if i_o is +ve.



In both cases, T_{A-} is on and

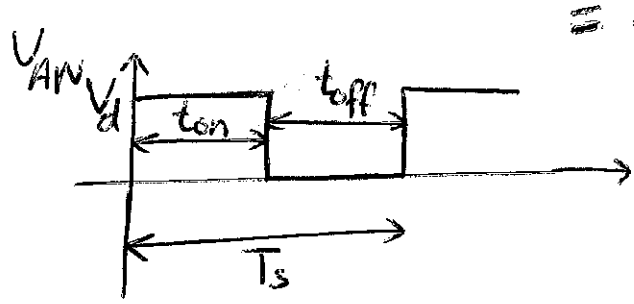
$V_{AN} = 0$ (T_{A-} is on & T_{A+} is off)

$\therefore$ V_{AN} depends only on the switch status and is independent of the direction of i_o.

average V_{AN}

$$\therefore V_{AN} = V_d \cdot \text{duty ratio of } T_{A+}.$$

$$= \frac{V_d \, t_{on}}{T_s}$$



$$\underbrace{V_{AN}}_{\text{area under } v_{AN}} = \frac{1}{T_s} V_d \, t_{on} \rightarrow \text{average value}$$

where t_{on} and t_{off} are the on and off intervals of T_{A+} respectively.

* Similarly for leg B:

$$V_{BN} = V_d \cdot \underbrace{\text{duty ratio of}}_{} \, T_{B+} \quad \text{and}$$

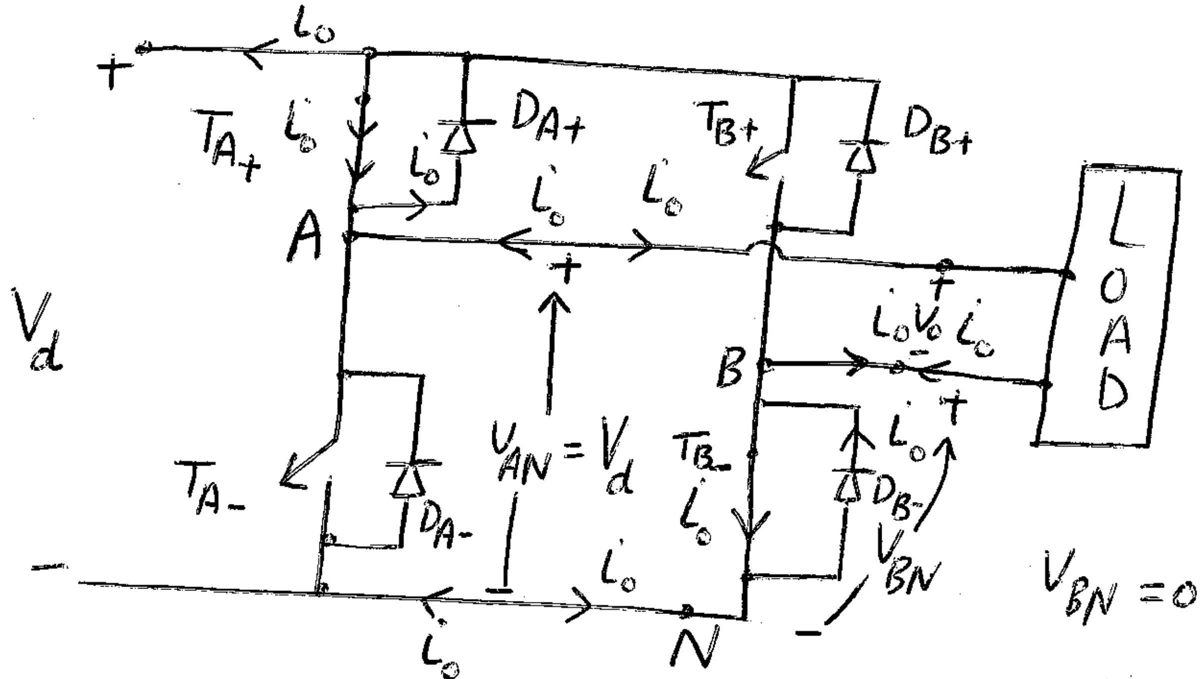
t_{on}/T_s

independent of the direction of i_o.

∴ Therefore :

$V_o = V_{AN} - V_{BN}$ can be controlled by controlling the switch duty ratios and is independent of the magnitude and the direction of i_o.

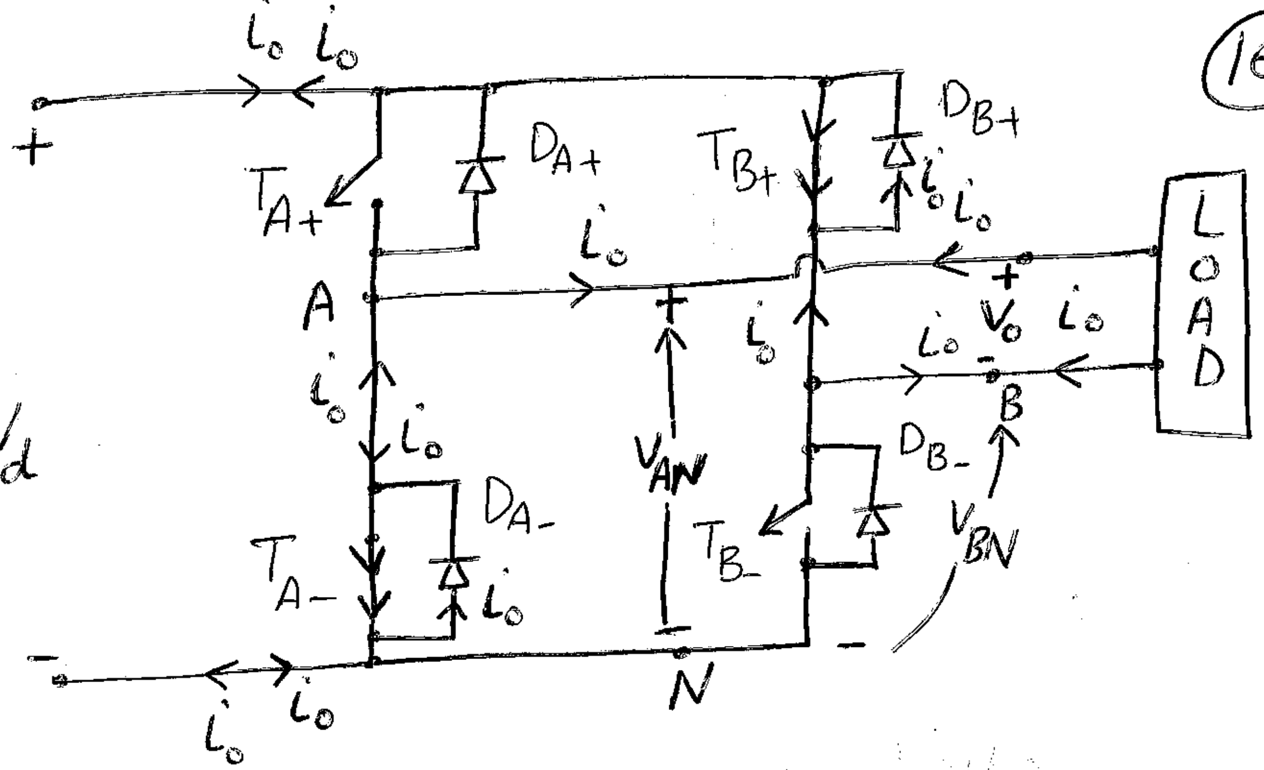
Summary:



In both cases T_{A+} & T_{B-} are "on".

$\boxed{\text{on-state}}$

$\begin{cases} +ve\ V_{AN}\ \&\ +ve\ i_o \Rightarrow T_{A+}\ \&\ T_{B-}\ \text{are conducting} \\ +ve\ V_{AN}\ \&\ -ve\ i_o \Rightarrow D_{A+}\ \&\ D_{B-}\ \text{are conducting} \end{cases}$



In both cases T_{B+} & T_{A-} are "on" on-state

+ve V_{BN} & +ve $i_o \Rightarrow D_{A-}$ & D_{B+} are conducting

+ve V_{BN} & −ve $i_o \Rightarrow T_{B+}$ & T_{A-} are conducting

* In the single-switch discussed previously (buck, boost, buck-boost and Cuk converters), the polarity of the output voltage is unidirectional and therefore the converter switch is pulse-width modulated by a switching-frequency saw tooth waveform with the control voltage $V_{control}$. In constrast, the output voltage of the full-bridge converter is reversible in polarity

and therefore, a switching-frequency triangular waveform is used for PWM of the converter switches. There are two PWM switching strategies :

1. PWM with "bipolar voltage switching" where (T_{A+}, T_{B-}) and (T_{A-}, T_{B+}) are turned on and off simultaneously.

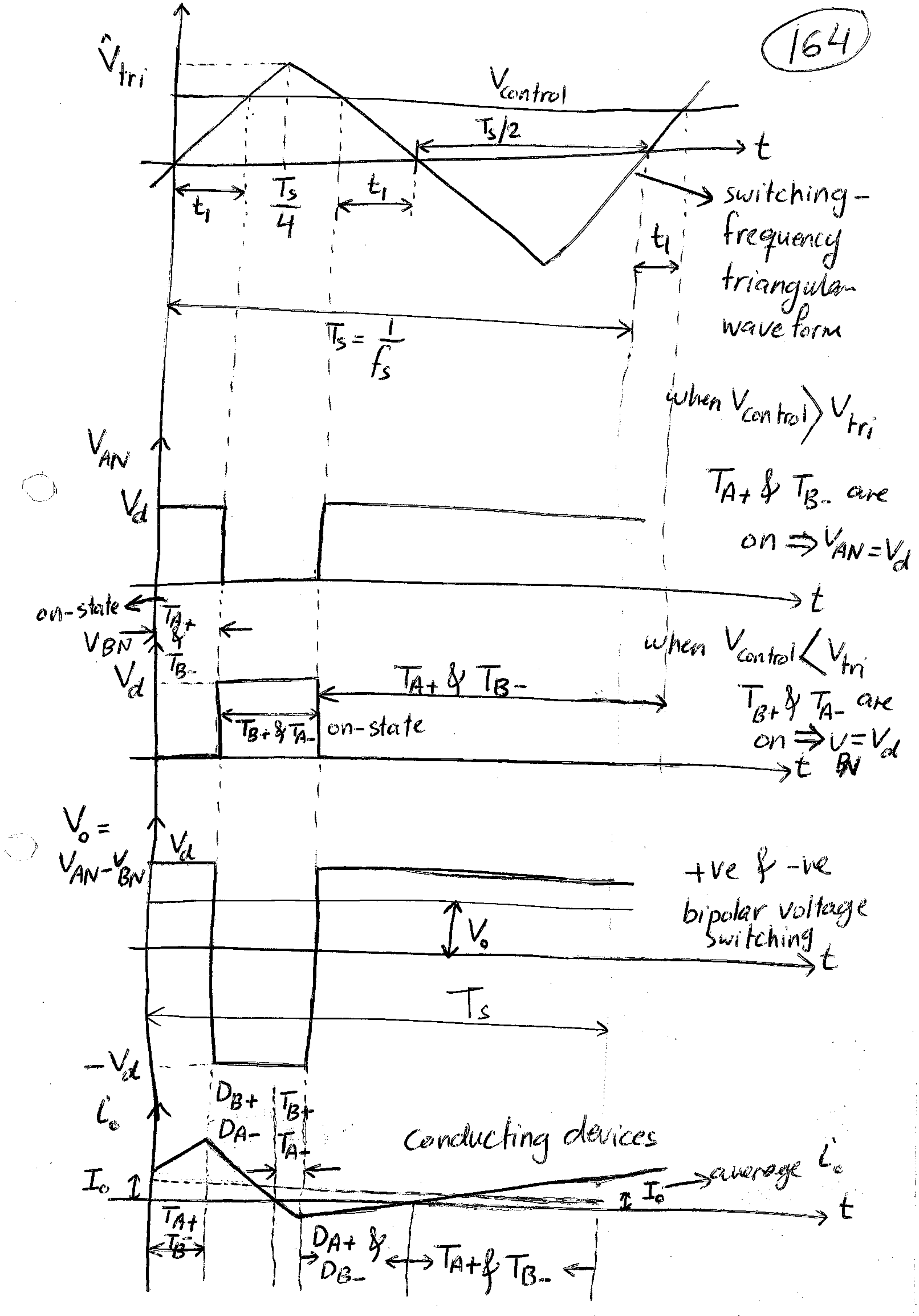
2. PWM with "unipolar voltage switching" referred to as double-PWM switching. The switches in each converter leg are controlled independently of the other leg.

* In full-bridge converter I_o does not have discontinuous mode of operation at low values of I_o as the topology allow for -ve values.

* In the full-bridge converter, the input current i_d changes direction instantaneously. Therefore, it is important that the input to this converter be a dc voltage source with a low internal impedance. In practice, a large filler-capacitor connected across V_d provides this low-impedance path.

PWM with Bipolar Voltage Switching

In this type (T_{A+}, T_{B-}) & (T_{B+}, T_{A-}) are switched on & off simultaneously. One of the two switch pairs is always on.


$\hat{V}_{tri}$
$V_{control}$
$T_s/2$
t
t_1
$\frac{T_s}{4}$
t_1
switching-frequency triangular wave form
t_1
$T_s = \frac{1}{f_s}$
when $V_{control} > V_{tri}$
T_{A+} & T_{B-} are on $\Rightarrow V_{AN} = V_d$
V_{AN}
V_d
t
on-state
T_{A+} & T_{B-}
V_{BN}
V_d
when $V_{control} < V_{tri}$
T_{B+} & T_{A-} are on $\Rightarrow V_{BN} = V_d$
T_{A+} & T_{B-}
T_{B+} & T_{A-} on-state
t
$V_o = V_{AN} - V_{BN}$
V_d
+ve & -ve bipolar voltage switching
V_o
t
T_s
$-V_d$
i_o
D_{B+} D_{A-}
T_{B+} T_{A+}
conducting devices
I_o
I_o average i_o
t
T_{A+} T_{B-}
D_{A+} & D_{B-}
T_{A+} & T_{B-}

Mathematically:

$$V_{tri} = \hat{V}_{tri} \frac{t}{T_s/4} = \frac{4 \hat{V}_{tri}\, t}{T_s} \qquad 0 < t < \frac{T_s}{4}$$

at $t = t_1$ $\quad V_{tri} = V_{control}$

$$V_{control} = \frac{4 \hat{V}_{tri}\, t_1}{T_s}$$

$$t_1 = \frac{V_{control}\, T_s}{4 \hat{V}_{tri}}$$

The on duration t_{on} of T_{A+} T_{B-} is:

$$t_{on} = 2t_1 + \frac{1}{2} T_s$$

$$D_1 = \frac{t_{on}}{T_s} = \frac{2t_1 + \frac{1}{2} T_s}{T_s} = \frac{2\left(\frac{V_{control}\, T_s}{4 \hat{V}_{tri}}\right) + \frac{1}{2} T_s}{T_s}$$

$$D_1 = \frac{1}{2} \frac{V_{control}}{\hat{V}_{tri}} + \frac{1}{2} = \frac{1}{2}\left(1 + \frac{V_{control}}{\hat{V}_{tri}}\right)$$

The on duration of T_{B+} & T_{A-} is:

$D_2 = 1 - \frac{t_{on}}{T_s}$

D_1

$$D_2 = \frac{T_s - t_{on}(T_{A+} \,\&\, T_{B-})}{T_s} = \frac{T_s - \left[2\left(\frac{V_{control}\, T_s}{4 \hat{V}_{tri}}\right) + \frac{1}{2} T_s\right]}{T_s}$$

$$D_2 = 1 - D_1$$

$$\therefore V_o = V_{AN} - V_{BN} = D_1 V_d - D_2 V_d$$

$$= D_1 V_d - (1-D_1) V_d = D_1 V_d - V_d + D_1 V_d$$

$$V_o = (2D_1 - 1) V_d$$

but $D_1 = \frac{1}{2}\left(1 + \frac{V_{control}}{\hat{V}_{tri}}\right)$

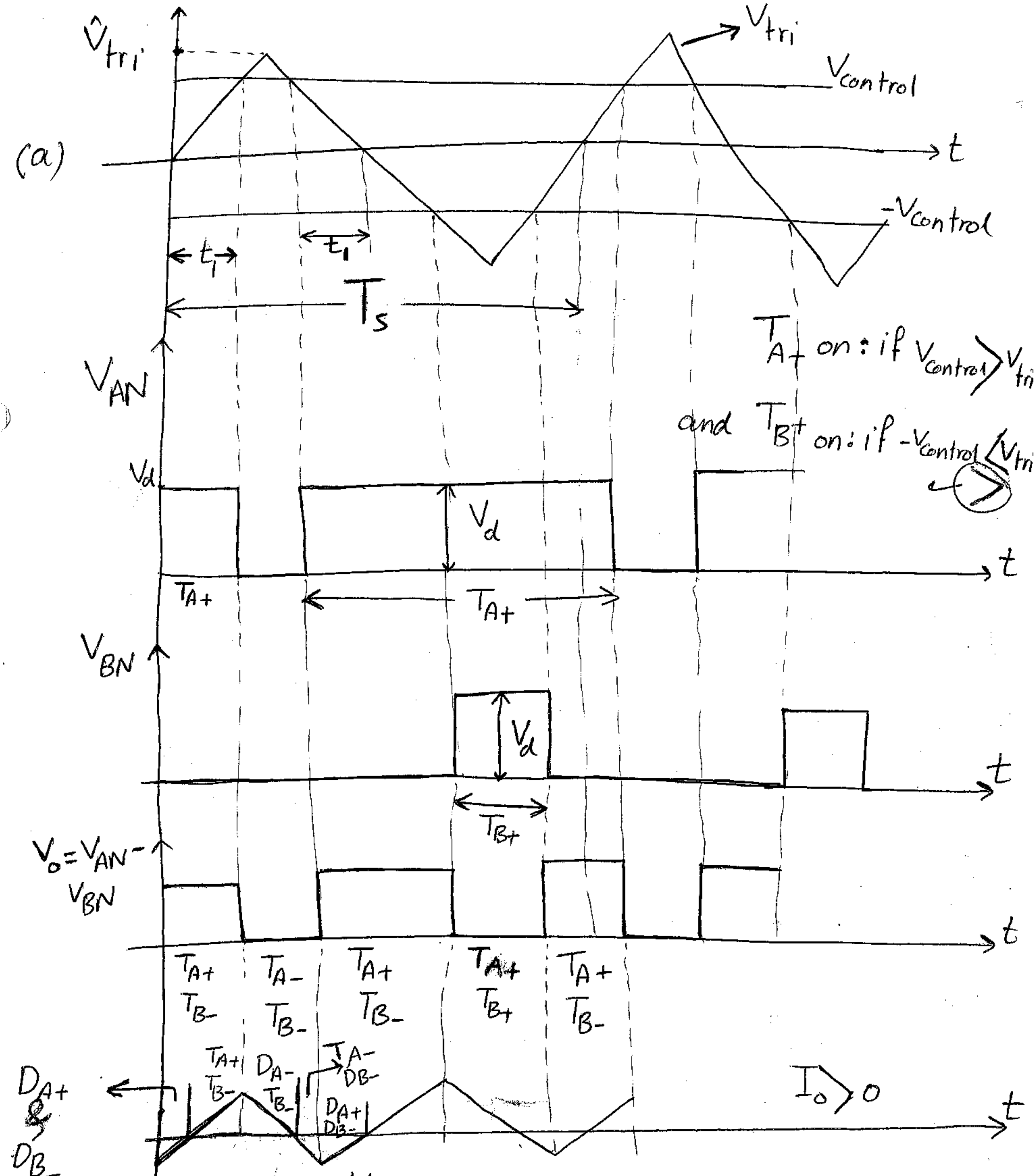
$$V_o = \left(2\left[\frac{1}{2}\left(1 + \frac{V_{control}}{\hat{V}_{tri}}\right)\right] - 1\right) V_d$$

$$V_o = \frac{V_{control}}{\hat{V}_{tri}} V_d = \frac{V_d}{\hat{V}_{tri}} V_{control} = R\, V_{control}$$

where $R = \frac{V_d}{\hat{V}_{tri}}$

* The average output voltage varies linearly with the input control signal (linear amplifier)

PWM with Unipolar Voltage Switching



$$D_1 = \frac{1}{2}\left(\frac{V_{control}}{\hat{V}_{tri}} + 1\right) \qquad T_{A+}$$

$$D_2 = 1 - D_1 \qquad T_{B+}$$

$$V_o = (2D_1 - 1)V_d = \frac{V_d}{\hat{V}_{tri}} V_{control}$$

* Therefore, the average output voltage V_o in this switching scheme is the same as in the bipolar voltage switching scheme and varies linearly with $V_{control}$.

forget it

<u>Ex.</u> Calculate the rms value of the ripple V_r in the output voltage as function of the average V_o for (a) PWM with bipolar voltage switching and (b) PMW with unipolar voltage switching of full-bridge dc-dc converter.

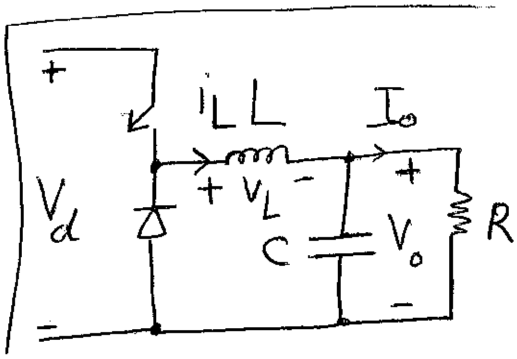
See the book.

7-1 PP. 197

step-down converter $V_o(t) = V_o = 5V$

L_{min} ? to keep the converter operation in a continuous-conduction mode under all conditions if V_d is 10-40V, $P_o \geq 5W$ $f_s = 50kHz$.

$$I_{LB} = \frac{T_s V_d}{2L} D(1-D)$$



$$P_o \geq V_o I_o$$

$$5 \geq 5 I_o \Rightarrow I_o = I_{LB} \cancel{\geq} 1A \quad \leq$$

if $V_d = 10V$ then $D = \frac{V_o}{V_d} = \frac{5}{10} = 0.5$.

$$1 = \frac{T_s V_d}{2L} D(1-D) = \frac{V_d}{f_s 2L} D(1-D)$$

$$1 = \frac{10}{(50 \times 10^3)(2)(L)} (0.5)(1-0.5)$$

$$L = 2.5 \times 10^{-5} \text{ H.}$$

if $\underline{V_d = 40V}$ then $D = \frac{5}{40} = 0.125$

$$1 = \frac{T_s V_d}{2L} D(1-D) = \frac{V_d}{f_s 2L} D(1-D)$$

$$1 = \frac{40}{(50\times10^3)(2)L}(0.125)(1-0.125)$$

$$L = 4.375 \times 10^{-5} H$$

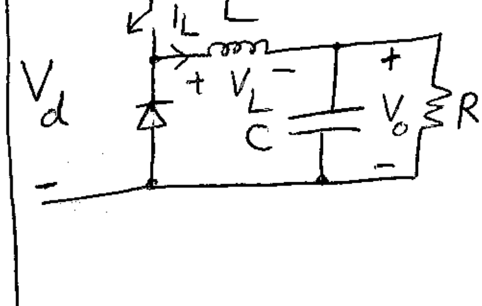
So, $\boxed{L_{min} = \overset{4.375}{\cancel{2.5}} \times 10^{-5} H.}$ → the higher value

$\underline{7\text{-}2}$ $\underline{\text{PP. } 197}$ Step-down converter

$V_o = 5V$, $f_s = 20KHz$, $L = 1mH$ $C = 470\mu F$

assume all elements are ideal. ΔV_o?

$V_d = 12.6$, $I_o = 200mA$



$$\Delta V_o = V_o \frac{1}{8} \frac{T_s^2(1-D)}{LC}$$

$$D = \frac{V_o}{V_d} = \frac{5}{12.6} = 0.397$$

$$\Delta V_o = 5 \, \frac{1}{8} \, \frac{\left(\frac{1}{20{,}000}\right)^2 (1-0.397)}{(1\times10^{-3})(470\times10^{-6})}$$

$$= \frac{5(1-0.397)}{(8)(20{,}000)^2(1\times10^{-3})(470\times10^{-6})}$$

$$= \frac{3.016}{1504} = 0.002\,V.$$

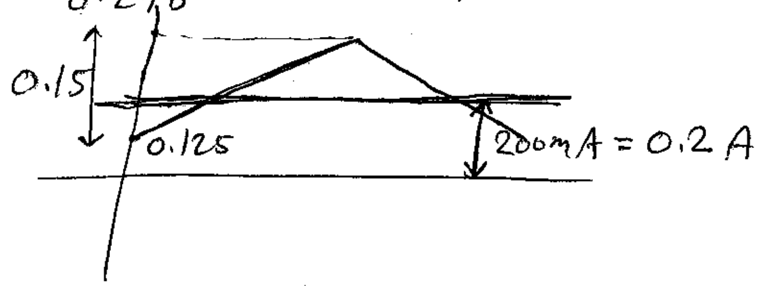
7-3 PP. 197

forget it?

rms of the ripple current through L and hence through C.

$$\Delta I_L = \frac{V_o}{L}(1-D)\,T_s = \frac{5}{1\times10^{-3}}(1-0.397)\frac{1}{20\times10^{3}}$$

$$= 0.15\,A.$$

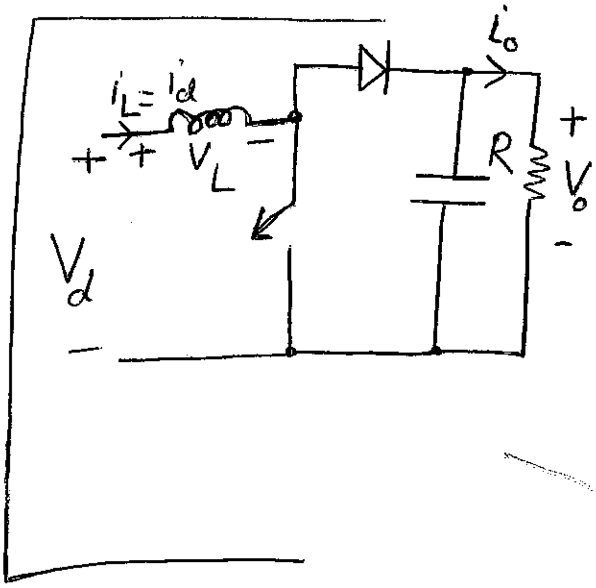


7-4 & 7-5 Can be solved

7-7 PP. 197 step-up converter

$V_d = 8-16\ V$, $V_o = 24V$ $f_s = 20kHz$

$C = 470\ \mu F$, L_{min}? $P_o \geq 5W$



$$I_{LB} = \frac{T_s V_o}{2L} D(1-D)$$

$$P_o \geq V_o I_o$$

$$\frac{V_o}{V_d} = \frac{1}{1-D}$$

if $V_d = 8$, $V_o = 24$

$$\frac{24}{8} = \frac{1}{1-D} \Rightarrow D = 0.6666$$

$$5 \geq 24\, I_o \Rightarrow 5 = 24\, I_o \Rightarrow I_o = \frac{5}{24} A$$

$$I_L = \frac{I_o}{1-D} = \frac{5/24}{1-0.6666} = \frac{0.08333}{0.3333} = 0.25A$$

$$0.25 = \frac{T_s V_o}{2L} D(1-D)$$

$$0.25 = \frac{\frac{1}{20{,}000}\, 24}{2L} (0.6666)(1-0.6666)$$

173

$$0.25 = \frac{24}{(2)(20{,}000)L}(0.6666)(0.3333)$$

$$10000\,L = 5.33$$

$$L = \frac{5.33}{10{,}000} = \underline{5.33 \times 10^{-4}}\ H.$$

if $\underline{\underline{V_d = 16}}$ $V_o = 24$

$$\frac{V_o}{V_d} = \frac{1}{1-D}$$

$$\frac{24}{16} = \frac{1}{1-D} \Rightarrow D = 0.3333$$

something wrong in numbers

$$5 \geq 24\, I_o \Rightarrow I_o = \frac{5}{24} \Rightarrow I_L = 0.25$$ the same as before

$$I_L = \frac{T_s V_o}{2L} D(1-D)$$

$$0.25 = \frac{\frac{1}{20{,}000}\, 24\,(0.3333)(1-0.3333)}{2L}$$

$$10000L = (24)(0.3333)(1-0.3333)$$

$$10{,}000L = 5.33$$

$$L = 5.33 \times 10^{-4}\, H.$$

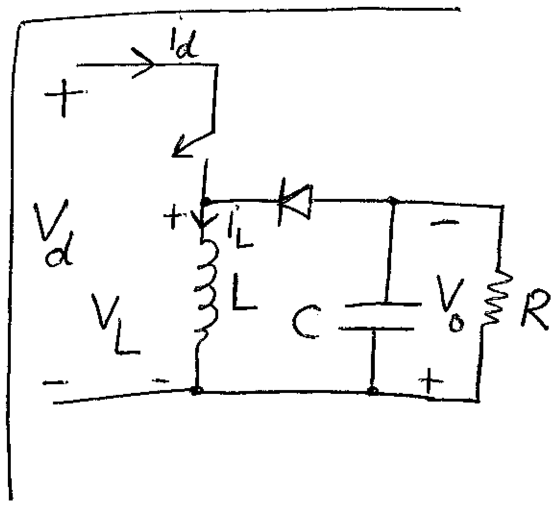
$L_{min} =$ the higher value

7-12 PP. 198

buck-boost converter $V_d = 8 \rightarrow 40\,V$ $V_o = 15V$

$f_s = 20\,kHz$ $C = 470\mu F$ L_{min}? that will

Keep the converter operating in a continuous-conduction mode if $P_o \geqslant 2W$.



$$\frac{V_o}{V_d} = D\,\frac{1}{1-D}$$

$$\frac{I_o}{I_d} = \frac{1-D}{D}$$

$$I_{LB} = \frac{T_s V_o}{2L}(1-D)$$

$$P_o \geqslant V_o I_o \Rightarrow P_o = V_o I_o = (15)(I_o) = 2$$

$$I_o = \frac{2}{15}A = I_L$$

if $\underline{V_d = 8} \Rightarrow$ $\dfrac{V_o}{V_d} = \dfrac{D}{1-D}$

$$\frac{15}{8} = \frac{D}{1-D}$$

$$15(1-D) = 8D$$

$$15 - 15D = 8D \Rightarrow 15 = 8D + 15D$$

$$D = 0.652$$

$$I_{LB} = \frac{T_s V_o}{2L}(1-D)$$

$$\cancel{\frac{I_o}{I_L} = \frac{1-D}{D} = \frac{1-0.652}{0.652} = 0.5337}$$

$$\cancel{I_L = \frac{I_o}{0.5337} = \frac{2/15}{0.5337} = 0.25\,A}$$

$$\frac{2}{15} = \frac{\frac{1}{20,000}(15)}{2L}(1-0.652)$$

$$\frac{2}{15} = \frac{(15)(1-0.652)}{(20,000)2L}$$

$$5333.33L = 5.22$$

$$\boxed{L = 9.78 \times 10^{-4}\,H.}$$

if $V_d = 40V \Rightarrow \frac{V_o}{V_d} = \frac{D}{1-D}$

$$\frac{15}{40} = \frac{D}{1-D} \Rightarrow 15 - 15D = 40D$$

$$D = 0.2727$$

$$\cancel{\frac{I_o}{I_L} = \frac{1-D}{D} = \frac{1-0.2727}{0.2727} = 2.667}$$

$$\cancel{I_L = \frac{I_o}{2.667} = \frac{2/15}{2.667} = 0.05\,A}$$

$$I_{LB} = \frac{T_s V_o}{2L}(1-D)$$

$$\frac{2}{15} = \frac{\frac{1}{20{,}000}(15)}{2L}(1-0.2727)$$

$$\left(\frac{2}{15}\right)(2)L(20{,}000) = (15)(1-0.2727)$$

$$5333.33L = 10.91$$

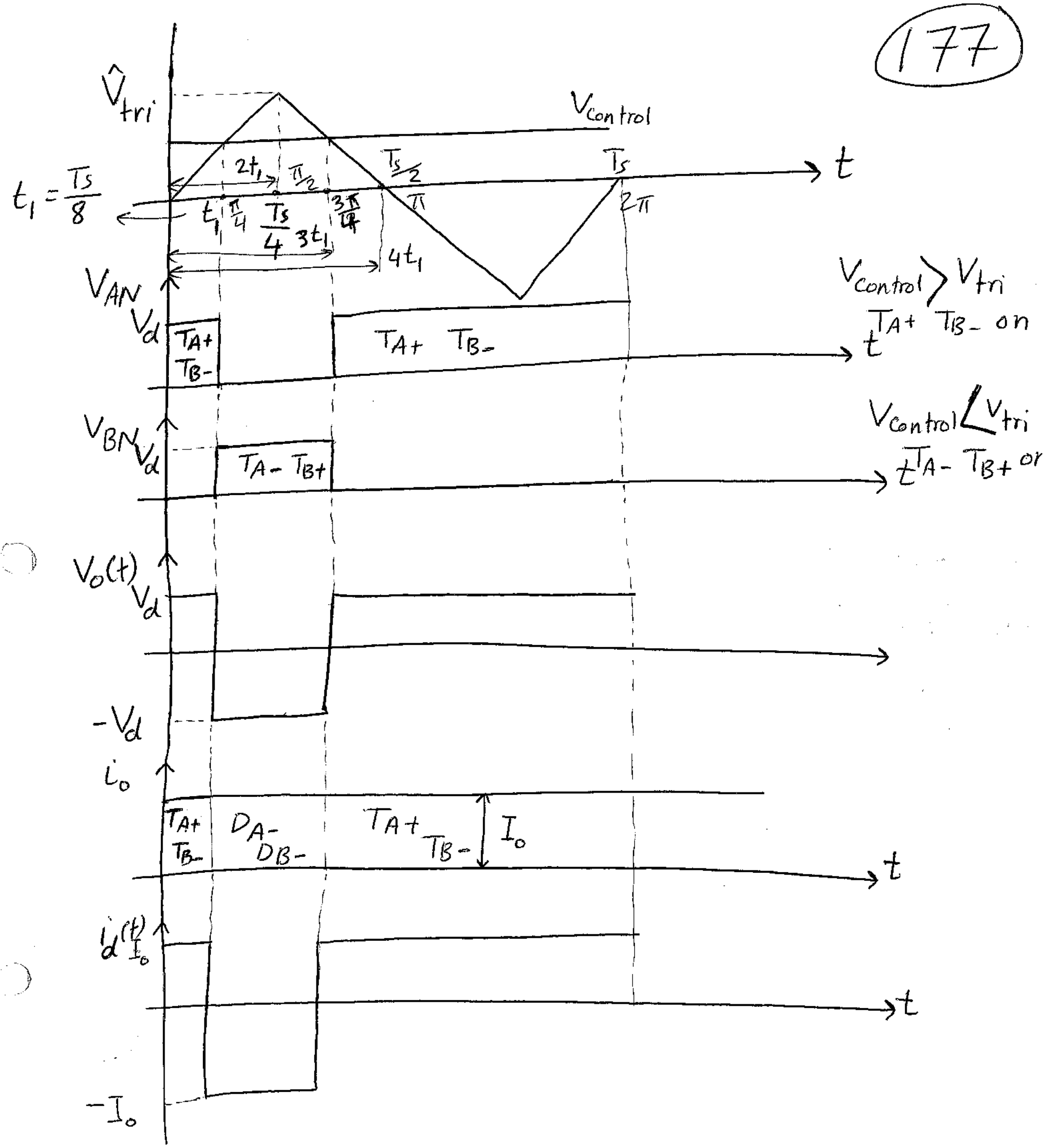
$$\boxed{L = 2.04 \times 10^{-3} H}$$

$$\boxed{\therefore L_{min} = 2.04 \times 10^{-3} H}$$ → the higher value

7-18 PP. 198

full- bridge dc-dc converter, PWM bipolar voltage switching, $V_{control} = 0.5\hat{V}_{tri}$ V_o & I_d? in terms of V_d & I_o. Calculate the amplitude of harmonics in $V_o(t)$ & $i_d(t)$. Assume $i_o(t) \cong I_o$.

by Fourier analysis



$$V_o = \frac{V_d}{\hat{V}_{tri}} V_{control} = \frac{V_d}{\hat{V}_{tri}} \underbrace{0.5\,\hat{V}_{tri}}_{} = 0.5\,V_d$$

$$= \frac{1}{2\pi}\left[V_d\frac{\pi}{4} - V_d\frac{\pi}{2} + V_d\frac{\pi}{4} + V_d\pi\right]$$

$$= 0.5\,V_d$$

Similarly, $I_d = 0.5\,I_o$

$$V_o(t) = V_o + \sum_{h=1}^{\infty} V_h(t)$$

$$= V_o + \sum_{h=1}^{\infty} \{a_h \cos(h\omega t) + b_h \sin(h\omega t)\}$$

$$a_h = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos(h\omega t)\,d(\omega t) \qquad h = 1, 2, 3, \ldots$$

$$a_h = \frac{1}{\pi}\left[\int_0^{t_1 \left(\frac{\pi}{4}\right)} V_d \cos(h\omega t)\,d(\omega t) + \int_{t_1 \left(\frac{\pi}{4}\right)}^{3t_1 \left(\frac{3\pi}{4}\right)} -V_d \cos(h\omega t)\,d(\omega t) + \int_{3t_1 \left(\frac{3\pi}{4}\right)}^{T_s (2\pi)} V_d \cos(h\omega t)\,d(\omega t)\right]$$

$$a_h = \frac{V_d}{\pi}\left[\int_0^{\pi/4} \cos(h\omega t)\,d(\omega t) + \int_{\pi/4}^{3\pi/4} -\cos(h\omega t)\,d(\omega t) + \int_{3\pi/4}^{2\pi} \cos(h\omega t)\,d(\omega t)\right]$$

$$= \frac{V_d}{h\pi}\left[\sin(h\omega t)\Big]_0^{\pi/4} + \sin(h\omega t)\Big]_{3\pi/4}^{\pi/4} + \sin(h\omega t)\Big]_{3\pi/4}^{2\pi}\right]$$

$$a_h = \frac{V_d}{h\pi}\left[\sin\left(h\frac{\pi}{4}\right) + \sin\left(h\frac{\pi}{4}\right) - \sin\left(h\frac{3\pi}{4}\right)\right.$$

$$\left. + \sin(h2\pi) - \sin\left(h\frac{3\pi}{4}\right)\right]$$

$$a_1 = \frac{V_d}{\pi}\left[\sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right) - \sin\left(\frac{3\pi}{4}\right)\right.$$

$$\left. + \sin(2\pi) - \sin\left(\frac{3\pi}{4}\right)\right]$$

$$a_1 = 0$$

$$a_2 = 0.6366 V_d$$

$$a_3 = 0$$

$$a_4 = 0$$

$$a_5 = 0$$

$$a_6 = -0.2122 V_d$$

$$b_h = \frac{1}{\pi}\left[\int_0^{2\pi} f(t)\sin(h\omega t)\, d(\omega t)\right] \quad h = 1, 2, 3 \ldots$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/4} V_d \sin(h\omega t) d(\omega t) + \int_{\pi/4}^{3\pi/4} -V_d \sin(h\omega t) d(\omega t)\right.$$

$$\left. + \int_{3\pi/4}^{2\pi} V_d \sin(h\omega t) d(\omega t)\right]$$

$b_1 = -0.9\,V_d$

$b_2 = 0$

$b_3 = 0.3\,V_d$

$b_4 = 0$

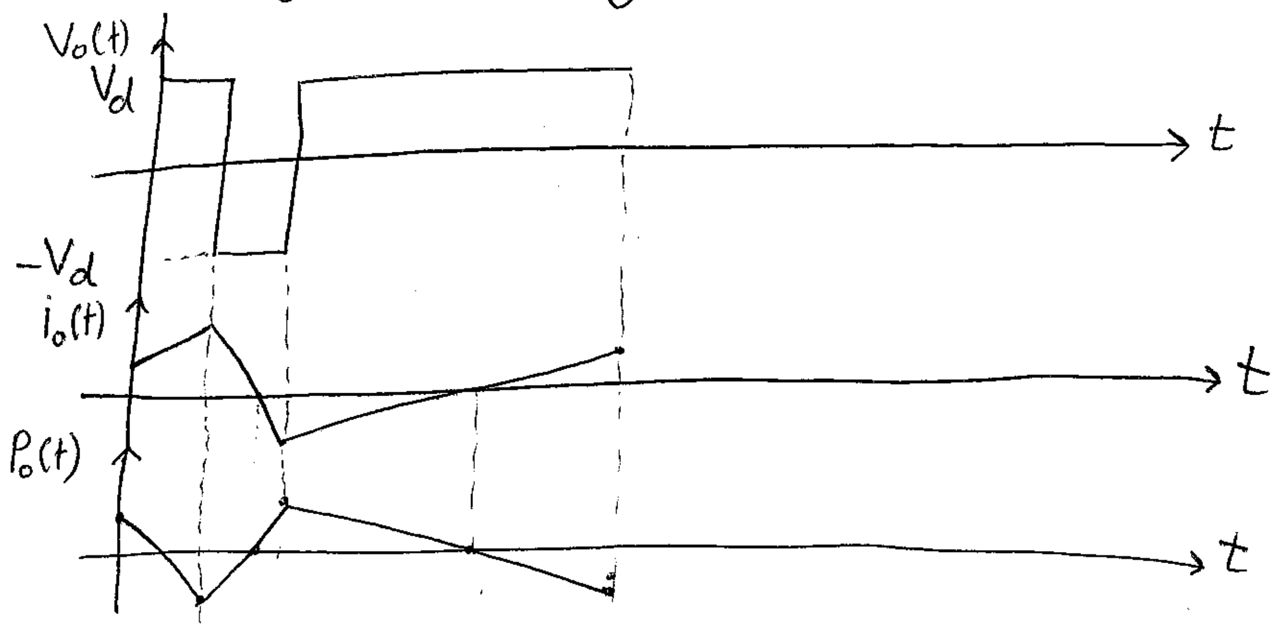
$b_5 = 0.1801\,V_d$

$\vdots$

The same thing is for i_d.

<u>7-19</u> <u>PP. 198</u>

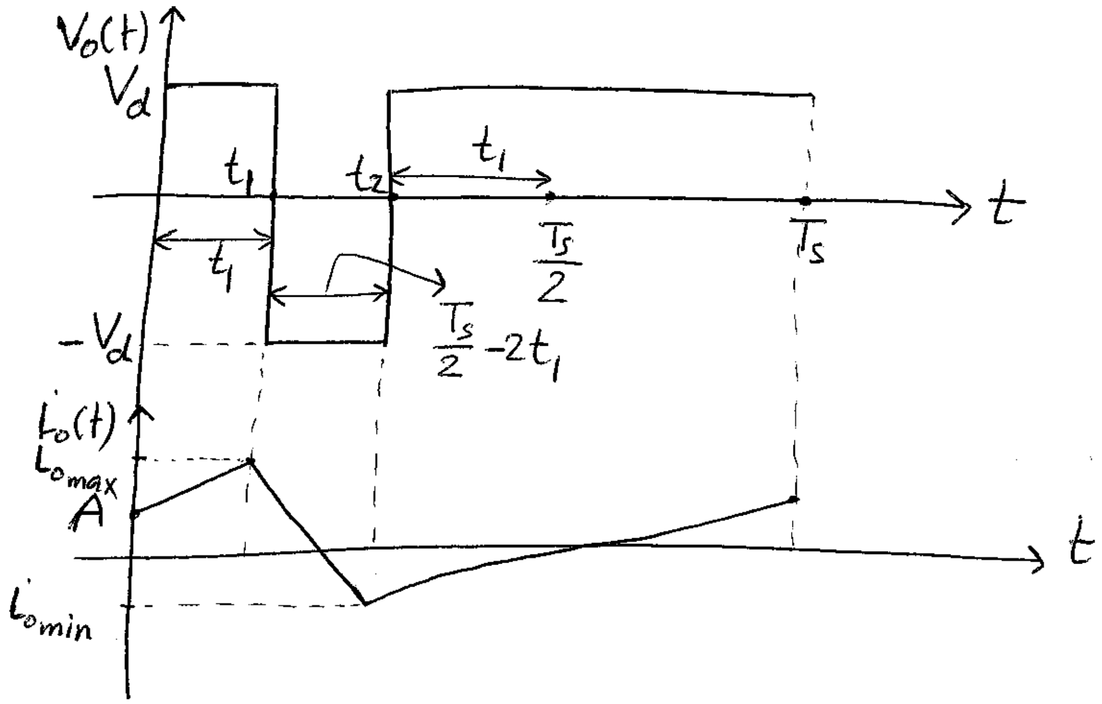
plot instantaneous power $P_o(t)$ and the average power P_o for the case of bipolar voltage switching:



7-22 PP. 198

In a full-bridge converter using PWM bipolar voltage switching, analytically obtain (V_o/V_d) which results in the maximum (peak-peak) ripple in the output current i_o. Calculate this ripple interms of V_d, L_a, f_s. Assume $R_a = 0$.

Sol. The waveforms of $V_o(t)$ & $i_o(t)$ are (in general)



$$i_{ripple} = i_{o_{max}} - i_{o_{min}}$$

$0 < t < t_1$

$$i_o(t) = \frac{V_d}{L_a} t + A$$

$$\boxed{i_{o_{max}} = \frac{V_d}{L_a} t_1 + A}$$

$t_1 < t < t_2$

$$i_o(t) = \frac{-V_d}{L_a} t + B$$

$$i_o(t_1) = i_{o_{max}} = \frac{V_d t_1}{L_a} + A = \frac{-V_d t_1}{L_a} + B$$

$$B = \frac{2 V_d t_1}{L_a} + A \quad \therefore \quad i_o(t) = \frac{-V_d t}{L_a} + \frac{2 V_d t_1}{L_a} + A$$

$$i_{ripple} = i_{o_{max}} - i_{o_{min}} \qquad \boxed{i_{o_{min}} = \frac{-V_d t_2}{L_a} + \frac{2 V_d t_1}{L_a} + A}$$

$$= \frac{V_d}{L_a} t_1 + A - \left[\frac{-V_d t_2}{L_a} + \frac{2 V_d}{L_a} t_1 + A \right]$$

$$= \frac{V_d t_1}{L_a} + A + \frac{V_d t_2}{L_a} - \frac{2 V_d t_1}{L_a} - A$$

$$i_{ripple} = \frac{V_d t_2}{L_a} - \frac{V_d t_1}{L_a} = \frac{V_d}{L_a}(t_2 - t_1)$$

i_{ripple} is max. if $t_2 - t_1$ is max. $\Rightarrow$ t_1 is min. & t_2 is max

$$t_2 = \frac{T_s}{2} - t_1$$

$$\therefore \ t_1 = 0 \ \& \ t_2 = \frac{T_s}{2}$$

$t_{on} = 2t_1 + \frac{1}{2} T_s = \frac{1}{2} T_s$ for max. i_{ripple} as $t_1 = 0$.

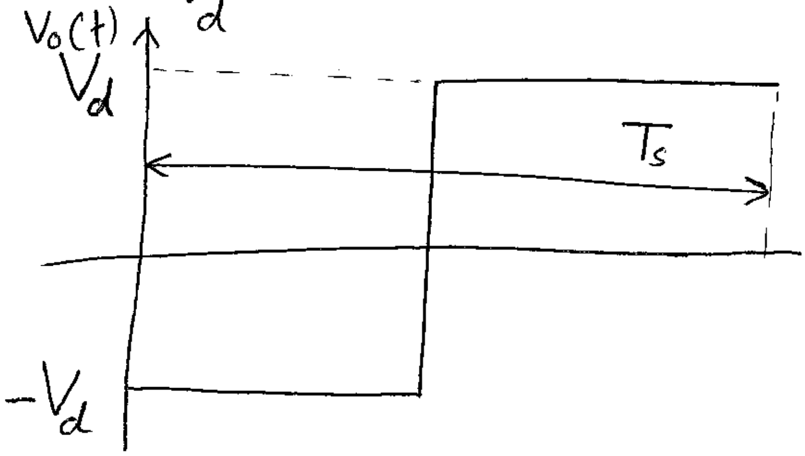
$$D_1 = \frac{t_{on}}{T_s} = \frac{\frac{1}{2} T_s}{T_s} = \frac{1}{2}$$

$$\therefore D_2 = \frac{1}{2}$$

$$\therefore V_o = (2D_1 - 1) V_d = ((2)(\tfrac{1}{2}) - 1) V_d$$

$$V_o = 0$$

$$\therefore \frac{V_o}{V_d} = 0.$$



Coming back to i_{ripple}:

$$i_{ripple} = \frac{V_d}{L_a}(t_2 - t_1)$$

For max. i_{ripple} $t_1 = 0$

$$i_{ripple_{max}} = \frac{V_d}{L_a} t_2 = \frac{V_d T_s}{2L_a} = \frac{V_d}{2L_a f_s}$$

Switch-Mode dc-ac

Inverters : dc ⟷ sinusoidal ac

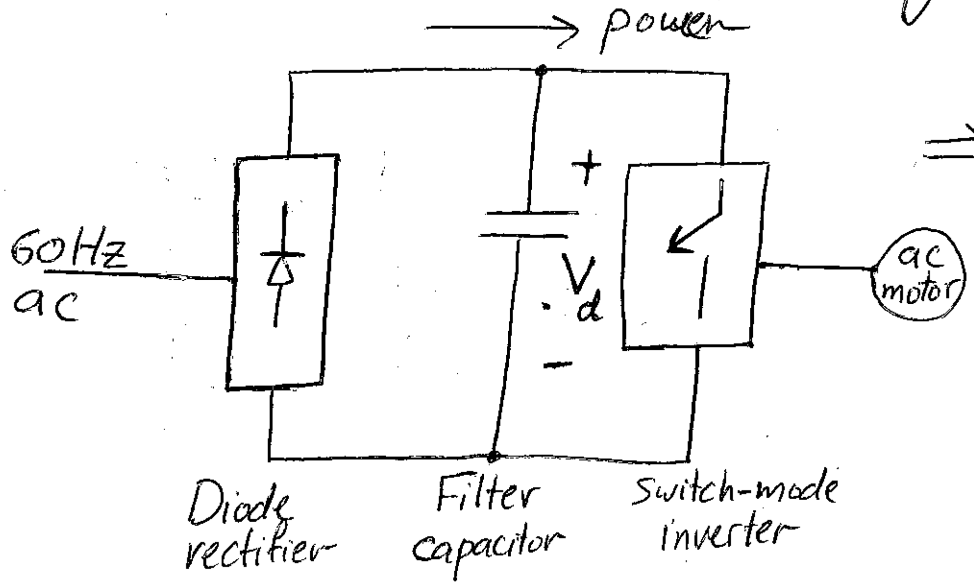
Applications :

1. ac motor drives → regenerative braking.
2. uninterruptible ac power supplies.
3. Photo-voltaic systems.
4. Wind energy power stations

Objective:

To produce a sinusoidal ac output whose magnitude and frequency are controlled.

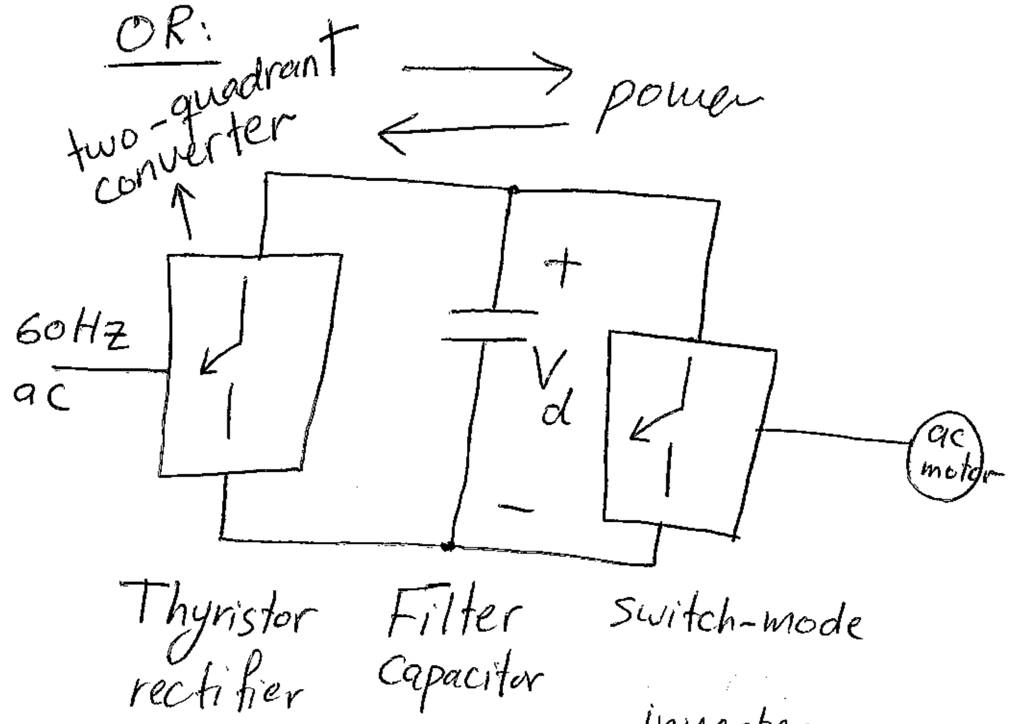
The block diagram of the system:



⇒ The regenerative braking is done by a parallel resistor with the capacitor.

Switch-mode inverter in ac motor drive

OR:



Switch-mode converters for motoring and regenerative braking in ac motor drive.

* During the regenerative braking where the energy recovered from the motor load inertia is fed back to the utility grid, the converter connecting the ac supply with the filter capacitor must be a two-quadrant converter like a thyristor rectifier circuit where the power flow can be reversible.

* The Voltage Source Inverters (VSIs) can be divided into:

① Pulse-Width-Modulated inverters (PWM inverters): the input dc voltage is essentially constant in magnitude, where a diode rectifier is used to rectify the line voltage. Therefore, the inverter must control the magnitude and the frequency of the ac output voltages. This is achieved by PWM of the inverter switches and hence such inverters are called PWM inverters.

→ special case of PWM inverters

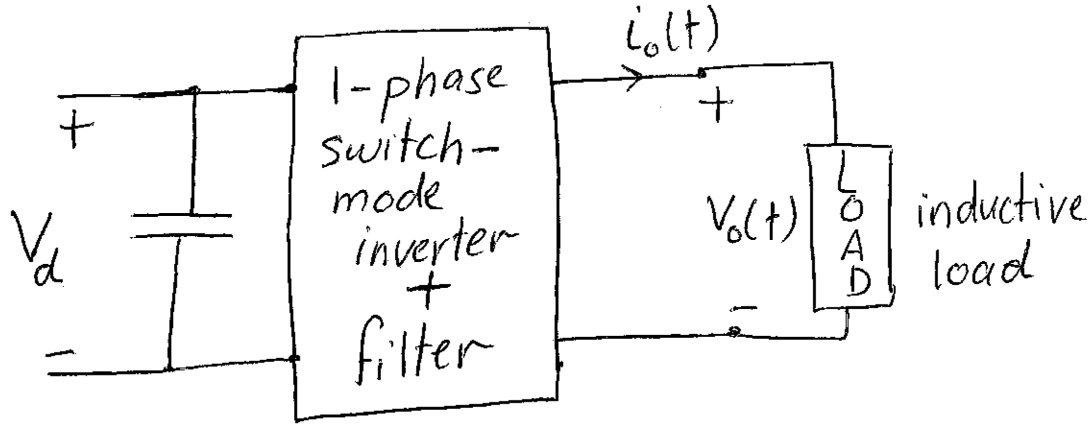
② Square-wave inverters: In these converters, the input dc voltage is controlled in order to control the magnitude of the output ac voltage and therefore the inverter has to control only the frequency of the output voltage. The output ac voltage has a waveform similar to a square wave and hence these inverters are called square-wave inverter.

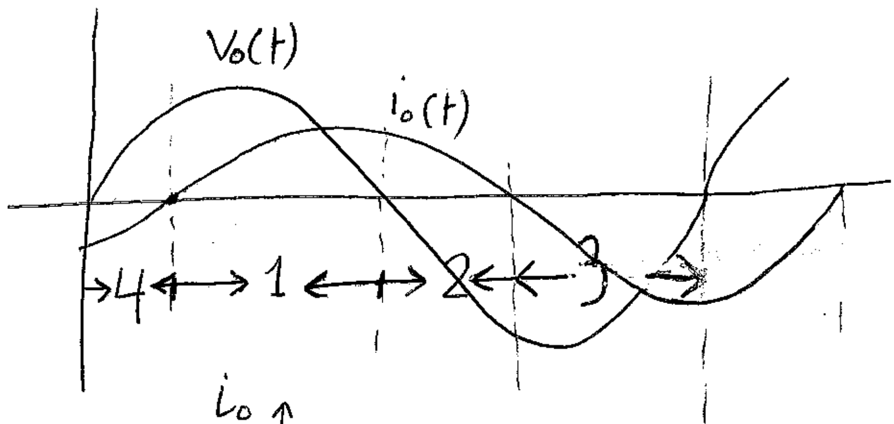
③ Single-phase inverters with voltage cancellation:

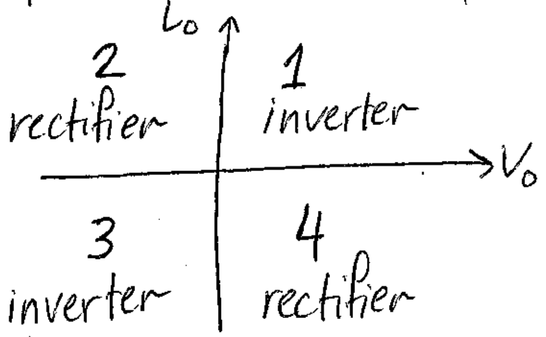
In this inverter, it is possible to control the magnitude and the frequency of the inverter output voltage even though the input to the inverter is a constant dc voltage and the inverter switches are not pulse-width modulated. (the output voltage waveform is like a square wave)

Basic Concept of Switch-Mode

Inverters

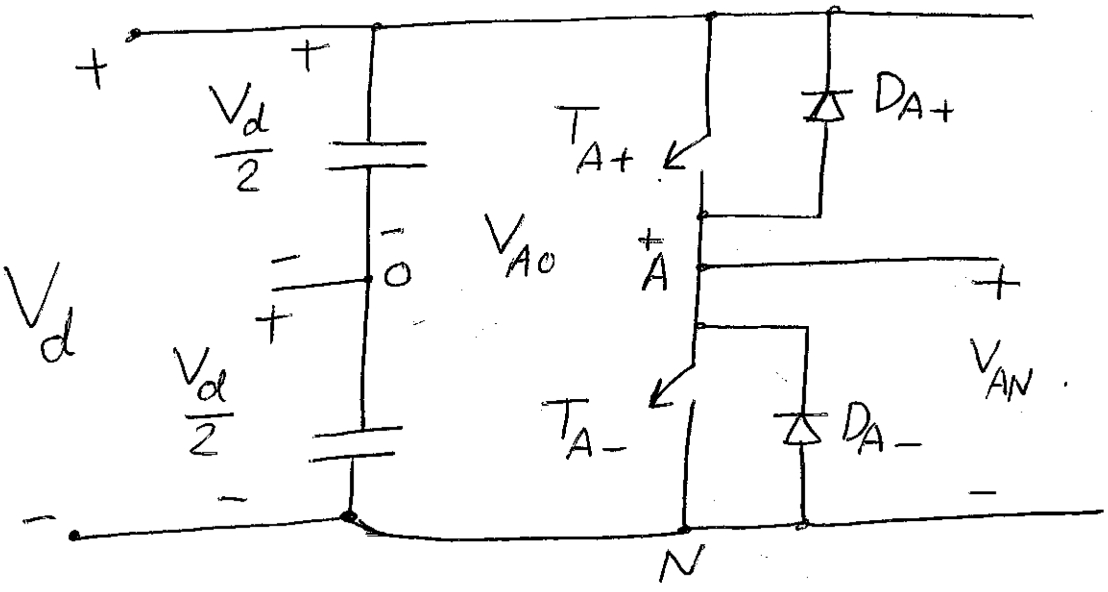






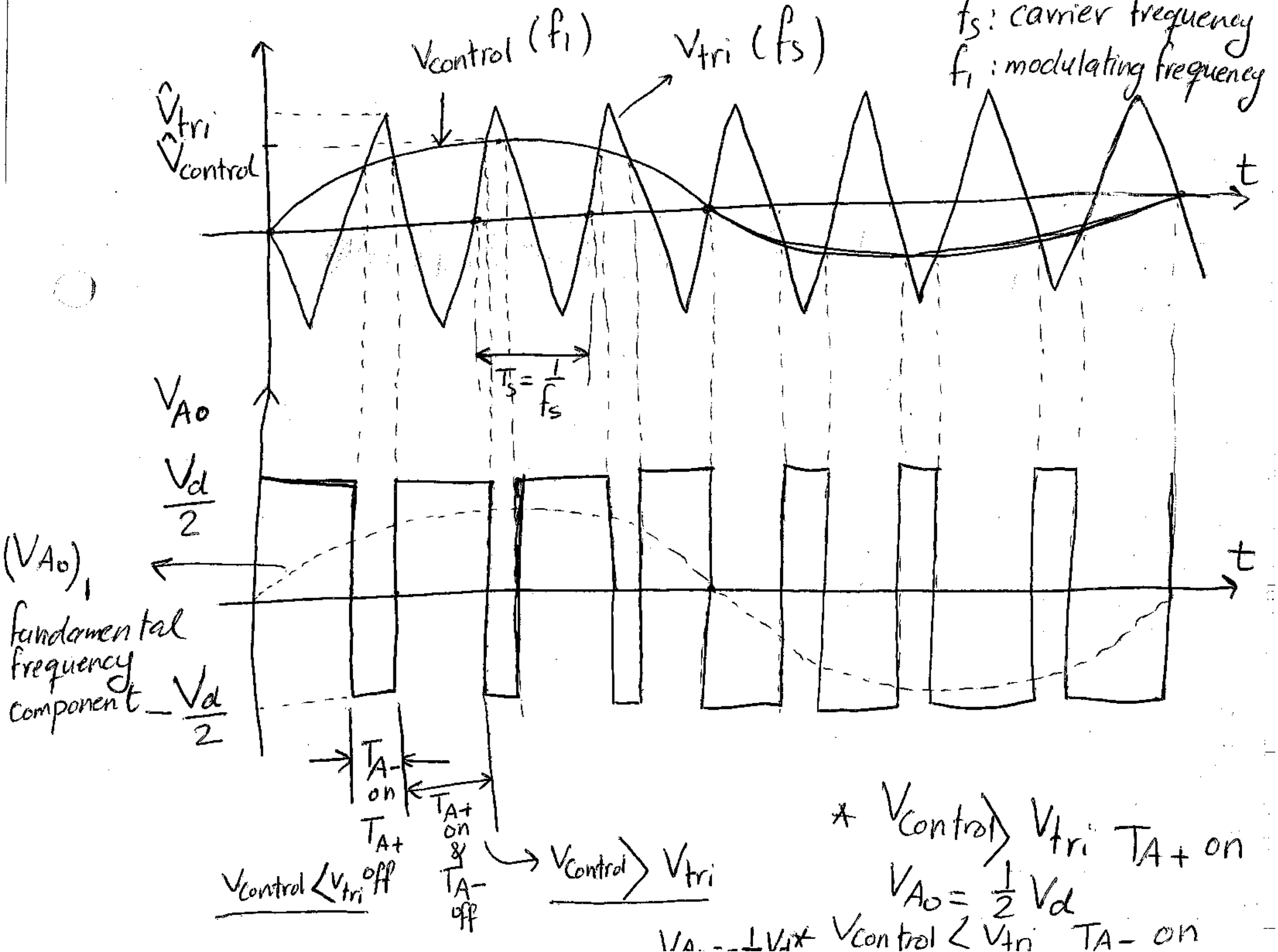
∴ The inverter operates in 4-quadrant scheme.

* The following figure is a One-leg switch-mode inverter:



One-leg switch-mode inverter

pulse-Width-Modulated Switching Scheme



* $V_{control} > V_{tri}$ T_{A+} on

$V_{Ao} = \frac{1}{2} V_d$

$V_{Ao} = -\frac{1}{2} V_d$ * $V_{control} < V_{tri}$ T_{A-} on

* f_s : the frequency with which the inverter switches are switched (carrier frequency).
* f_1 : the frequency which is the desired fundamental frequency of the inverter voltage output (modulating frequency)

* The amplitude modulation ratio m_a is:

$$m_a = \frac{\hat{V}_{control}}{\hat{V}_{tri}}$$

* The frequency modulation ratio m_f is:

$$m_f = \frac{f_s}{f_1}$$

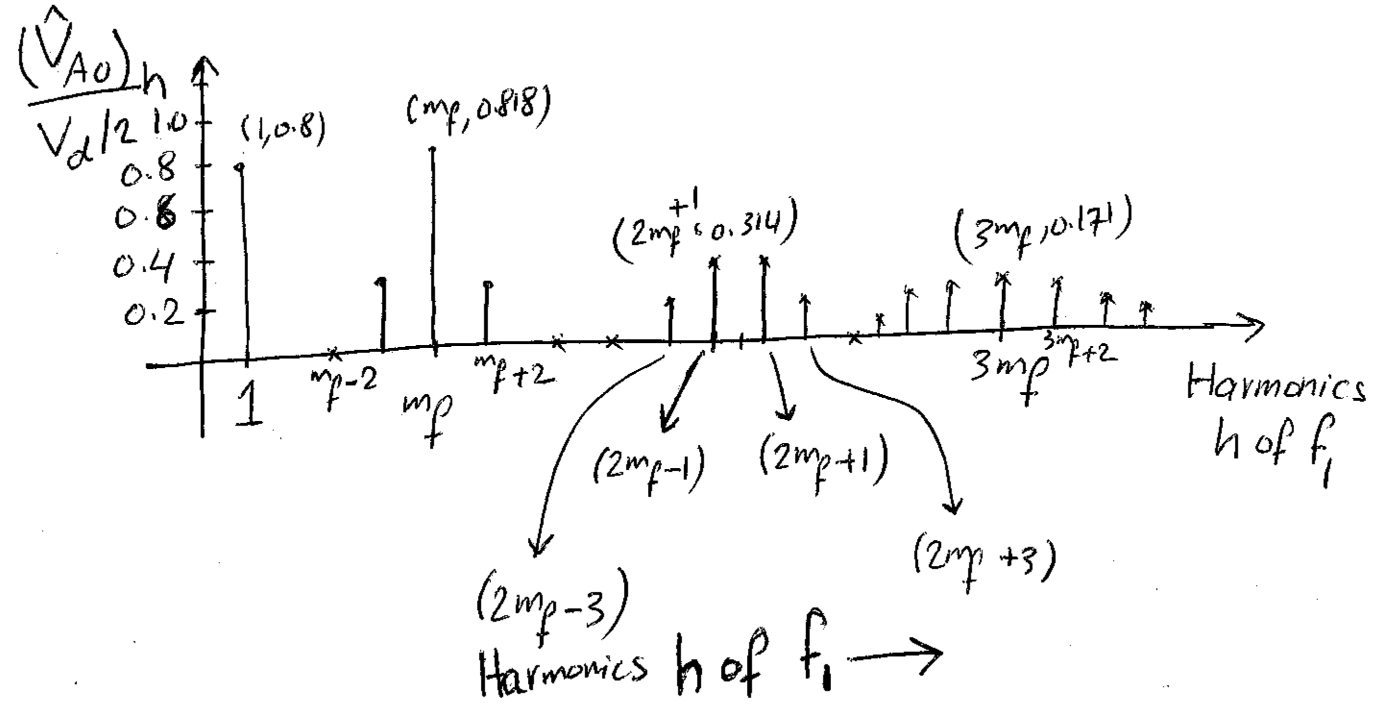
* For the above disussion, one-leg switch-mode inverter, T_{A+} & T_{A-} are controlled based on the comparison of $V_{control}$ & V_{tri} :

$V_{control} > V_{tri}$ T_{A+} is on $\Rightarrow V_{Ao} = \frac{1}{2} V_d$

or

$V_{control} < V_{tri}$ T_{A-} is on $\Rightarrow V_{Ao} = -\frac{1}{2} V_d$

* The following is harmonic spectrum of V_{Ao} for $m_a = 0.8$ & $m_f = 15$ for one-leg switch-mode inverter :



Harmonic spectrum of V_{Ao} for $m_a = 0.8$ & $m_f = 15$

Notes on PWM

① The peak amplitude of the fundamental-frequency component $(\hat{V}_{Ao})_1$ is m_a times $\frac{1}{2} V_d$.

$$(\hat{V}_{Ao})_1 = m_a \left(\frac{1}{2} V_d\right)$$

$$(\hat{V}_{Ao})_1 = \frac{\hat{V}_{control}}{\hat{V}_{tri}} \frac{V_d}{2}$$

$$(V_{Ao})_1(t) = \frac{\hat{V}_{control}}{\hat{V}_{tri}} \sin\omega_1 t \ \frac{V_d}{2}$$

$$= m_a \frac{V_d}{2} \sin\omega_1 t \quad \text{for } m_a \leq 1.0$$

$$\therefore (V_{AO})_1 = m_a \frac{V_d}{2} \quad m_a \leq 1.0$$

(2) The harmonics in the inverter output voltage waveform appear as sidebands centered around the switching frequency and its multiples, that is around harmonics m_f, $2m_f$, $3m_f$ and so on. This general pattern holds true for all values of m_a in the range 0–1.

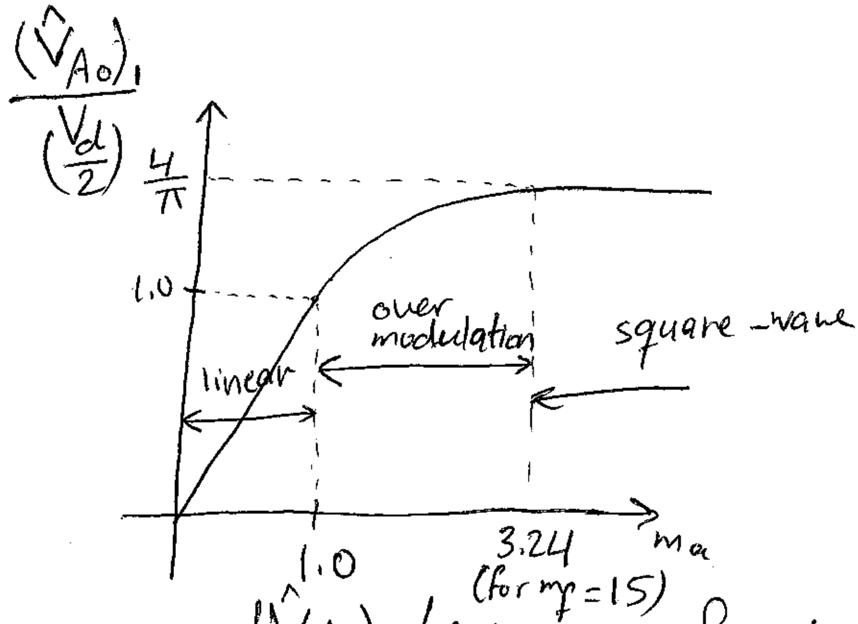
(3) For a frequency modulation ratio $m_f \leq 9$ (which is always the case, except in very high power ratings) the harmonic amplitudes are almost independent of m_f. The frequencies at which voltage harmonics occur can be indicated as:

$$f_h = \underbrace{(j m_f \pm K)}_{h} f_1$$

where for odd values of j, the harmonics exist only for even values of K. For even values of j, the harmonics exist only for odd values of K.

③ The harmonic m_f should be an odd integer.

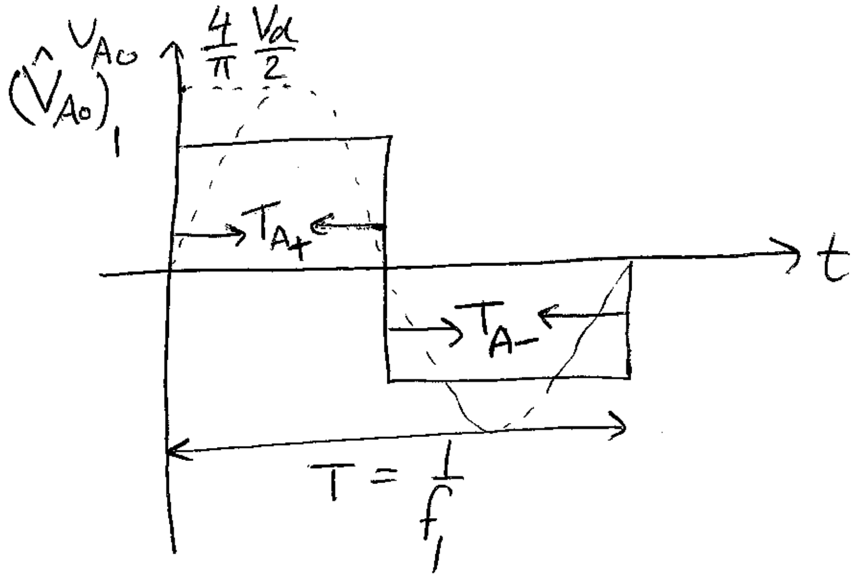
Over modulation ($m_a > 1.0$)



$(\hat{V}_{Ao})_1 / (V_d/2)$ as function of m_a for small m_f ($m_f < 9$)

Square-Wave switching Scheme

In square-wave switching scheme, each switch of the inverter leg is on for one half-cycle (180°) of the desired output frequency. The output voltage waveform is:



$$(\hat{V}_{Ao})_1 = \frac{4}{\pi}\frac{V_d}{2} = 1.273\left(\frac{V_d}{2}\right)$$

$$(\hat{V}_{Ao})_h = \frac{(\hat{V}_{Ao})_1}{h} \quad \text{for odd "}h\text{"}.$$

* It should be noted that the square-wave switching is also a special case of the sinusoidal PWM switching when m_a becomes so large that the control voltage waveform intersects with the triangular waveform only at the zero crossing of $v_{control}$. Therefore, the output voltage is independent of m_a in the square-wave region.

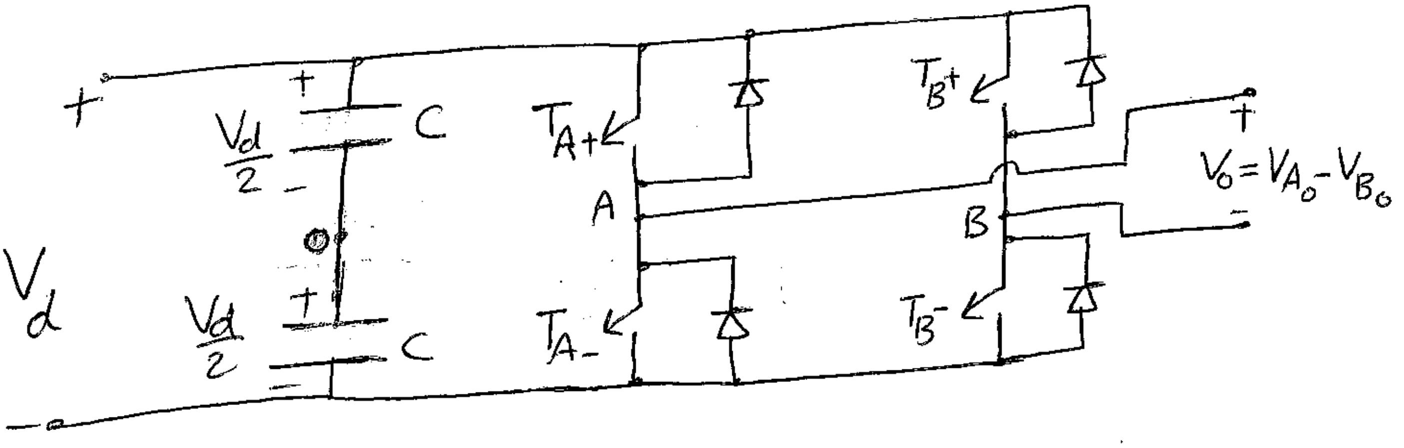
Advantages

(1) The inverter switches change its state only twice per cycle which in turn increases the efficiency of the inverter as the switching losses decrease (switching losses proportional with f_s). Additionally, at very high power levels, the solid-state switches generally have slower turn-on and turn-off speeds.

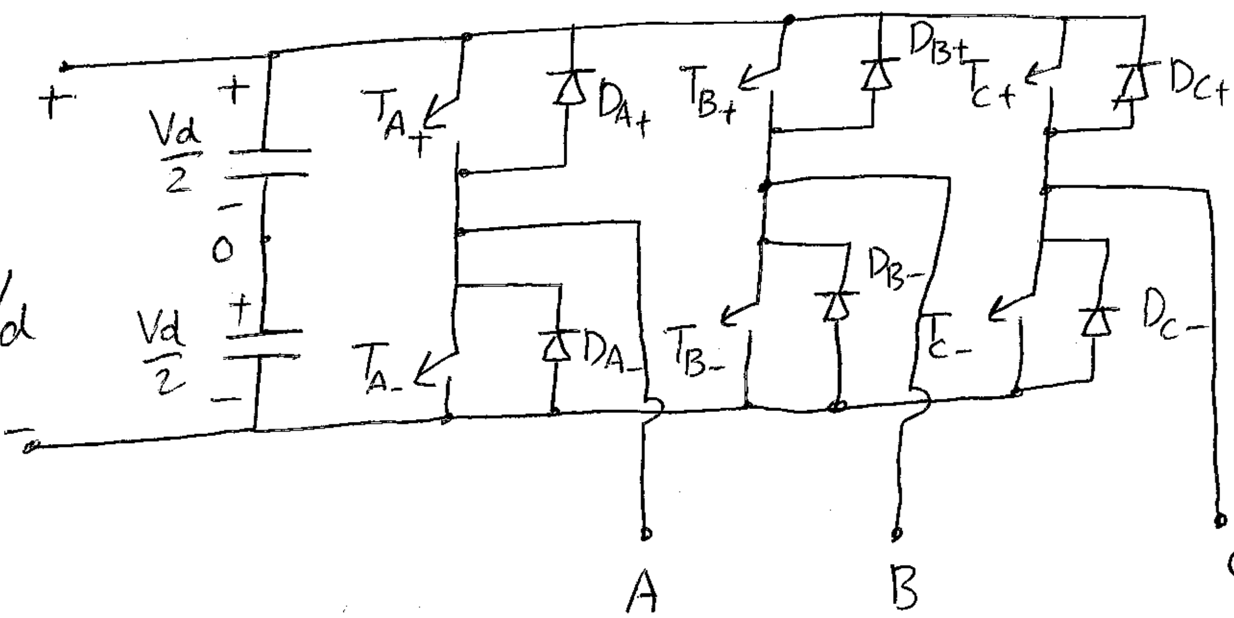
disadvantages

The output ~~is~~ voltage is not function of m_a. Therefore, the dc input voltage V_d ~~is~~ must be adjusted in order to control the magnitude of the inverter output voltage.

Full-Bridge Inverter (Single-phase)



Full-Bridg Inverters (Three-phase)



* In three-phase converter, three control signals shifted by 120° are needed and only one triangular signal is needed