Transformers
Ch. 2

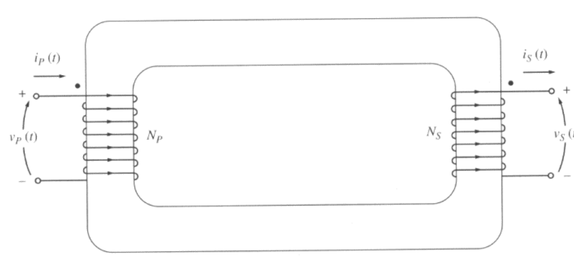
Dr. Feras Alasali

Introduction

A **transformer** is a device that converts one AC voltage to another AC voltage at the same frequency.

- It consists of one or more coil(s) of wire wrapped around a common ferromagnetic core.
- These coils are usually not connected electrically together.
- However, they are connected through the common magnetic flux confined to the core.

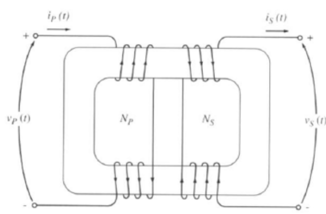
Assuming that the transformer has at least two windings, one of them (**primary**) is connected to a source of AC power; the other (**secondary**) is connected to the loads.



Types and construction

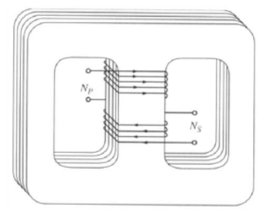
Power transformers

Core form



Windings are wrapped around two sides of a laminated square core.

Shell form



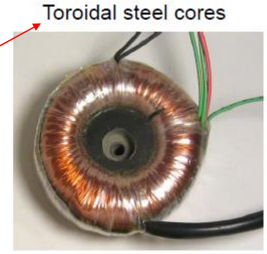
Windings are wrapped around the center leg of a laminated core.

Usually, windings are **wrapped on top of each other** to decrease flux leakage and, therefore, increase efficiency.

Lamination types

Laminated steel cores

Toroidal steel cores

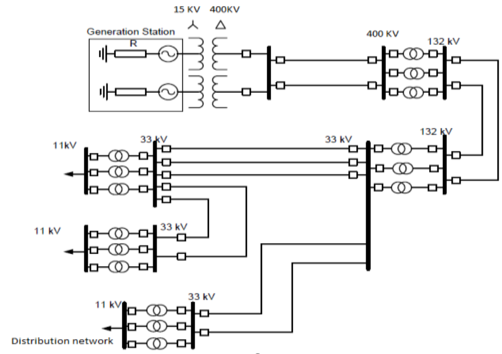


Efficiency of transformers with toroidal cores is usually higher.

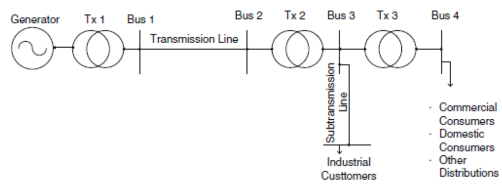
- **Overview of Electrical Power Grid.**

Power transformers used in power distribution systems are sometimes referred as follows:

1.Unit transformer: a power transformer connected to the output of a generator and used to step its voltage up to the transmission level (110 kV and higher).
2.Substation transformer: a transformer used at a substation to step the voltage from the transmission level down to the distribution level (2.3 -34.5 kV).
3.Distribution transformer: a transformer converting the distribution voltage down to the final level (110 V, 220 V).



SLD



Ideal transformer

We consider a lossless transformer with an input (primary) winding having N_p turns and a secondary winding of N_s turns.

The relationship between the voltage applied to the primary winding $v_p(t)$ and the voltage produced on the secondary winding $v_s(t)$ is

$$\frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$$

Here a is the turn ratio of the transformer.

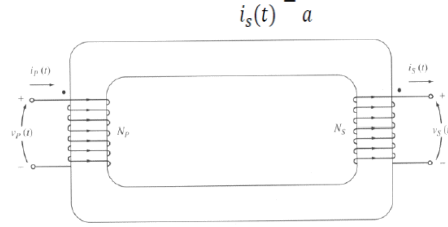
In the phasor notation:

$$\frac{V_p}{V_s} = a$$

$$\frac{I_p}{I_s} = \frac{1}{a}$$

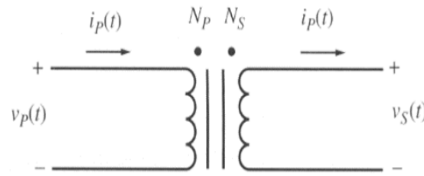
The relationship between the primary $i_p(t)$ and secondary $i_s(t)$ currents is

$$\frac{i_p(t)}{i_s(t)} = \frac{1}{a}$$



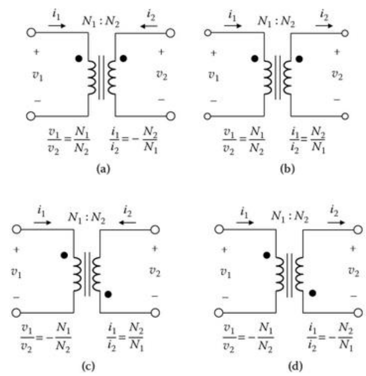
The **phase angles** of primary and secondary voltages **are the same**. The phase angles of primary and secondary currents are the same also. ***The ideal transformer changes magnitudes of voltages and currents but not their angles.***

One winding's terminal is usually marked by a **dot** used to determine the polarity of voltages and currents.



If the **voltage is positive at the dotted end of the primary winding** at some moment of time, the **voltage at the dotted end of the secondary winding will also be positive** at the same time instance.

If the **primary current flows into the dotted end of the primary** winding, the **secondary current will flow out of the dotted end of the secondary** winding.



Power in an ideal transformer

Assuming that θ_p and θ_s are the angles between voltages and currents on the primary and secondary windings respectively, the power supplied to the transformer by the primary circuit is:

$$P_{in} = V_p I_p \cos\theta_p$$

The power supplied to the output circuits is

$$P_{out} = V_s I_s \cos\theta_s$$

Since ideal transformers do not affect angles between voltages and currents:

$$\theta_p = \theta_s = \theta$$

Both windings of an ideal transformer have the same power factor.

Since for an ideal transformer the following holds: $V_s = \frac{V_p}{a}$ $\quad I_s = aI_p$

Therefore: $$P_{out} = V_s I_s \cos\theta_s = \frac{V_p}{a} aI_p \cos\theta_p = P_{in}$$

The output power of an ideal transformer equals to its input power – to be expected since assumed no loss. Similarly, for reactive and apparent powers:

$$Q_{out} = V_s I_s \sin\theta_s = \frac{V_p}{a} a I_p \sin\theta_p = Q_{in}$$

$$S_{out} = V_s I_s = \frac{V_p}{a} a I_p = S_{in}$$

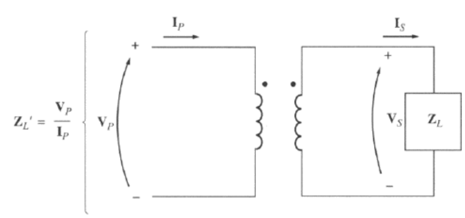
Impedance transformation

The **impedance** is defined as a following ratio of phasors:

$$Z_L = \frac{V_L}{I_L}$$

A transformer changes voltages and currents and, therefore, an apparent impedance of the load that is given by

$$Z_L = \frac{V_S}{I_S}$$



The apparent impedance of the primary circuit is:

$$\grave{Z}_L = \frac{V_P}{I_P}$$

which is

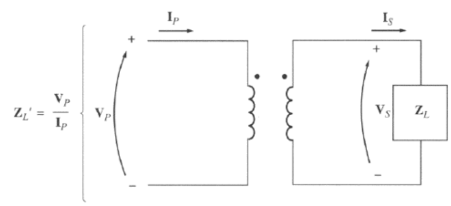
$$\grave{Z}_L = \frac{V_P}{I_P} = \frac{aV_s}{I_S/a} = a^2\frac{V_s}{I_S} = a^2 Z_L$$

It is possible to match magnitudes of impedances (load and a transmission line) by selecting a transformer with the proper turn ratio.

Analysis of circuits containing ideal transformers

- A simple method to analyze a circuit containing an ideal transformer is by replacing the portion of the circuit on one side of the transformer by an equivalent circuit with the same terminal characteristics.
- Next, we exclude the transformer from the circuit and solve it for voltages and currents.
- The solutions obtained for the portion of the circuit that was not replaced will be the correct values of voltages and currents of the original circuit.
- Finally, the voltages and currents on the other side of the transformer (in the original circuit) can be found by considering the transformer s turn ratio.

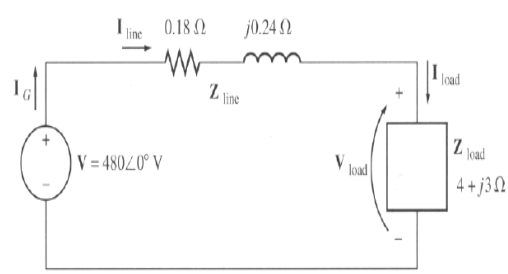
This process is called referring of transformer s sides.



Example

Example 4.1: A single-phase power system consists of a 480-V 60-Hz generator that is connected to the load $Z_{load} = 4 + j3\ \Omega$ through the transmission line with $Z_{line} = 0.18 + j0.24\ \Omega$.

a) What is the voltage at the load? What are the transmission line losses?
b) If a 1:10 step up transformer and a 10:1 step down transformer are placed at the generator and the load ends of the transmission line respectively, what are the new load voltage and the new transmission line losses?

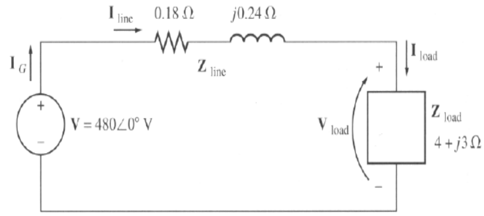


a) $I_G = I_{line} = I_{load} = \dfrac{V}{Z_{line} + Z_{load}}$

$$= \frac{480\angle 0^o}{0.18 + j0.24 + 4 + j3}$$

$$= \frac{480\angle 0^o}{5.29\angle 37.8^o}$$

$$= 90.8\angle -37.8^o \quad A$$



The load voltage:

$$V_{load} = I_{load} Z_{load} = (90.8\angle -37.8^o)(4 + j3) = (90.8\angle -37.8^o)(5\angle -36.9^o) = 454\angle -0.9^o$$

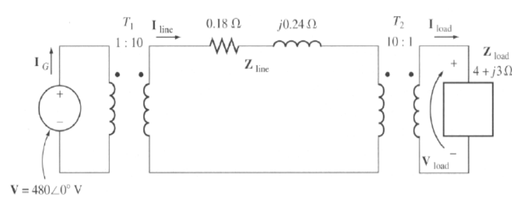
The line losses are:

$$P_{loss} = I_{line}^2 R_{line} = 90.8^2 \cdot 0.18 = 1484\ W$$

b) We will

1) eliminate transformer T_2 by referring the load over to the transmission line's voltage level.

2) Eliminate transformer T_1 by referring the transmission line's



The load impedance when referred to the transmission line (while the transformer T_2 is eliminated) is:

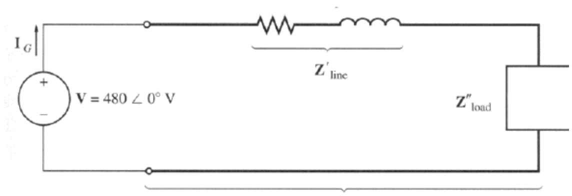
$$\grave{Z}_{load} = {a_2}^2 Z_{load} = \left(\frac{10}{1}\right)^2 (4 + j3)$$
$$= 400 + j300$$

The total impedance on the transmission line level is

$$Z_{equ} = Z_{line} + \grave{Z}_{load} = 400.18 + j300.24$$
$$= 500.3\angle 36.88^o$$

The total impedance is now referred across T_1 to the source's voltage level:

$$\grave{Z}_{equ} = {a_1}^2 Z_{load} = \left(\frac{1}{10}\right)^2 500.3\angle 36.88^o = 5.003\angle 36.88^o$$



Equivalent circuit

The generator's current is

$$I_G = \frac{V}{\tilde{Z}_{equ}} = \frac{480\angle 0^o}{5.003\angle} = 95.94\angle -36.88^o$$

Knowing transformers' turn ratios, we can determine line and load currents:

$$I_{line} = a_1 I_G = 0.1(95.94\angle -36.88^o) = 9.594\angle -36.88^o \quad A$$

$$I_{load} = a_2 I_{line} = 10(9.594\angle -36.88^o) = 95.94\angle -36.88^o \quad A$$

Therefore, the load voltage is:

$$V_{load} = I_{load} Z_{load} = (95.94\angle -36.88^o)(5\angle -36.7^o) = 479.7\angle -0.01^o \quad V$$

The losses in the line are:

$$P_{loss} = I_{line}^2 R_{line} = 9.594^2 \cdot 0.18 = 16.7 \; W$$

Note: transmission line losses are reduced by a factor nearly 90, the load voltage is much closer to the generator's voltage – effects of increasing the line's voltage.

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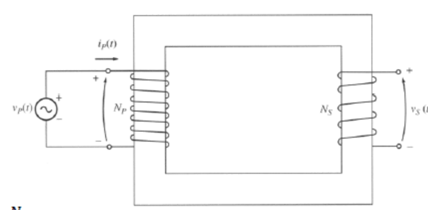
Theory of operation of real single phase transformers

Real transformers approximate ideal ones to some degree.

The basis transformer operation can be derived from Faraday's law:

$$e_{ind} = \frac{d\lambda}{dt}$$



Here λ is the flux linkage in the coil across which the voltage is induced:

$$\lambda = \sum_{i=1}^{N} \phi_i$$

where ϕ_i is the flux passing through the i^{th} turn in a coil – slightly different for different turns. However, we may use an average flux per turn in the coil having N turns:

$$\bar{\phi} = \lambda / N$$

Therefore:

$$e_{ind} = N\frac{d\bar{\phi}}{dt}$$

If the source voltage $v_p(t)$ is applied to the primary winding, the average flux in the primary winding will be:

$$\bar{\phi} = \frac{1}{N_p}\int v_p(t)dt$$

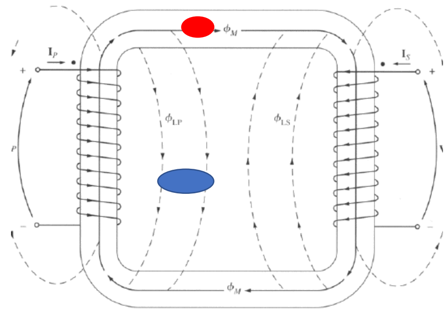
A portion of the flux produced in the primary coil passes through the secondary coil (mutual flux); the rest is lost (leakage flux):

$$\overline{\phi_p} = \phi_m + \phi_{Lp}$$

average primary flux, mutual flux, leakage flux

Similarly, for the secondary coil:

Average secondary flux $\overline{\phi_s} = \phi_m + \phi_{Ls}$



From the Faraday's law, the primary coil's voltage is: $v_p(t) = N_p\frac{d\bar{\phi}_p}{dt} = N_p\frac{d\phi_m}{dt} + N_p\frac{d\phi_{Lp}}{dt}$

The secondary coil's voltage is: $v_s(t) = N_s\frac{d\bar{\phi}_s}{dt} = N_s\frac{d\phi_m}{dt} + N_s\frac{d\phi_{Ls}}{dt}$

The primary and secondary voltages due to the mutual flux are:

$$e_p(t) = N_p\frac{d\phi_m}{dt}$$

$$e_s(t) = N_s\frac{d\phi_m}{dt}$$

Combining the last two equations:

$$\frac{e_p(t)}{N_p} = \frac{d\phi_m}{dt} = \frac{e_s(t)}{N_s}$$

Therefore:

$$\frac{e_p(t)}{e_s(t)} = \frac{N_p}{N_s} = a$$

That is, the ratio of the primary voltage to the secondary voltage both caused by the mutual flux is equal to the turns ratio of the transformer.

For well-designed transformers: $\Phi_m \gg \Phi_{LP}$ and $\Phi_m \gg \Phi_{LS}$

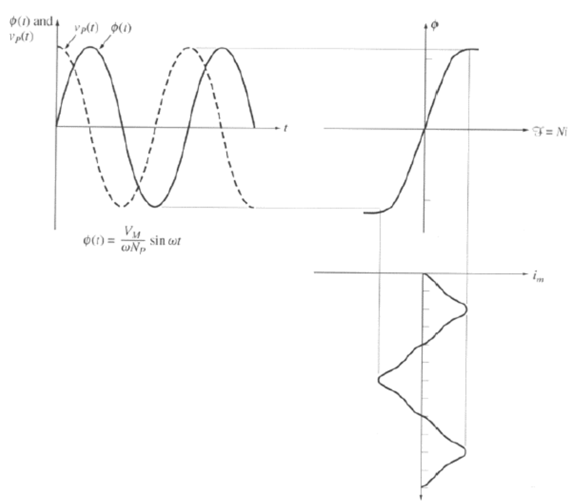
Therefore, the following approximation normally holds:

$$\frac{v_p(t)}{v_s(t)} \approx \frac{N_p}{N_s} \approx a$$

The magnetization current in a real transformer

Even when no load is connected to the secondary coil of the transformer, a current will flow in the primary coil. This current consists of:

1. The magnetization current i_m needed to produce the flux in the core;
2. The core-loss current i_{h+e} hysteresis and eddy current losses.



- If the values of current are comparable to the flux they produce in the core, it is possible to sketch a magnetization current. We observe:

1.Magnetization current is not sinusoidal: there are high frequency components;

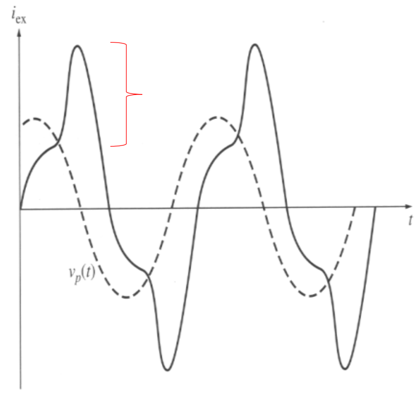
2. Once saturation is reached, a small increase in flux requires a large increase in magnetization current;

Core-loss current is:

1. Nonlinear due to nonlinear effects of hysteresis;
2. In phase with the voltage.

The total no-load current in the core is called the excitation current of the transformer:

$$i_{ex} = i_m + i_{h+e}$$

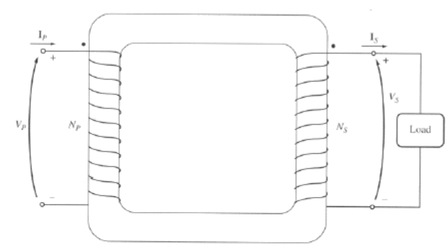


The current ratio on a transformer

- If a load is connected to the secondary coil, there will be a current flowing through it.
- A current flowing into the dotted end of a winding produces a positive magnetomotive force F
- The net magnetomotive force in the core
- where (R) is the reluctance of the transformer core. For well designed transformer cores, the reluctance is very small if the core is not saturated. Therefore:

$$F_p = N_p i_p$$

$$F_s = N_s i_s$$



$$F_{net} = N_p i_p - N_s i_s = \phi \Re$$

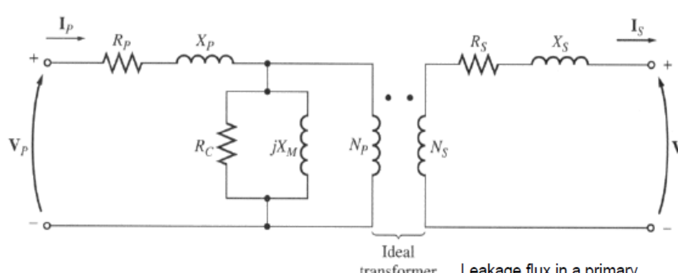
$$F_{net} = N_p i_p - N_s i_s \approx 0$$

The transformers equivalent circuit & Transformer losses

To model a real transformer accurately, we need to account for the following losses:

1. Copper losses resistive heating in the windings: I^2R
2. Eddy current losses resistive heating in the core: proportional to the square of voltage applied to the transformer.
3. Hysteresis losses energy needed to rearrange magnetic domains in the core: nonlinear function of the voltage applied to the transformer.
4. Leakage flux that escapes from the core and flux that passes through one winding only.

The exact equivalent circuit of a real transformer



Copper losses are modeled by the resistors R_p and R_s.

Leakage flux in a primary winding produces the voltage:

$$e_{Lp}(t) = N_p \frac{d\Phi_{LP}}{dt}$$

Since much of the leakage flux pass through air, and air has a constant reluctance that is much higher than the core reluctance, the primary coil's leakage flux is:

$$\Phi_{LP} = PN_p i_p$$

Where **P** permeance of flux path

Therefore:

$$e_{Lp}(t) = N_p \frac{d}{dt}(PN_p i_p) = N_p{}^2 P \frac{di_p}{dt}$$

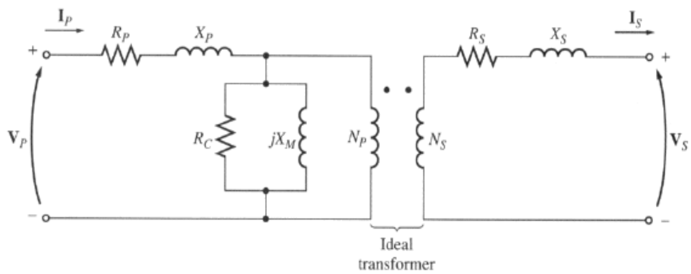
Recognizing that the self-inductance of the primary coil is

$$L = N_p^{\,2}\, P$$

The induced voltages are:

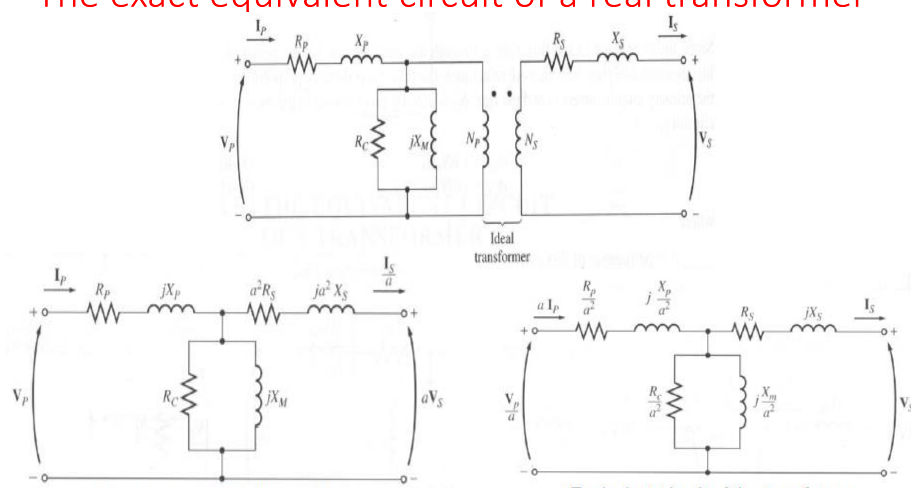
Primary coil: $e_{Lp}(t) = L_P \dfrac{di_P}{dt}$

Secondary coil: $e_{Ls}(t) = L_s \dfrac{di_s}{dt}$



The leakage flux can be modeled by primary and secondary inductors.
The magnetization current can be modeled by a reactance X_M connected across the primary voltage source.
The core-loss current can be modeled by a resistance R_C connected across the primary voltage source.
Both currents are nonlinear; therefore, X_M and R_C are just approximations.

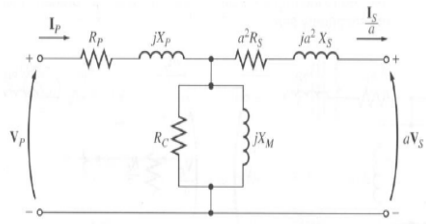
The exact equivalent circuit of a real transformer

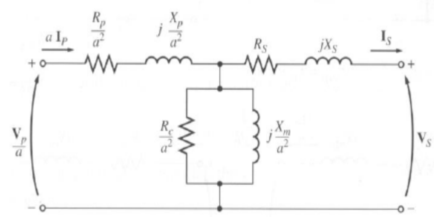


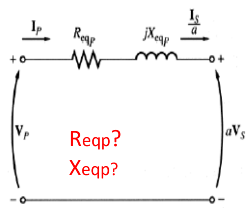
Equivalent circuit of the transformer referred to its primary side.

Equivalent circuit of the transformer referred to its secondary side.

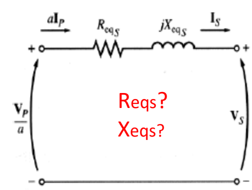
Approximate equivalent circuit of a transformer







Without an excitation branch referred to the primary side.



Without an excitation branch referred to the secondary side.

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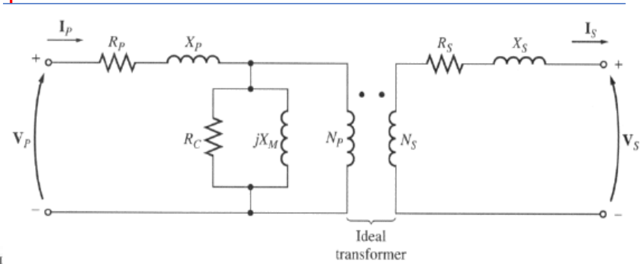
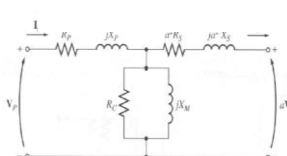
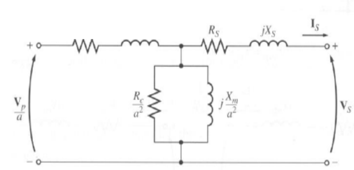
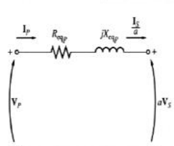
equivalent circuit of a transformer

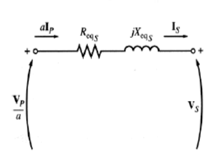








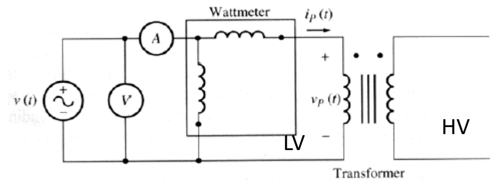
Without an excitation branch referred to the primary side.



Without an excitation branch referred to the secondary side.

Determining the values of Tr. components

- **The open-circuit test.**
  - Full line voltage is applied to the primary side of the transformer. The input voltage, current, and power are measured.



- From this information, the power factor of the input current and the magnitude and the angle of the excitation impedance can be determined.
- To evaluate R_C and X_M, we determine the conductance of the core-loss resistor is:

$$G_c = \frac{1}{R_c}$$

- The susceptance of the magnetizing inductor is:

$$B_M = \frac{1}{X_M}$$

- Since both elements are in parallel, their admittances add. Therefore, the total excitation admittance is:

$$Y_E = G_C - jB_M$$

$$Y_E = \frac{1}{R_c} - j\frac{1}{X_M}$$

- The **magnitude of the excitation admittance** in the open-circuit test is:

$$|Y_E| = \frac{I_{OC}}{V_{OC}}$$

- The **angle of the admittance** in the open-circuit test can be found from the circuit power factor (PF):

$$\cos(\theta) = PF = \frac{P_{OC}}{V_{OC} I_{OC}}$$

- In real transformers, the power factor is always lagging, so the angle of the current always lags the angle of the voltage by θ degrees. The admittance is:

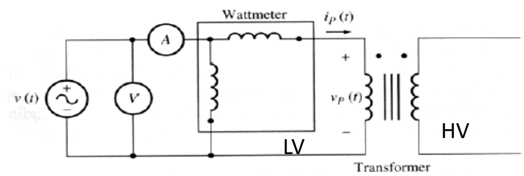
$$|Y_E| = \frac{I_{OC}}{V_{OC}} \angle -\theta$$

- Therefore, it is possible to determine values of R_C and X_M in the open-circuit test.

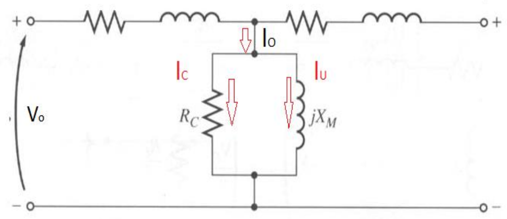
$$= \frac{I_{OC}}{V_{OC}} \angle -cos^{-1} PF$$

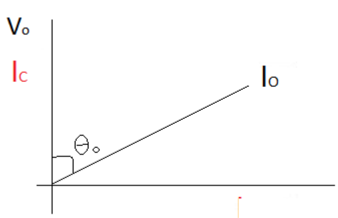
O.C Test

- Vo= rated voltage
- Io= small value
- Wo = core losses (small value)



- From O.C test (Vo, Io, Wo)
- Where Wo= (Vo) (Io) (cosϴo) ➡ cosϴo = Wo/(Vo)(Io)
- ϴo is no load PF angle





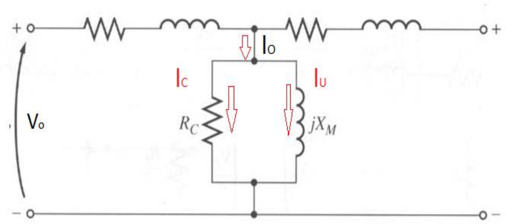
Ic= Io cosϴo

Iu= Io sinϴo

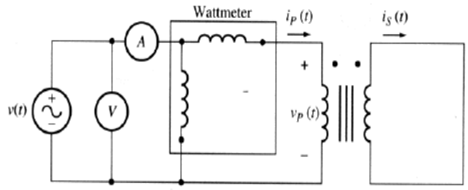
⬇

Rc = Ro = Vo/ Ic

Xm = Xo = Vo/Iu



The short circuit test.



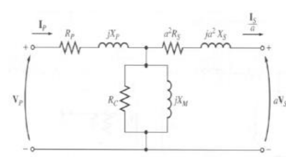
- Fairly low input voltage is applied to the primary side of the transformer. This voltage is adjusted until the current in the secondary winding equals to its rated value.
- The input voltage, current, and power are again measured.
- Since the input voltage is low, the current flowing through the excitation branch is negligible; therefore, all the voltage drop in the transformer is due to the series elements in the circuit. The magnitude of the series impedance referred to the primary side of the transformer is:

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}}$$

- The power factor of the current is given by: $\cos(\theta) = PF = \dfrac{P_{SC}}{V_{SC} I_{SC}}$

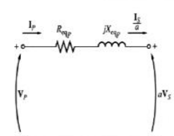
- Therefore:

$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle \theta$$



- Since the serial impedance Z_{SE} is equal to

$$Z_{SE} = R_{eq} + jX_{eq}$$



Without an excitation branch referred to the primary side.

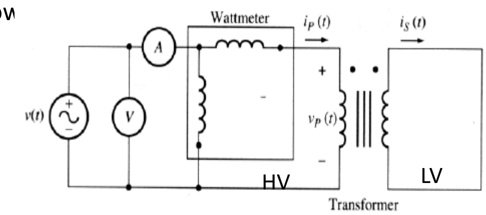
- it is possible to determine the total series impedance referred to the primary side of the transformer. However, there is no easy way to split the series impedance into primary and secondary components.

$$Z_{SE} = (R_p + a^2 R_s) + j(X_p + a^2 X_s)$$

- The **same tests can be performed on the secondary** side of the transformer. The results will yield the equivalent circuit impedances referred to the secondary side of the transformer.

S.C Test

- Vsc= short circuit voltage (small value)
- Isc= short circuit current
- Wsc = copper losses (short circuit pow

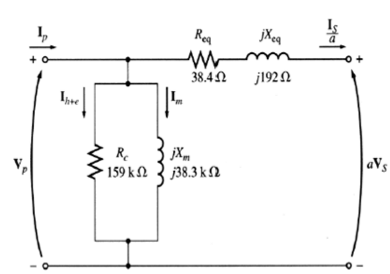


- From S.C test (Vsc, Isc, Wsc)
- Where Wsc= $(I_{sc})^2 R_{eq}$
- $R_{seq} = \frac{W_{sc}}{(I_{sc})^2}$
- $Z_{seq} = \frac{V_{sc}}{I_{sc}}$ → $Z_{SE} = (R_p + a^2 R_s) + j(X_p + a^2 X_s)$
- $X_{seq} = \sqrt{(Z_{seq})^2 - (R_{seq})^2}$

Example

Example: We need to determine the equivalent circuit impedances of a **20 kVA, 8000/240 V, 60 Hz** transformer. The open-circuit and short-circuit tests led to the following data:

V_{OC} = 240 V	V_{SC} = 489 V
I_{OC} = 7.133 A	I_{SC} = 2.5 A
P_{OC} = 400 W	P_{SC} = 240 W



- **The open-circuit test.**

The power factor during the open-circuit test is $\cos(\theta) = PF = \frac{P_{OC}}{V_{OC} I_{OC}} = 0.234\ lagging$

The excitation admittance is $Y_E = \frac{I_{OC}}{V_{OC}} \angle -cos^{-1} PF = 0.0297 \angle -cos^{-1} 0.234$

$$= 0.00693 - j0.0288$$

$$Y_E = \frac{1}{R_c} - j\frac{1}{X_M}$$

$$R_C = \frac{1}{0.00693} = 144\ \Omega \qquad X_M = \frac{1}{0.0288} = 34.63\ \Omega$$

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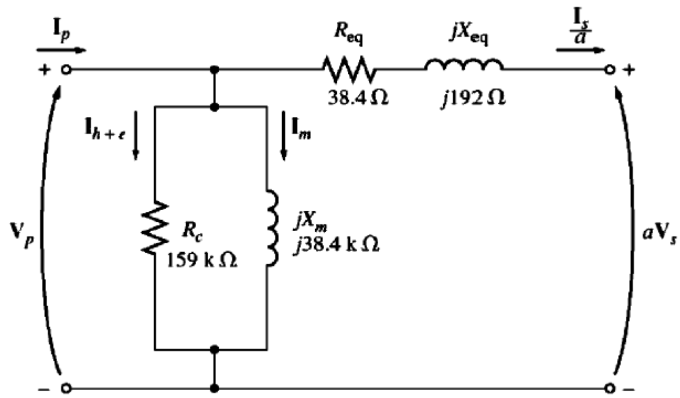
$$= 38.4 - j192$$

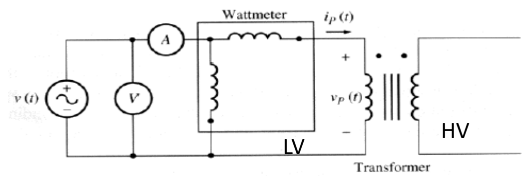
$$R_{eq} = 38.4\Omega \qquad X_{eq} = 192\Omega$$



Rc,p = a^2 Rc,s = 33.3^2 (144) = 159 k ohm
Xm,p = a^2 Xm,s = 33.3^2 (34.63) = 38.4 k ohm

O.C Test

- V_o= rated voltage
- I_o= small value
- W_o = core losses (small value)



- From O.C test (V_o, I_o, W_o)
- Where W_o= (V_o) (I_o) ($\cos\Theta_o$) → $\cos\Theta_o = W_o/(V_o)(I_o)$
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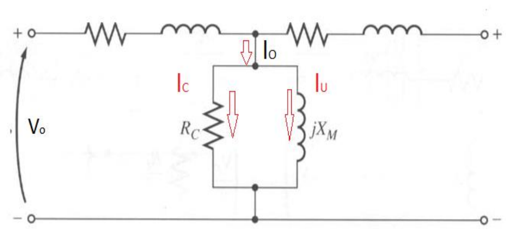
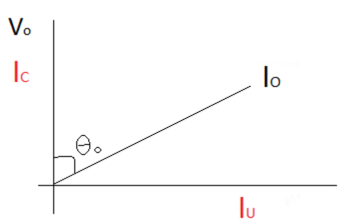


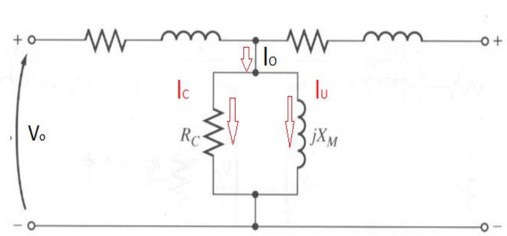


$I_c = I_o \cos\Theta_o$

$I_u = I_o \sin\Theta_o$

$R_c = R_o = V_o / I_c$

$X_m = X_o = V_o / I_u$



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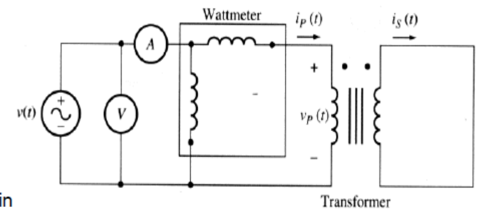

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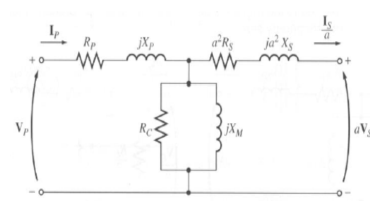

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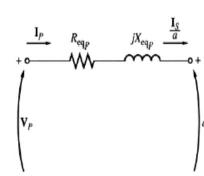

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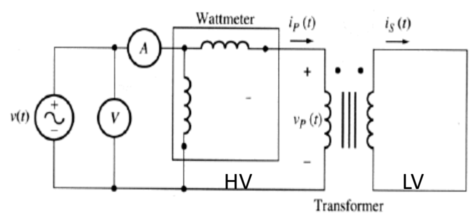
- $Z_{seq} = \frac{V_{sc}}{I_{sc}}$ ⇒ $Z_{SE} = (R_p + a^2 R_s) + j(X_p + a^2 X_s)$
- $X_{seq} = \sqrt{(Z_{seq})^2 - (R_{seq})^2}$



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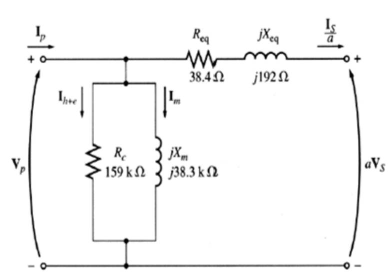

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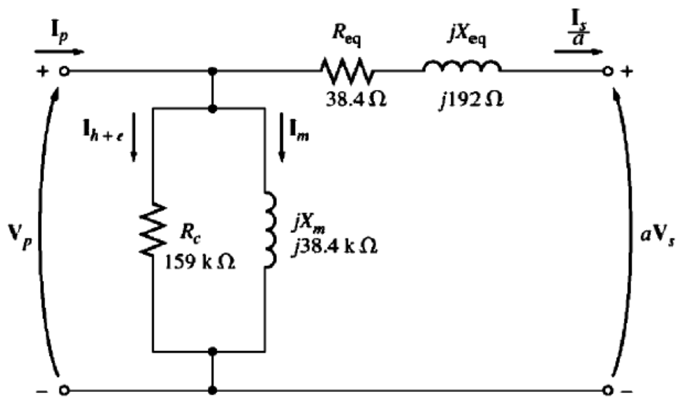
$$R_{eq} = 38.4\Omega \qquad X_{eq} = 192\Omega$$

Quiz



- Rc,p = a^2 Rc,s = 33.3^2 (144) = 159 k ohm
- Xm,p = a^2 Xm,s = 33.3^2 (34.63) = 38.4 k ohm

The per unit system

- Another approach to solve circuits containing transformers is the per-unit system. Impedance and voltage-level conversions are avoided. Also, machine and transformer impedances fall within fairly narrow ranges for each type and construction of device while the per-unit system is employed.

- The voltages, currents, powers, impedances, and other electrical quantities are measured as fractions of some base level instead of conventional units.

$$Quantity\ per\ unit = \frac{actual\ value}{base\ value\ quantity}$$

$$P_{base}, Q_{base}, or\ S_{base} = V_{base} I_{base}$$

- Usually, two base quantities are selected to define a given per-unit system. Often, such quantities are voltage and power (or apparent power). In a 1-phase system:

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{{V_{base}}^2}{S_{base}}$$

$$Y_{base} = \frac{I_{base}}{V_{base}}$$

- Ones the base values of P (or S) and V are selected, all other base values can be computed form the above equations.
- In a power system, a base apparent power and voltage are selected at the specific point in the system . Note that a transformer has no effect on the apparent power of the system, since the apparent power into a transformer equals the apparent power out of a transformer. As a result, the base apparent power remains constant everywhere in the power system.
- On the other hand, voltage (and, therefore, a base voltage) changes when it goes through a transformer according to its turn ratio. Therefore, the process of referring quantities to a common voltage level is done automatically in the per unit system.

Example

A simple power system is given by the circuit

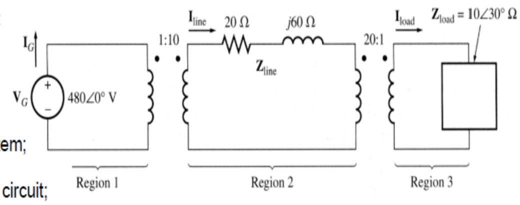
The generator is rated at 480 V and 10 kVA.

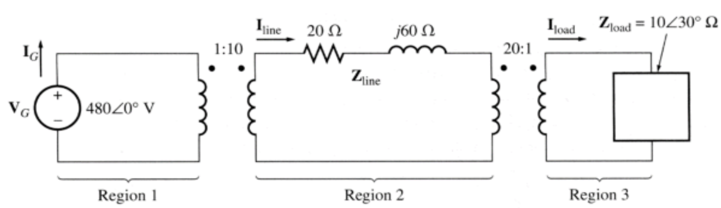
a) Find the base voltage, current, impedance, and apparent power at every points in the system;

b) Convert the system to its per-unit equivalent circuit;

c) Find the power supplied to the load in this system;

e) Find the power lost in the transmission line (Region 2).





a. In the generator region: $V_{base\,1} = 480\ V$ and $S_{base} = 10\ kVA$

$$I_{base1} = \frac{S_{base1}}{V_{base1}} = \frac{10000}{480} = 20.83A \qquad Z_{base1} = \frac{V_{base1}}{I_{base1}} = \frac{480}{20.83} = 23.04\Omega$$

The turns ratio of the transformer T_1 is $a_1 = 0.1$; therefore, the voltage in the transmission line region is

$$V_{base2} = \frac{V_{base1}}{a_1} = \frac{480}{0.1} = 4800V \qquad S_{base2} = 10000\ VA$$

$$I_{base2} = \frac{S_{base2}}{V_{base2}} = \frac{10000}{4800} = 2.083A \qquad Z_{base2} = \frac{V_{base2}}{I_{base2}} = \frac{4800}{2.083} = 2304\Omega$$

The turns ratio of the transformer T_2 is $a_2 = 20$, therefore, the voltage in the load region is

$$V_{base3} = \frac{V_{base2}}{a_2} = \frac{4800}{20} = 240V \qquad S_{base3} = 10000\,VA$$

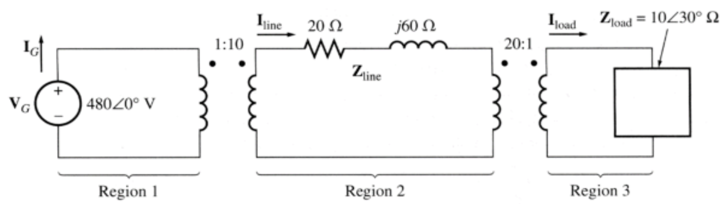
$$I_{base3} = \frac{S_{base3}}{V_{base3}} = \frac{10000}{240} = 41.67A \qquad Z_{base3} = \frac{V_{base3}}{I_{base3}} = \frac{240}{41.67} = 5.76\Omega$$

b. To convert a power system to a per-unit system, each component must be divided by its base value in its region. The generator's per-unit voltage is

$$V_{G,pu} = \frac{V_G}{V_{base1}} = \frac{480\angle 0°}{480} = 1\angle 0°$$

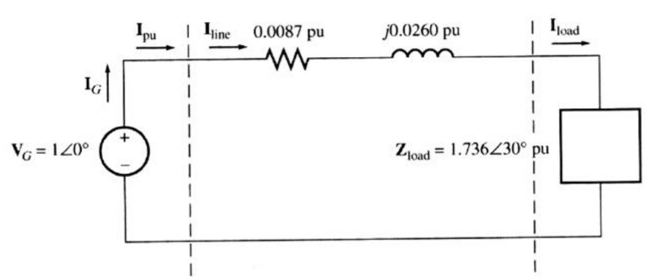
The transmission line's per-unit impedance is

$$Z_{line,pu} = \frac{Z_{line}}{Z_{base2}} = \frac{20 + j60}{2304} = 0.0087 + j0.026pu$$



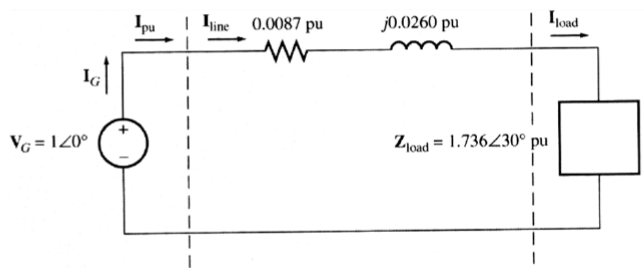
The load's per-unit impedance is

$$Z_{load,pu} = \frac{Z_{load}}{Z_{base3}} = \frac{10\angle 30°}{5.76} = 1.736\angle 30°$$



c. The current flowing in this per-unit power system is

$$I_{pu} = \frac{V_{pu}}{Z_{total,pu}} = \frac{1\angle 0°}{0.0087 + j0.026 + 1.736\angle 30°} = 0.569\angle 30.6° pu$$



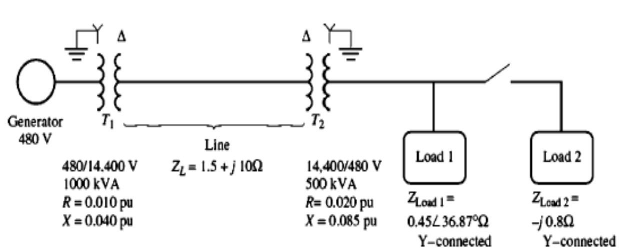
Therefore, the per-unit power on the load is $P_{load,pu} = I_{pu}{}^2 R_{pu} = 0.569^2 \cdot 1.503$ =0.487pu

The actual power on the load is $P_{load} = P_{load,pu} S_{base} = 0.487 \cdot 10000$ =487W

d. The per-unit power lost in the transmission line is $P_{line,pu} = I_{pu}{}^2 R_{line,pu} = 0.569^2 \cdot 0.0087$ =0.00282pu

The actual power lost in the transmission line $P_{line} = P_{line,pu} S_{base} = 0.00282 \cdot 10000$ =28.2W

Problem 2-23 in the book



Notes

- When only one device (transformer or motor) is analyzed, its own ratings are used as the basis for per unit system. When considering a transformer in a per unit system, transformer s characteristics will not vary much over a wide range of voltages and powers.
- For example, the series resistance is usually from 0.02 to 0.1 pu ; the magnetizing reactance is usually from 10 to 40 pu ; the core loss resistance is usually from 50 to 200 pu . Also, the per unit impedances of synchronous and induction machines fall within relatively narrow ranges over quite large size ranges.
- **If more than one transformer is present in a system, the system base voltage and power can be chosen arbitrary. However, the entire system must have the same base power, and the base voltages at various points in the system must be related by the voltage ratios of the transformers**.
- System base quantities are commonly chosen to the base of the largest component in the system.

Notes

Per-unit values given to another base can be converted to the new base either through an intermediate step (converting them to the actual values) or directly as follows:

$$(P, Q, S)_{pu,base2} = (P, Q, S)_{pu,base1} \frac{S_{base2}}{S_{base1}} \qquad V_{pu,base2} = V_{pu,base1} \frac{V_{base2}}{V_{base1}}$$

$$(R, X, Z)_{pu,base2} = (R, X, Z)_{pu,base1} \frac{Z_{base2}}{Z_{base1}} = (R, X, Z)_{pu,base1} \frac{V_{base2}^2 S_{base2}}{V_{base1}^2 S_{base1}}$$

Example 2

Sketch the appropriate per-unit equivalent circuit for the 8000/240 V, 60 Hz, 20 kVA transformer with R_c = 159 kΩ, X_M = 38.4 kΩ, R_{eq} = 38.3 Ω, X_{eq} = 192 Ω.

To convert the transformer to per-unit system, the primary circuit base impedance needs to be found.

$$S_{base1} = 20000\,VA$$

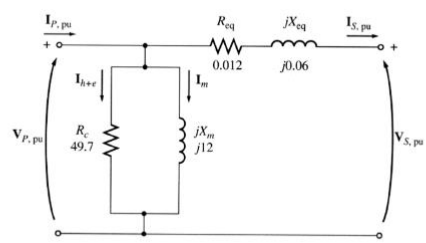
$$V_{base1} = 8000V$$

$$Z_{base1} = \frac{V_{base1}^2}{S_{base1}} = \frac{8000}{20000} = 3200\Omega$$

$$Z_{Rc,pu} = \frac{159000}{3200} = 49.7pu$$

$$Z_{XM,pu} = \frac{38400}{3200} = 12pu$$

$$Z_{Eq,pu} = \frac{38.3 + j192}{3200} = 0.012 + j0.06pu$$



Voltage regulation and efficiency

Since a real transformer contains series impedances, the transformer's output voltage varies with the load even if the input voltage is constant. To compare transformers in this respect, the quantity called a full-load voltage regulation (VR) is defined as follows:

$$VR = \frac{V_{s,nl} - V_{s,fl}}{V_{s.fl}} \cdot 100\% = \frac{V_p/a - V_{s,fl}}{V_{s.fl}} \cdot 100\%$$

In a per-unit system: $$VR = \frac{V_{p,pu} - V_{s,fl,pu}}{V_{s.fl,pu}} \cdot 100\%$$

Where $V_{s,nl}$ and $V_{s,fl}$ are the secondary no load and full load voltages.

Note, the VR of an ideal transformer is zero.

The transformer phasor diagram

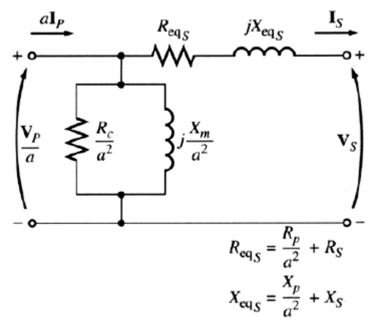
- To determine the VR of a transformer, it is necessary to understand t**<u>he voltage drops within it</u>**. Usually, the effects of the excitation branch on transformer VR can be ignored and, therefore, **<u>only the series impedances</u>** need to be considered. The VR depends **<u>on the magnitude of the impedances and on the current phase angle</u>**.

A phasor diagram is often used in the VR determinations. The phasor voltage V_s is assumed to be at 0^0 and all other voltages and currents are compared to it.

Considering the diagram and by applying the Kirchhoff's voltage law, the primary voltage is:

$$\frac{V_P}{a} = V_s + R_{eq}I_s + jX_{eq}I_s$$

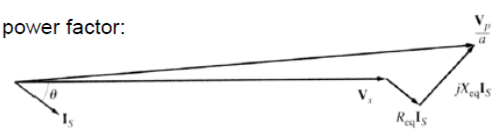
A transformer phasor diagram is a graphical representation of this equation.



$$\frac{V_P}{a} = V_s + R_{eq}I_s + jX_{eq}I_s$$

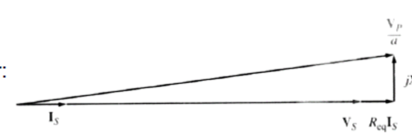
A transformer operating at a lagging power factor:

It is seen that $V_p/a > V_s$, VR > 0



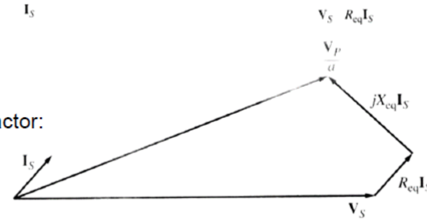
A transformer operating at a unity power factor:

It is seen that $VR > 0$



A transformer operating at a leading power factor:

If the secondary current is leading, the secondary voltage can be higher than the referred primary voltage; VR < 0.



The transformer efficiency

The efficiency of a transformer is defined as: $$\eta = \frac{P_{out}}{P_{in}} \cdot 100\% = \frac{P_{out}}{P_{out} + P_{loss}} \cdot 100\%$$

Note: the same equation describes the efficiency of motors and generators.

Considering the transformer equivalent circuit, we notice three types of losses:

1. Copper (I^2R) losses – are accounted for by the series resistance
2. Hysteresis losses – are accounted for by the resistor R_c.
3. Eddy current losses – are accounted for by the resistor R_c.

Since the output power is $$P_{out} = V_s I_s cos\theta$$

The transformer efficiency is $$\eta = \frac{V_s I_s cos\theta}{P_{cu} + P_{core} + V_s I_s cos\theta} \cdot 100\%$$

Transformers Ch. 2

Dr. Feras Alasali

The transformer efficiency

The efficiency of a transformer is defined as: $\eta = \frac{P_{out}}{P_{in}} \cdot 100\% = \frac{P_{out}}{P_{out} + P_{loss}} \cdot 100\%$

Note: the same equation describes the efficiency of motors and generators.

Considering the transformer equivalent circuit, we notice three types of losses:

1. Copper (I^2R) losses – are accounted for by the series resistance
2. Hysteresis losses – are accounted for by the resistor R_c.
3. Eddy current losses – are accounted for by the resistor R_c.

Since the output power is $P_{out} = V_s I_s cos\theta$

The transformer efficiency is $\eta = \frac{V_s I_s cos\theta}{P_{cu} + P_{core} + V_s I_s cos\theta} \cdot 100\%$

The copper losses are: $P_{cu} = I^2{}_s R_{eq}$

The core losses are: $P_{core} = \frac{(V_p/a)^2}{R_c}$;

Example

A 15 kVA, 2300/230 V transformer was tested to by open-circuit and closed-circuit tests. The following data was obtained:

V_{OC} = 230 V	V_{SC} = 47 V
I_{OC} = 2.1 A	I_{SC} = 6.0 A
P_{OC} = 50 W	P_{SC} = 160 W

a. Find the equivalent circuit of this transformer referred to the high-voltage side.

b. Find the equivalent circuit of this transformer referred to the low-voltage side.

c. Calculate the full-load voltage regulation at 0.8 lagging power factor, at 1.0 power factor, and at 0.8 leading power factor.

d. Plot the voltage regulation as load is increased from no load to full load at power factors of 0.8 lagging, 1.0, and 0.8 leading.

e. What is the efficiency of the transformer at full load with a power factor of 0.8 lagging?

a. The excitation branch values of the equivalent circuit can be determined as:

$$\theta = \cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}} = \cos^{-1}\frac{50}{(230)(2.1)} = 84°$$

Ic = $I_{OC}\cos 84°$ ⇨ Rc,s = Voc/Ic = 1050 Ω

Iu = $I_{OC}\sin 84°$ Xm = Voc/Iu = 110 Ω

From the test result , we know that the OC test is done on secondary side (so result referred to secondary)

From the short-circuit test data, the short-circuit impedance angle is

$$\theta_{SC} = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}} = \cos^{-1}\frac{160}{(47)(6)} = 55.4°$$

The equivalent series impedance is thus

$$Z_{SE} = \frac{V_{SC}}{I_{SC}}\angle\theta_{SC} = \frac{47}{6}\angle 55.4° = 4.45 + j6.45\ \Omega$$

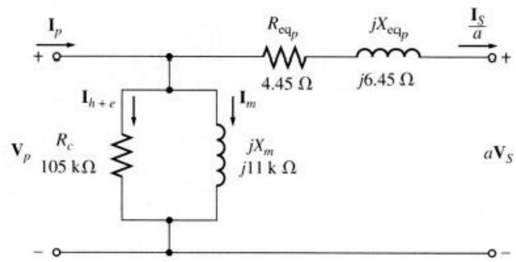
The series elements referred to the primary winding are:

$$R_{eq} = 4.45\Omega$$
$$X_M = 6.45\Omega$$

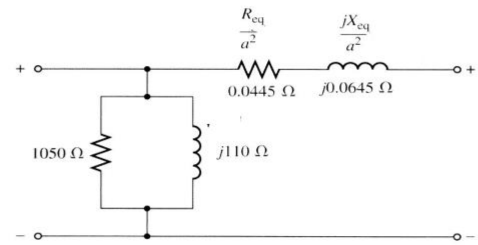
In (a) we need to find equivalent circuit referred to HV

$R_{c,p} = a^2 R_{c,s} = 105k\ ohm$

$X_{m,p} = a^2 X_{m,s} = 11k\ ohm$



b) Referred to LV



c. The full-load current on the secondary side of the transformer is

$$\frac{V_P}{a} = V_s + R_{eq}I_s + jX_{eq}I_s$$

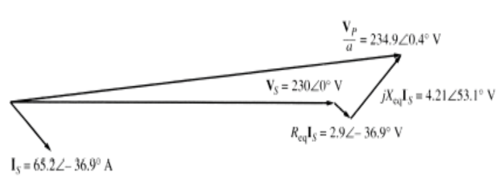
$$I_{S,rated} = \frac{S_{rated}}{V_{S,rated}} = \frac{15000}{230} = 65.2A$$

At PF = 0.8 lagging, current $I_{S,rated} = 65.2\angle - cos^{-1}(0.8) = 65.2\angle - 36.9°$

$$\frac{V_P}{a} = 230\angle 0° + 0.0445(65.2\angle - 36.9°) + j0.0645(65.2\angle - 36.9°)$$
$$= 234.85\angle 0.40°\ V$$

The voltage regulation is $VR = \frac{|V_p/a| - V_{s,fl}}{V_{s,fl}} \cdot 100\%$

$$= \frac{234.85 - 230}{230} \cdot 100\%$$
$$= 2.1\%$$



At PF = 1, current $I_{S,rated} = 65.2\angle - cos^{-1}(1) = 65.2\angle - 0°$

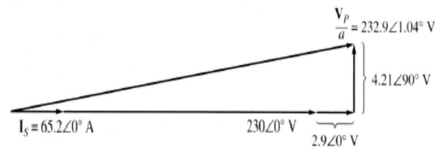
$$\frac{V_P}{a} = 230\angle 0° + 0.0445(65.2\angle 0°) + j0.0645(65.2\angle 0°)$$

$$= 232.94\angle 1.04°\, V$$

The voltage regulation is $VR = \frac{|V_p/a| - V_{s,fl}}{V_{s,fl}} \cdot 100\%$

$$= \frac{232.94 - 230}{230} \cdot 100\%$$

$$= 1.28\%$$



$$\frac{V_P}{a} = V_s + R_{eq}I_s + jX_{eq}I_s$$

At PF = 0.8 leading, current $I_{S,rated} = 65.2\angle cos^{-1}(0.8) = 65.2\angle 36.9°$

$$\frac{V_P}{a} = 230\angle 0° + 0.0445(65.2\angle 36.9°) + j0.0645(65.2\angle 36.9°)$$

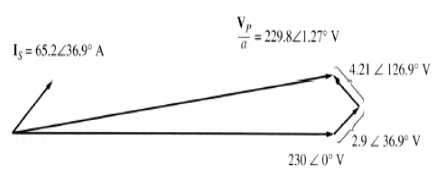
$$= 229.85\angle 1.27°\, V$$

The voltage regulation is

$$VR = \frac{|V_p/a| - V_{s,fl}}{V_{s,fl}} \cdot 100\%$$

$$= \frac{229.85 - 230}{230} \cdot 100\%$$

$$= -0.062\%$$



$$\frac{V_P}{a} = V_s + R_{eq}I_s + jX_{eq}I_s$$

e. To find the efficiency of the transformer, first calculate its losses.

The efficiency of the transformer is

$$\eta = \frac{V_s I_s cos\theta}{P_{cu} + P_{core} + V_s I_s cos\theta} \cdot 100\%$$

The output power of the transformer at the given Power Factor is:

$$P_{out} = V_s I_s cos\theta = 230 \cdot 65.2 \cdot \cos(36.9°) = 12000W$$

The copper losses are: $P_{cu} = I^2{}_s R_{eq} = 65.2^2 - 0.0445 = 189W$

The core losses are: $P_{core} = \frac{(V_p/a)^2}{R_c} = \frac{234.85^2}{1050} = 52.5W$

The efficiency of the transformer is

$$\eta = \frac{V_s I_s cos\theta}{P_{cu} + P_{core} + V_s I_s cos\theta} \cdot 100\% = 98.03\%$$

Transformer taps and voltage regulation

- The transformer **turns ratio is a fixed (constant)** for the given transformer. However, distribution transformers have a **series of taps in the windings** to permit **small changes in their turns ratio**. Typically, transformers may have **4 taps** in addition to the nominal setting with spacing of **2.5 % of full load** voltage. Therefore, adjustments **up to 5 % above** or below the nominal voltage rating of the transformer are possible

Example: **A 500 kVA**, **13 200/480 V** transformer has **four 2.5 %** taps on its primary winding. What are the transformer's voltage ratios at each tap setting?

+ 5.0% tap	13 860/480 V
+ 2.5% tap	13 530/480 V
Nominal rating	13 200/480 V
- 2.5% tap	12 870/480 V
- 5.0% tap	12 540/480 V

Transformer taps and voltage regulation

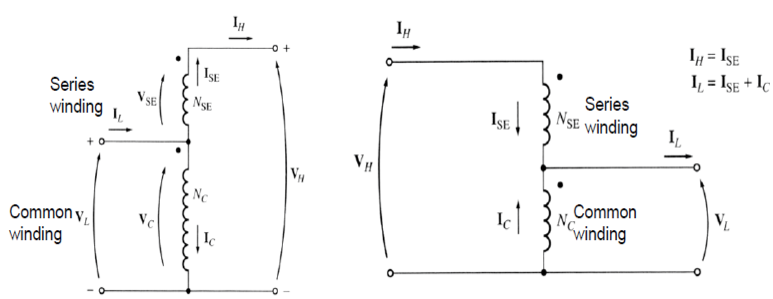
- Taps allow adjustment of the transformer in the field to **accommodate for local voltage variations.**
- Sometimes, transformers are used on a power line, whose voltage varies widely with the load (due to high line impedance, for instance). Normal loads need fairly constant input voltage though.
- One possible solution to this problem is to use a special transformer called a **tap changing under load (TCUL)** transformer or voltage regulator . TCUL is a transformer with the ability to change taps while power is connected to it. A voltage regulator is a TCUL with build in voltage sensing circuitry that automatically changes taps to keep the system voltage constant.
- These **self adjusting** transformers *are very common in modern power systems*.

The autotransformer

It is desirable to change the voltage by a small amount (for instance, when the consumer is far away from the generator and it is needed to raise the voltage to compensate for voltage drops).

In such situations, it would be expensive to wind a transformer with two windings of approximately equal number of turns. An autotransformer (a transformer with only one winding) is used instead.

Diagrams of step-up and step-down autotransformers:

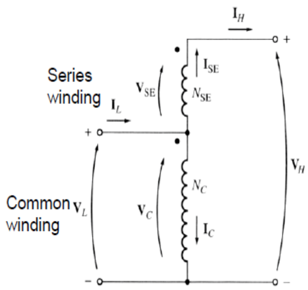


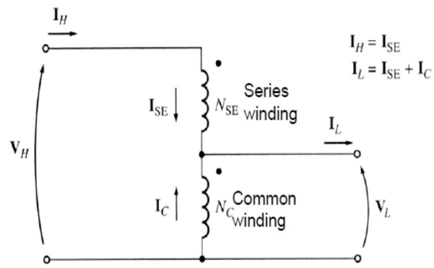
Since the autotransformer's coils are physically connected, a different terminology is used for autotransformers:

The voltage across the common winding is called a common voltage V_C, and the current through this coil is called a common current I_C. The voltage across the series winding is called a series voltage V_{SE}, and the current through that coil is called a series current I_{SE}.

The voltage and current on the low-voltage side are called V_L and I_L; the voltage and current on the high-voltage side are called V_H and I_H.

For the autotransformers:





$$\frac{V_C}{V_{SE}} = \frac{N_C}{N_{SE}}$$

$$N_C I_C = N_{SE} I_{SE}$$

$$V_L = V_C \qquad I_L = I_C + I_{SE}$$

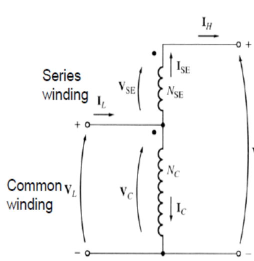
$$V_H = V_C + V_{SE} \qquad I_H = I_{SE}$$

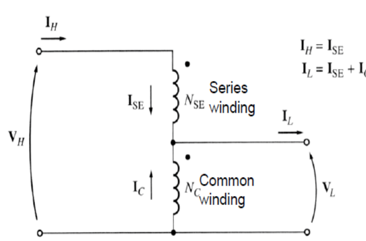
$$V_H = V_C + \frac{N_{SE}}{N_C} V_C = V_L + \frac{N_{SE}}{N_C} V_L$$

$$\boxed{\frac{V_L}{V_H} = \frac{N_C}{N_C + N_{SE}}}$$

$$I_L = I_{SE} + \frac{N_{SE}}{N_C} I_{SE} = I_H + \frac{N_{SE}}{N_C} I_H$$

$$\boxed{\frac{I_L}{I_H} = \frac{N_C + N_{SE}}{N_C}}$$





The apparent power advantage

Not all the power traveling from the primary to the secondary winding of the autotransformer goes through the windings. As a result, an autotransformer can handle much power than the conventional transformer (with the same windings).

Considering a step-up autotransformer, the apparent input and output powers are:

$$S_{in} = V_L I_L$$

$$S_{out} = V_H I_H$$

It is easy to show that

$$S_{in} = S_{out} = S_{IO}$$

where S_{IO} is the input and output apparent powers of the autotransformer.

However, the apparent power in the autotransformer's winding is

$$S_W = V_C I_C = V_{SE} I_{SE}$$

$$S_W = V_L (I_L - I_H) = V_L I_L - V_L I_H$$

$$V_H = V_C + \frac{N_{SE}}{N_C} V_C = V_L + \frac{N_{SE}}{N_C} V_L$$

$$\boxed{\frac{V_L}{V_H} = \frac{N_C}{N_C + N_{SE}}}$$

$$I_L = I_{SE} + \frac{N_{SE}}{N_C} I_{SE} = I_H + \frac{N_{SE}}{N_C} I_H$$

$$\boxed{\frac{I_L}{I_H} = \frac{N_C + N_{SE}}{N_C}}$$

$$S_W = V_L I_L - V_L I_L \frac{N_C}{N_C + N_{SE}} = S_{IO} \frac{N_{SE}}{N_C + N_{SE}}$$

Therefore, the ratio of the apparent power in the primary and secondary of the autotransformer to the apparent power **actually** traveling through its windings is

$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}}$$

The last equation described the apparent power rating advantage of an autotransformer over a conventional transformer.

S_W is the apparent power actually passing through the windings. The rest passes from primary to secondary parts without being coupled through the windings.

Note that the smaller the series winding, the greater the advantage!

example

a 5 MVA autotransformer that connects a 110 kV system to a 138 kV system would have a turns ratio (common to series) 110:28. Such an autotransformer would actually have windings rated at:

$$S_W = S_{IO}\frac{N_{SE}}{N_{SE}+N_C} \qquad = 5\cdot\frac{28}{28+110} = 1.015MVA$$

Therefore, the autotransformer would have windings rated at slightly over 1 MVA instead of 5 MVA, which makes is 5 times smaller and, therefore, considerably less expensive.

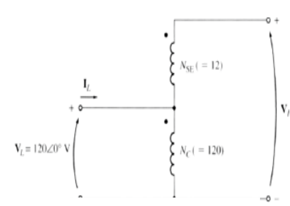
However, the construction of autotransformers is usually slightly different. In particular, the insulation on the smaller coil (the series winding) of the autotransformer is made as strong as the insulation on the larger coil to withstand the full output voltage.

The primary disadvantage of an autotransformer is that there is a direct physical connection between its primary and secondary circuits. Therefore, the electrical isolation of two sides is lost.

Example

A 100 VA, 120/12 V transformer will be connected to form a step-up autotransformer with the primary voltage of 120 V.

a. What will be the secondary voltage?
b. What will be the maximum power rating?
c. What will be the power rating advantage?



a. The secondary voltage: $V_H = \frac{N_{SE}+N_C}{N_c}V_L = 120\cdot\frac{120+12}{120} = 132V$

b. The max series winding current: $I_{SE,max} = \frac{S_{max}}{V_{SE}} = \frac{100}{12} = 8.33A$

c. The power rating advantage:

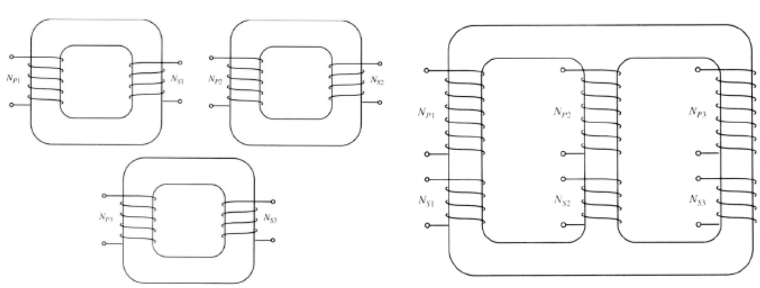
The secondary apparent power: $S_{out} = V_S I_S = V_H I_H = 132 \cdot 8.33 = 1100VA$

$$\frac{S_{IO}}{S_W} = \frac{1100}{100} = 11$$

$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}} = \frac{120 + 12}{12} = 11$$

3 phase transformers

- The majority of the power generation/distribution systems in the world are 3 phase systems. The transformers for such circuits can be constructed either as a systems.
- The transformers for such circuits can be constructed either as a 3 phase bank of independent identical transformers (can be replaced independently) or as a **single transformer** wound on a single 3 legged core (lighter, cheaper, more efficient).
- We assume that any single transformer in a 3 phase transformer (bank) behaves exactly as a single phase transformer. The impedance, voltage regulation, efficiency, and other calculations for 3 phase transformers are done on a per phase basis, using the techniques studied previously for single phase transformers.



Three-phase transformer connections

- Y-Y (Wye-Wye)
- Y-Δ (Wye-Delta)
- Δ-Y (Delta-Wye)
- Δ-Δ (Delta-Delta)

1. Y-Y connection:

The primary voltage on each phase of the transformer is

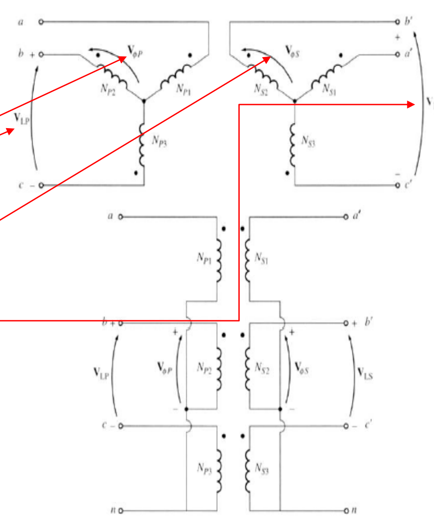
$$V_{\phi P} = \frac{V_{LP}}{\sqrt{3}}$$

The secondary phase voltage is

$$V_{LS} = \sqrt{3} V_{\phi S}$$

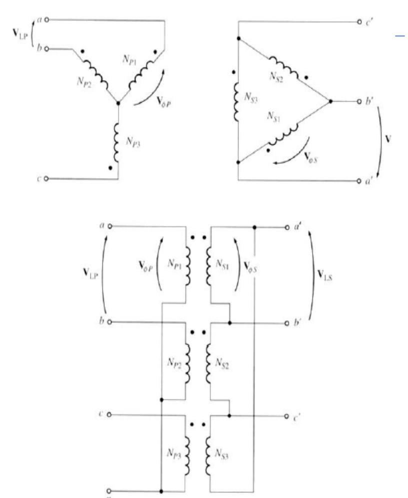
The overall voltage ratio is

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3} V_{\phi P}}{\sqrt{3} V_{\phi S}} = a$$



Advantages	Disadvantages
Requires less turns per winding ie cheaper Phase voltage is 1/√3 times of line voltage	If the load on the secondary side unbalanced then the shifting of neutral point is possible
Cross section of winding is large *i.e.* stronger to bear stress during short circuit Line current is equal to phase current	The third harmonic present in the alternator voltage may appear on the secondary side. This causes distortion in the secondary phase voltages
Less dielectric strength in insulating materials phase voltage is less	Magnetizing current of transformer has 3rd harmonic component

2. Y-Δ connection:



The primary voltage on each phase of the transformer is

$$V_{\emptyset P} = \frac{V_{LP}}{\sqrt{3}}$$

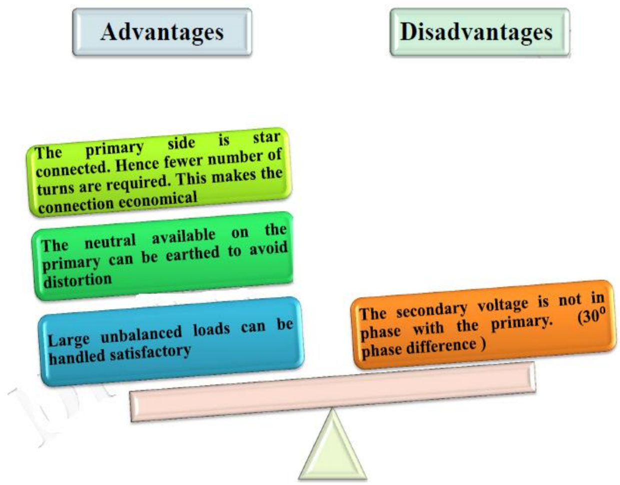
The secondary phase voltage is

$$V_{LS} = V_{\emptyset S}$$

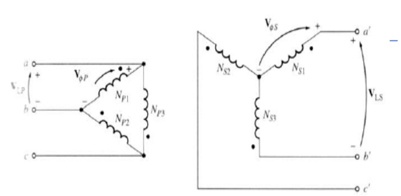
The overall voltage ratio is

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\emptyset P}}{V_{\emptyset S}} = \sqrt{3}a$$

- The Y-Delta connection has no problem with third harmonic components due to circulating currents in Delta . It is also more stable to unbalanced loads since the Delta partially redistributes any imbalance that occurs.
- One problem associated with this connection is that the secondary voltage is shifted by 30 with respect to the primary voltage. This can cause problems when paralleling 3 phase transformers since transformers secondary voltages must be in phase to be paralleled. Therefore, we must pay attention to these shifts.



3. Δ-Y connection:

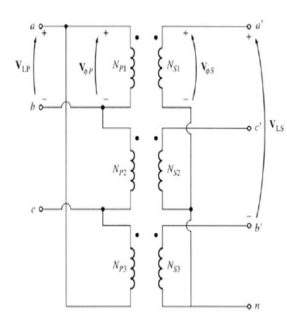


The primary voltage on each phase of the transformer is

$$V_{\emptyset P} = V_{LP}$$

The secondary phase voltage is

$$V_{LS} = \sqrt{3} V_{\emptyset S}$$

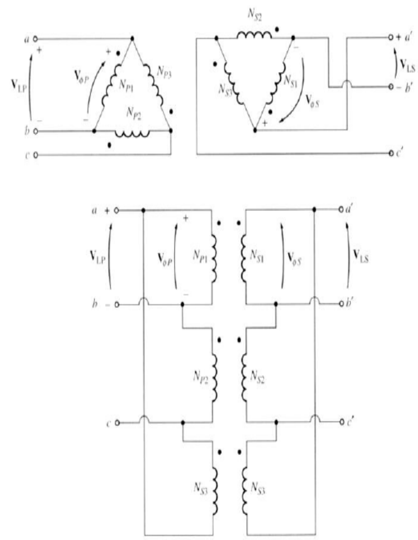


The overall voltage ratio is

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\emptyset P}}{\sqrt{3} V_{\emptyset S}} = \frac{a}{\sqrt{3}}$$

The same advantages and the same phase shift as the Y-Δ connection.

4. Δ-Δ connection:



The primary voltage on each phase of the transformer is

$$V_{\emptyset P} = V_{LP}$$

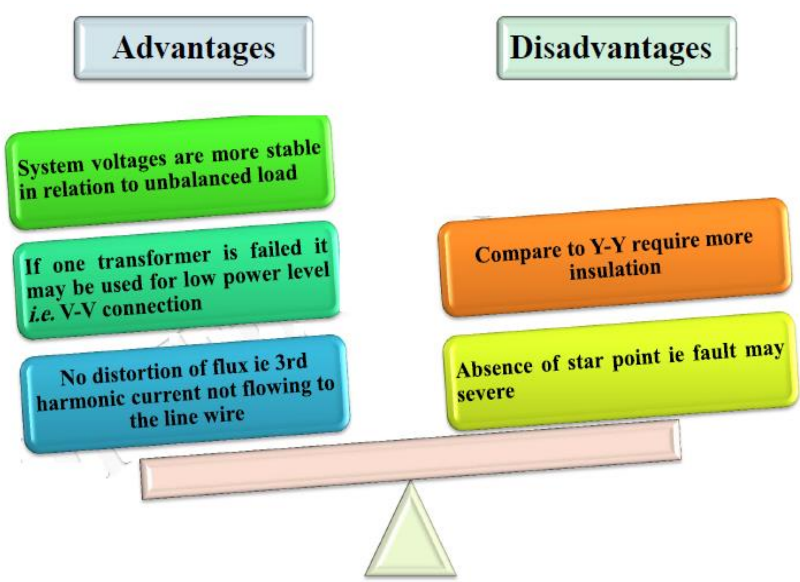
The secondary phase voltage is

$$V_{LS} = V_{\emptyset S}$$

The overall voltage ratio is

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\emptyset P}}{V_{\emptyset S}} = a$$

No phase shift, no problems with unbalanced loads or harmonics.



3 phase transformer: per unit system

The per-unit system applies to the 3-phase transformers as well as to single-phase transformers. If the total base VA value of the transformer bank is S_{base}, the base VA value of one of the transformers will be

$$S_{1\emptyset,base} = \frac{S_{base}}{3}$$

Therefore, the base phase current and impedance of the transformer are

$$I_{\emptyset,base} = \frac{S_{1\emptyset,base}}{V_{\emptyset,base}} = \frac{S_{base}}{3V_{\emptyset,base}}$$

$$Z_{base} = \frac{(V_{\emptyset,base})^2}{S_{1\emptyset,base}} = \frac{3(V_{\emptyset,base})^2}{S_{base}}$$

The line quantities on 3-phase transformer banks can also be represented in per-unit system. If the windings are in Δ:

$$V_{L,base} = V_{\emptyset,base}$$

If the windings are in Y:

$$V_{L,base} = \sqrt{3}V_{\emptyset,base}$$

And the base line current in a 3-phase transformer bank is

$$I_{\emptyset,base} = \frac{S_{1\emptyset,base}}{V_{\emptyset,base}} = \frac{S_{base}}{3V_{\emptyset,base}} \qquad I_{L,base} = \frac{S_{base}}{\sqrt{3}V_{L,base}}$$

The application of the per-unit system to 3-phase transformer problems is similar to its application in single-phase situations. The voltage regulation of the transformer bank is the same.

example

A 50 kVA, 13 800/208 V Δ-Y transformer has a resistance of 1% and a reactance of 7% per unit.

a. What is the transformer's phase impedance referred to the high voltage side?
b. What is the transformer's voltage regulation at full load and 0.8 PF lagging, using the calculated high-side impedance?
c. What is the transformer's voltage regulation under the same conditions, using the per-unit system?

a. The high-voltage side of the transformer has the base voltage 13 800 V and a base apparent power of 50 kVA. Since the primary side is Δ-connected, its phase voltage and the line voltage are the same. The base impedance is:

$$Z_{base} = \frac{3\left(V_{\emptyset,base}\right)^2}{S_{base}} = \frac{3(13800)^2}{50000} = 114226\ \Omega$$

$$Z_{eq} = 0.01 + j\ 0.07\ pu$$

$$Z_{eq} = Z_{base} \text{ x Zpu} = 114.2 + \text{j}800 \text{ ohm}$$

b)

The rated phase voltage on the secondary of the transformer is

$$V_{\phi S} = \frac{208}{\sqrt{3}} = 120V$$

When referred to the primary (high-voltage) side, this voltage becomes

$$V_{\phi S}{}' = aV_{\phi S} = 13800V$$

Assuming that the transformer secondary winding is working at the rated voltage and current, the resulting primary phase voltage is

$$V_{\phi P} = aV_{\phi S} + R_{eq}I_\phi + jX_{eq}I_\phi$$

$$V_{\phi P} = 13800\angle 0° + 114.2 \cdot 1.208\angle cos^{-1}(-0.8) + 800 \cdot 1.208\angle cos^{-1}(-0.8) = 14506\angle 2.73°$$

The voltage regulation, therefore, is

$$VR = \frac{|V_{\phi P}| - aV_{\phi S}}{aV_{\phi S}} \cdot 100\% = \frac{14506 - 13800}{13800} \cdot 100\% = 5.1\%$$

c. In the per-unit system, the output voltage is $1\angle 0^0$, and the current is $1\angle\cos^{-1}(-0.8)$. Therefore, the input voltage is

$$V_{\phi P} = 1\angle 0° + 0.01 \cdot 1\angle cos^{-1}(0.8) + j0.07 \cdot 1\angle cos^{-1}(0.8) = 1.051\angle 2.73°$$

Thus, the voltage regulation in per-unit system will be

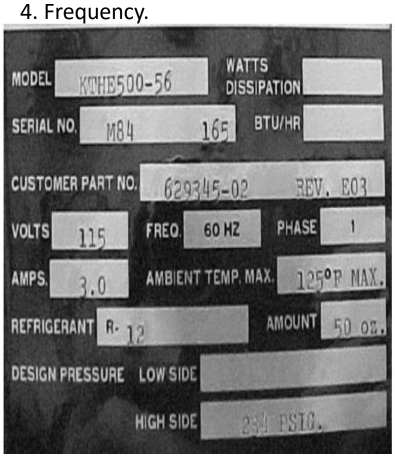
$$VR = \frac{1.051 - 1}{1} \cdot 100\% = 5.1\%$$

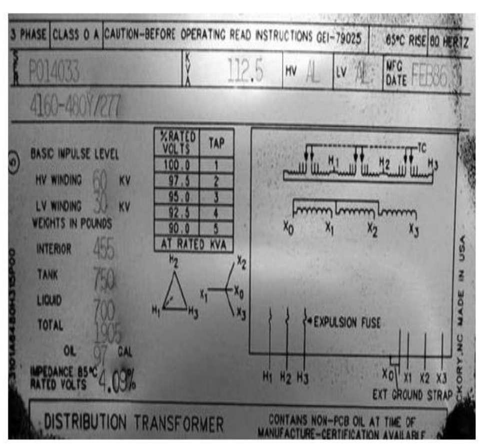
The voltage regulation in per-unit system is the same as computed in volts…

Transformer ratings

Transformers have the following major ratings:

1. Apparent power;
2. Voltage;
3. Current;
4. Frequency.





The **voltage** rating is a) used to protect the winding insulation from breakdown; b) related to the magnetization current of the transformer (more important)

An increase in voltage will lead to a proportional increase in flux. However, after some point (in a saturation region), such increase in flux would require an unacceptable increase in magnetization current!

Therefore, the maximum applied voltage (and thus the rated voltage) is set by the maximum acceptable magnetization current in the core.

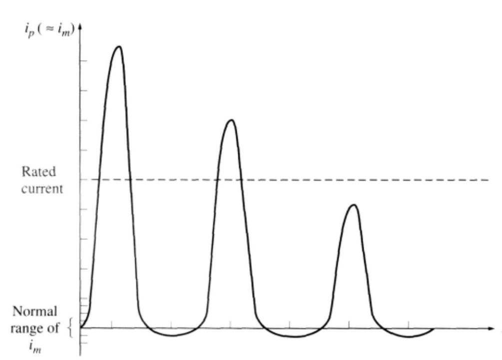
The apparent power rating sets (together with the voltage rating) the current through the windings. The current determines the I^2R losses and, therefore, the heating of the coils. Remember, overheating shortens the life of transformer's insulation!

In addition to apparent power rating for the transformer itself, additional (higher) rating(s) may be specified if a forced cooling is used. Under any circumstances, the temperature of the windings must be limited.

Note, that if the transformer's voltage is reduced (for instance, the transformer is working at a lower frequency), the apparent power rating must be reduced by an equal amount to maintain the constant current.

Transformer ratings: Current inrush

- **Inrush current** is the instantaneous high input **current** drawn by a power supply or electrical equipment at turn-on. This **arises** due to the high initial **currents** required to charge the capacitors and inductors or transformers.
- The maximum flux reached on the first half cycle depends on the phase of the voltage at the instant the voltage is applied.



$v(t) = V_m \sin \omega t$

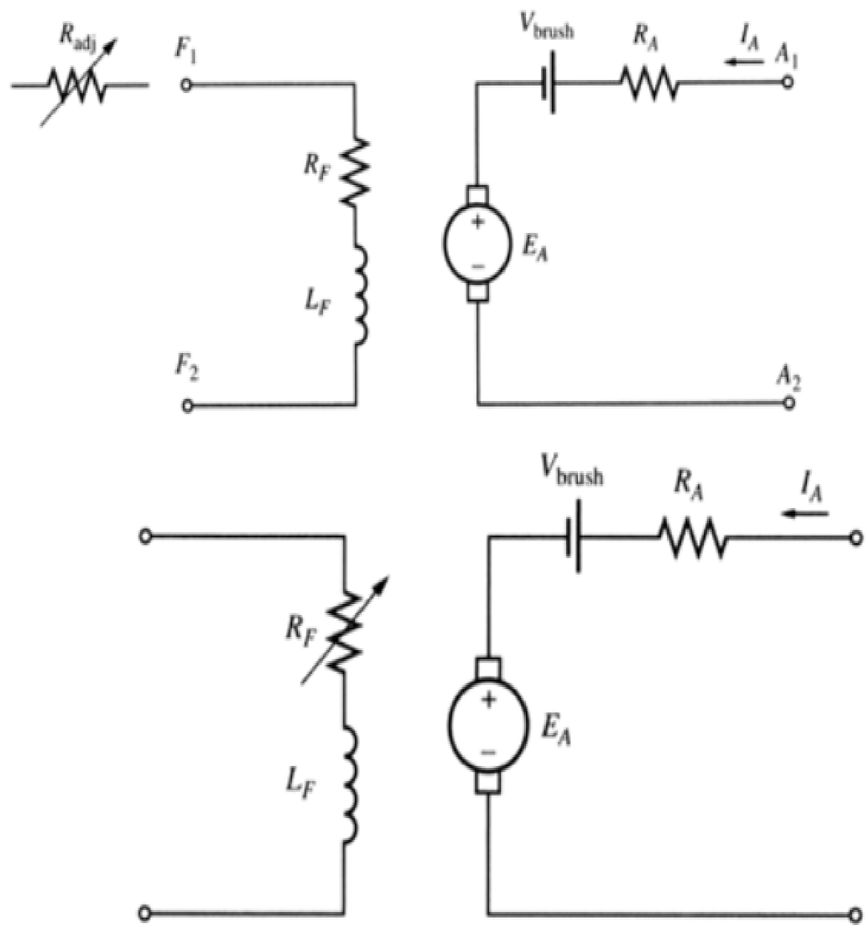
DC Motors

Dr. Feras Alasali

The Equivalent Circuit of a DC Motor

The armature circuit (the entire rotor structure) is represented by an ideal voltage source $\boldsymbol{E_A}$ and a resistor $\boldsymbol{R_A}$. A battery $\boldsymbol{V_{brush}}$ in the opposite to a current flow in the machine direction indicates brush voltage drop.

The field coils producing the magnetic flux are represented by inductor $\boldsymbol{L_F}$ and resistor $\boldsymbol{R_F}$. The resistor $\boldsymbol{R_{adj}}$ represents an external variable resistor (sometimes lumped together with the field coil resistance) **used to control the amount of current in the field circuit**.



The internal generated voltage in the machine is

$$E_A = K\emptyset\omega$$

The induced torque developed by the machine is

$$\tau_{ind} = K\emptyset I_A$$

Here ***K*** is the constant **depending on the design of a particular DC machine** (*number and commutation of rotor coils*, etc.) and $\boldsymbol{\phi}$ is the total flux inside the machine.

Introduction to DC motors

- **DC motors are driven from DC power supply, there are five major types of DC motors in general use:**

1- The separately excited dc motor.

2- The shunt dc motor.

3- The permanent –magnet DC motor

4- The series DC motor.

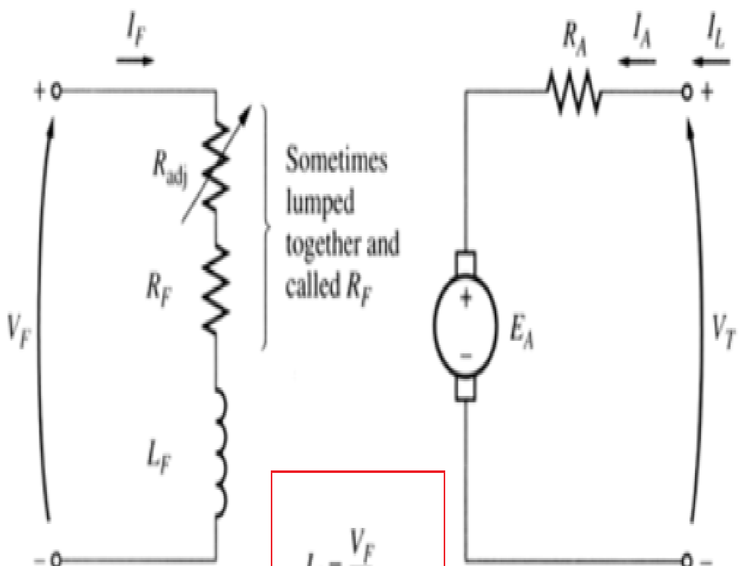
5- The compounded dc motor

Separately excited and Shunt DC motors

- **Separately DC motor**: is a motor whose filed circuit is supplied from a separate constant voltage power supply.

- **Shunt DC motor**: is a motor whose filed circuit gets its power directly across the armature terminals of motor.

Separately excited DC motor:
a field circuit is supplied from a separate constant voltage power source.

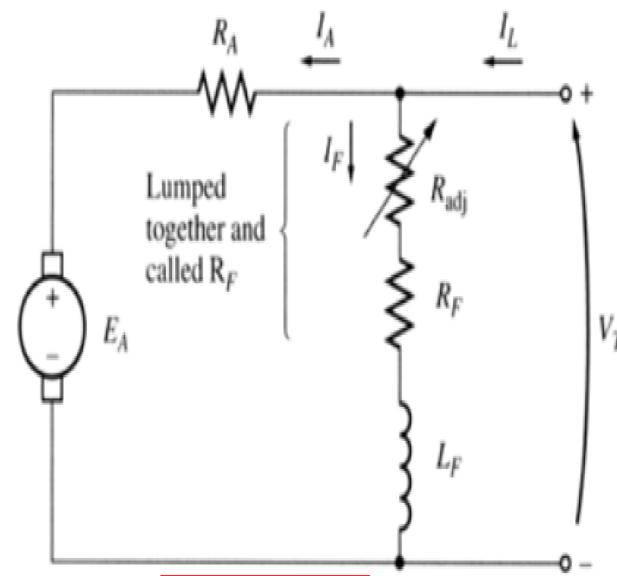


$$I_F = \frac{V_F}{R_F}$$

$$V_T = E_A + I_A R_A$$

$$I_L = I_A$$

Shunt DC motor:
a field circuit gets its power from the armature terminals of the motor.



$$I_F = \frac{V_T}{R_F}$$

$$V_T = E_A + I_A R_A$$

$$I_L = I_A + I_F$$

For the armature circuit of these motors:

$$V_T = E_A + I_A R_A$$

The terminal characteristic of a Shunt motor:

A **terminal characteristic** of a machine is a plot of the machine's output quantities vs. each other.

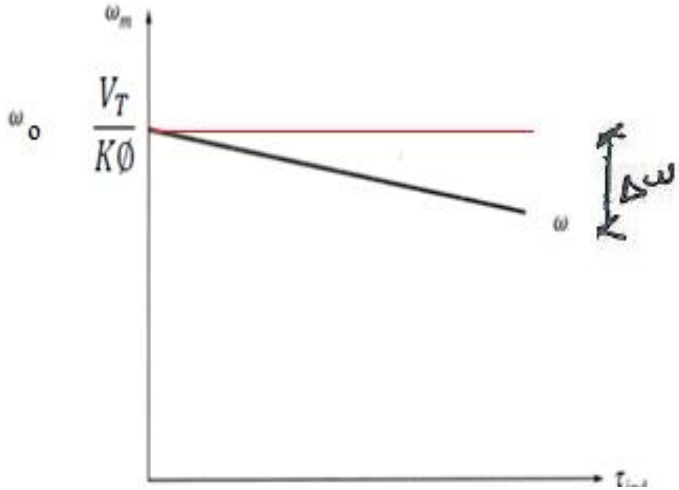
For a motor, the output quantities are shaft torque and speed. Therefore, the **terminal characteristic of a motor** is its **output torque vs. speed**.

If the load on the shaft increases, the load torque τ_{load} will exceed the induced torque τ_{ind}, and the motor will **slow down**. Slowing down the motor will **decrease its internal generated voltage** (since $E_A = K\phi\omega$), so the **armature current increases** ($I_A = (V_T - E_A)/R_A$).
As the **armature current increases**, the **induced torque in the motor increases** (since $\tau_{ind} = K\phi I_A$), and the induced torque will equal the load torque at a lower speed ω.

This is linked to Ch. simple DC machine and also to Ch 8 (Section 1).

$$\omega = \frac{V_T}{K\emptyset} - \frac{R_A}{(K\emptyset)^2}\tau_{ind}$$

$$\omega = \frac{V_T}{K\emptyset} - \frac{R_A}{(K\emptyset)^2}\tau_{ind}$$



$$\omega = \omega_o - \Delta\omega$$

- ω → speed at Load
- ω_o → no Load speed
- $\Delta\omega$ → drop speed

$$\omega = \frac{V_T}{K\phi} - \frac{T_{ind}\,R_A}{(K\phi)^2}$$

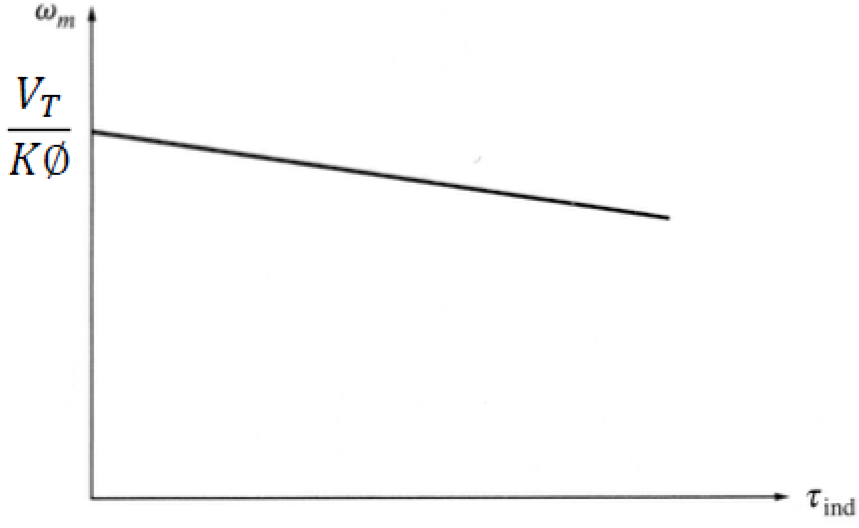
$$T_{ind} = K\phi I_A \Rightarrow I_A = \frac{T_{ind}}{K\phi}$$

$$\omega = \frac{V_T}{K\phi} - \frac{I_A R_A}{K\phi}$$

- The terminal characteristic of a Shunt motor

This equation is just a straight line with a negative slope.

$$\omega = \frac{V_T}{K\emptyset} - \frac{R_A}{(K\emptyset)^2}\tau_{ind}$$



The resulting torque –speed characteristic of a shunt DC motor.

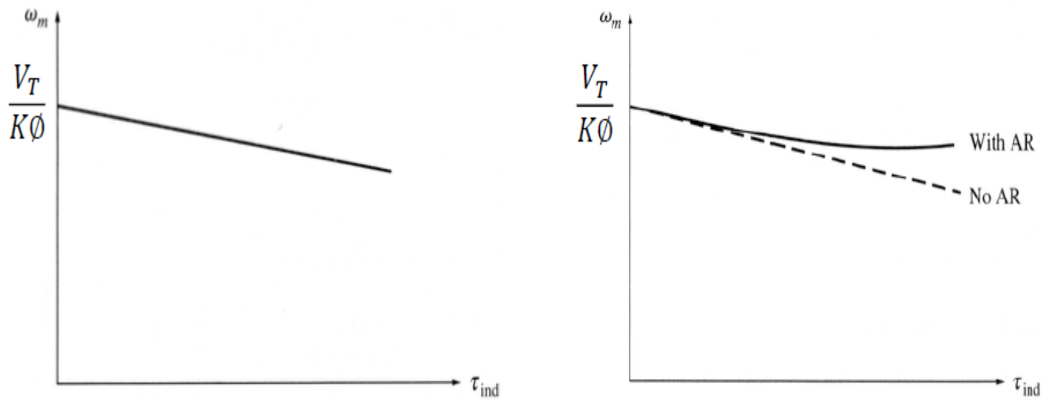
In order for motor speed to vary **linearly** with the torque, the other terms in the equation must be constant , otherwise the shape of the curve will be effected.

So, the curve can be effected by

1- terminal voltage 2- Armature reaction

• The terminal characteristic of a Shunt motor

Assuming that the terminal voltage and other terms are constant, the motor's speed vary linearly with torque.



armature reaction

However, if a motor has an **armature reaction**, flux-weakening reduces the flux when torque increases. Therefore, the **motor's speed will increase**.

If a shunt (or separately excited) motor has **compensating windings**, and the motor's speed and armature current are known for any value of load, it's possible to calculate the speed for any other value of load.

If a shunt DC motor has compensating winding so that its flux is constant regardless of load, so there will be no flux-weakening problem in the machine .

A compensation **winding** in a **DC** shunt **motor** is a **winding** in the field pole face plate that carries armature current to reduce stator field distortion. Its purpose is to reduce brush arcing and erosion in **DC motors** that are operated with weak fields, variable heavy loads or reversing operation such as steel-mill **motors**.

Speed Control of Shunt DC+ separately excited motor:

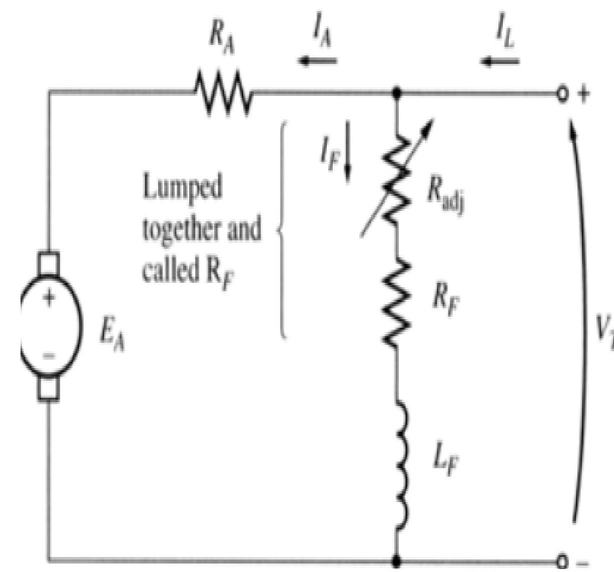
Shunt DC motor:
a field circuit gets its power from the armature terminals of the motor.

- ➢How can the speed of a shunt DC motor be controlled?

There are two common methods to control the speed of a shunt DC motor:

1. Adjusting the field resistance *RF* (and thus the field flux)

2. Adjusting the terminal voltage applied to the armature



$$\omega = \frac{V_T}{K\phi} - \frac{T_{ind}\, R_A}{(K\phi)^2}$$

$$T_{ind} = K\phi I_A \Rightarrow I_A = \frac{T_{ind}}{K\phi}$$

$$\omega = \frac{V_T}{K\phi} - \frac{I_A R_A}{K\phi}$$

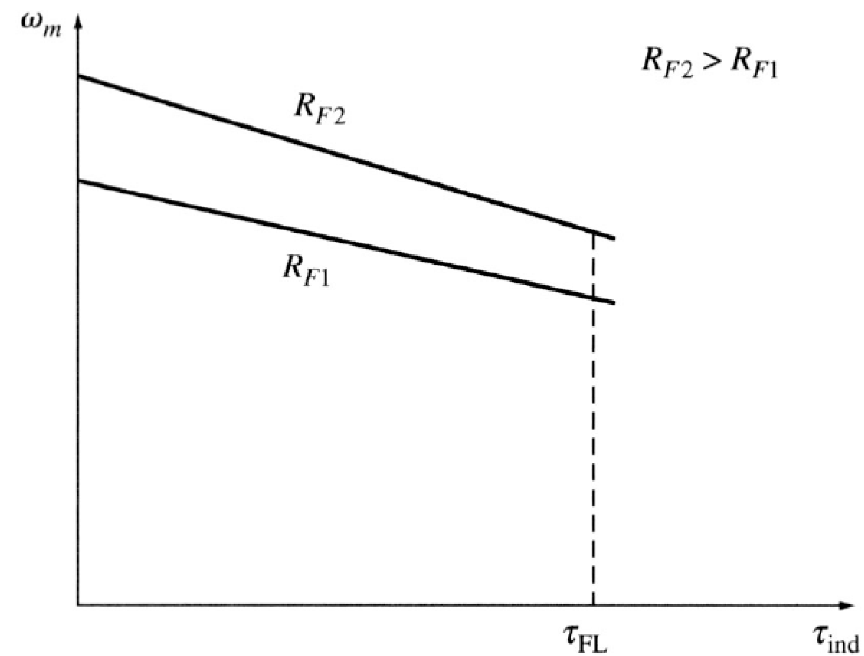
1.Adjusting the field resistance

- Firstly, to understand what happens when the filed resistance of a DC motor is changed, assume that filed resistor increases and observe the response.

1) Increasing field resistance R_F decreases the field current ($I_F = V_T/R_F$);
2) Decreasing field current I_F decreases the flux ϕ;
3) Decreasing flux decreases the internal generated voltage ($E_A = K\phi\omega$);
4) Decreasing E_A increases the armature current ($I_A = (V_T - E_A)/R_A$);
5) Changes in armature current dominate over changes in flux; therefore, increasing I_A increases the induced torque ($\tau_{ind} = K\phi I_A$);
6) Increased induced torque is now larger than the load torque τ_{load} and, therefore, the speed ω increases;
7) Increasing speed increases the internal generated voltage E_A;
8) Increasing E_A decreases the armature current I_A…
9) Decreasing I_A decreases the induced torque until $\tau_{ind} = \tau_{load}$ at a higher speed ω.

The effect of increasing the field resistance within a normal load range: from no load to full load.

Increase in the field resistance increases the motor speed. Observe also that the slope of the speed-torque curve becomes steeper when field resistance increases.

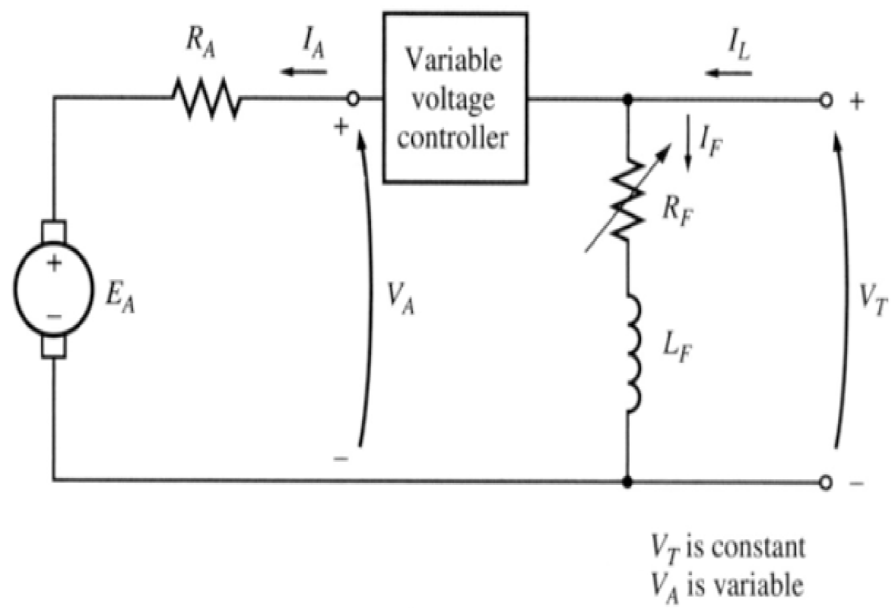


2. Changing the armature voltage

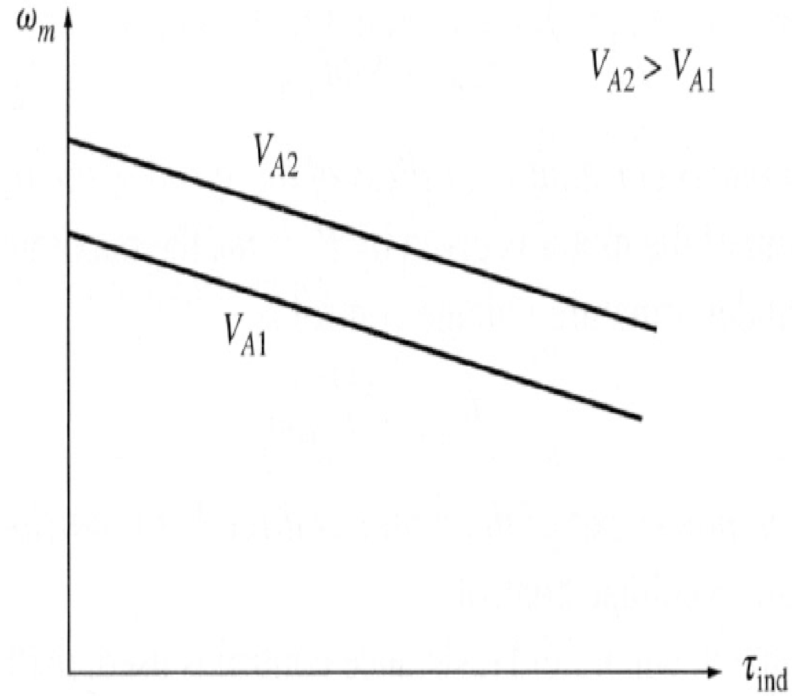
2. Changing the armature voltage

This method implies changing the voltage applied to the armature of the motor without changing the voltage applied to its field. Therefore, the motor must be separately excited to use armature voltage control.

Armature voltage speed control



1) Increasing the armature voltage V_A increases the armature current ($I_A = (V_A - E_A)/R_A$);
2) Increasing armature current I_A increases the induced torque τ_{ind} ($\tau_{ind} = K\phi I_A$);
3) Increased induced torque τ_{ind} is now larger than the load torque τ_{load} and, therefore, the speed ω;
4) Increasing speed increases the internal generated voltage ($E_A = K\phi\omega$);
5) Increasing E_A decreases the armature current I_A…
6) Decreasing I_A decreases the induced torque until $\tau_{ind} = \tau_{load}$ at a higher speed ω.



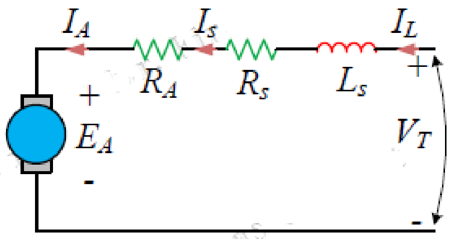
Increasing the armature voltage of a separately excited DC motor does not change the slope of its torque-speed characteristic.

Note: the no load speed of the motor is shifted by method of speed control but the slope of the curve remains constant.

The series DC motors

- A series Dc motor is a dc motor whose field windings consists of a relatively few turns connected in series with armature circuit.

this motor has field coil connected in series to the armature winding. For this reason relatively higher current flows through the field coils, and its designed accordingly as mentioned below.



1. The field coils of **DC series motor** are wound with relatively fewer turns as the current through the field is its armature current and hence for required mmf less numbers of turns are required.

2. The wire is heavier, as the diameter is considerable increased to provide minimum electrical resistance to the flow of full armature current.

3. In spite of the above mentioned differences, about having fewer coil turns the running of this DC motor remains unaffected, as the current through the field is reasonably high to produce a field strong enough for generating the required amount of torque.

Since the entire supply current flows through both the armature and field conductor.

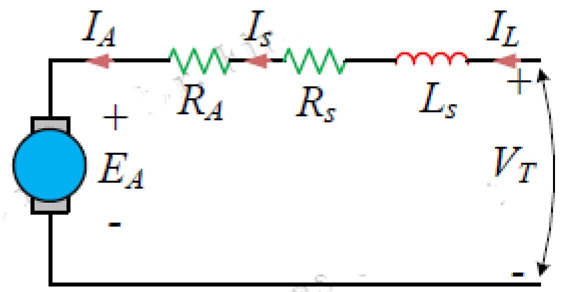
$$Therefore,\ I_{total} = I_{s} = I_a$$

Where, I_s is the series current in the field coil and I_a is the armature current.

$$I_A = I_S = I_L$$

$$V_T = E_A + I_A(R_A + R_S)$$

$$E_A = k\phi\omega_m$$



$$\tau_{ind} = K\emptyset I_A$$

The terminal characteristic if a Series DC motor Speed and Torque of Series DC Motor

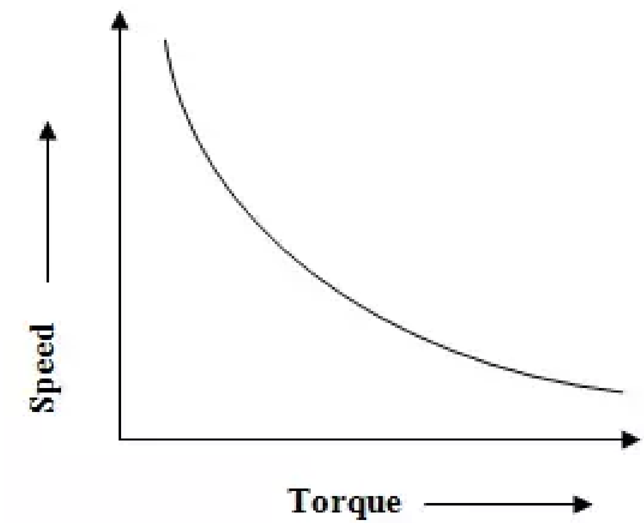
1- A series wound motors has relationship existing between the field current and the amount of torque produced. As in this case relatively higher current flows through the heavy series field winding with thicker diameter.

2- The electromagnetic torque produced here is much higher than normal. This high electromagnetic torque produces motor speed, strong enough to lift heavy load overcoming its initial inertial of rest.

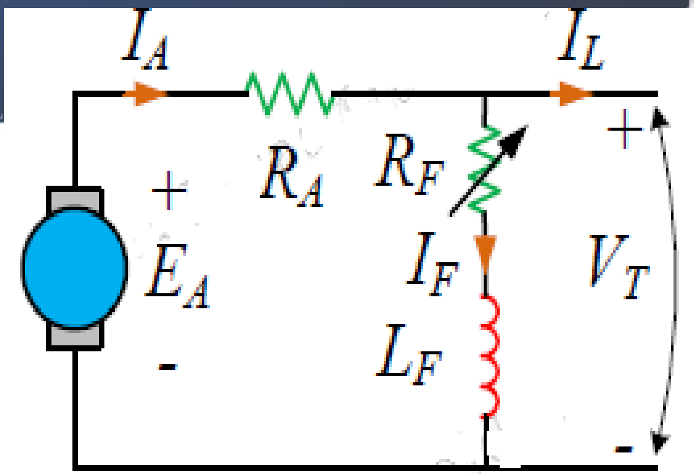
3- Series motors are generally operated for a very small duration, about only a few seconds, just for the purpose of starting. Because if its run for too long, the high series current might burn out the series field coils thus leaving the motor useless.

The resulting torque- speed relationship is

$$\omega_m = \frac{V_T}{\sqrt{K_c}} \frac{1}{\sqrt{\tau_{ind}}} - \frac{R_A + R_c}{K_c}$$



The shunt DC Generators



$I_A = I_F + I_L$
$V_T = E_A - I_A R_A$
$V_F = I_F R_F = V_T$
$E_A = k\varphi\omega_m$

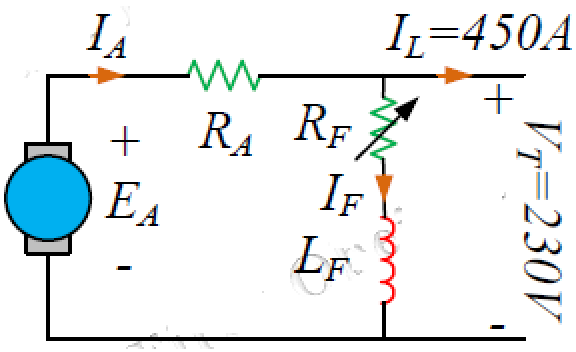
The terminal voltage can be controlled by:

1. Change the speed of rotation: If ω increases, then $E_A = k\varphi\omega m$ increases, so $V_T = EA - IA\ RA$ increases as well.

2. Change the field current. If R_F is decreased. then the field current increases ($V_F = I_F R_F$). Therefore, the flux in the machine increases. As the flux rises, $E_A = k\varphi\omega m$ must rise too, so V_T increases.

Example

Example: A shunt DC generator delivers 450A at 230V and the resistance of the shunt field and armature are 50Ω and 0.3 Ω respectively. Calculate emf.



$I_f = 230/50 = 4.6A$

$I_A = I_F + I_L = 4.6 + 450 = 454.6A$

$E_A = V_T + I_A R_A = 230 + 454.6 \times 0.3 = 243.6\text{V}$

Example

Example: A shunt DC generator, if $E_A = V_T$ = 140 V, rated speed is 1500 rpm, what is the E_A if the speed decrease to 1200 rpm. Assume the filed current is constant.

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

EA = 140 * (1200/1500) = 112 V

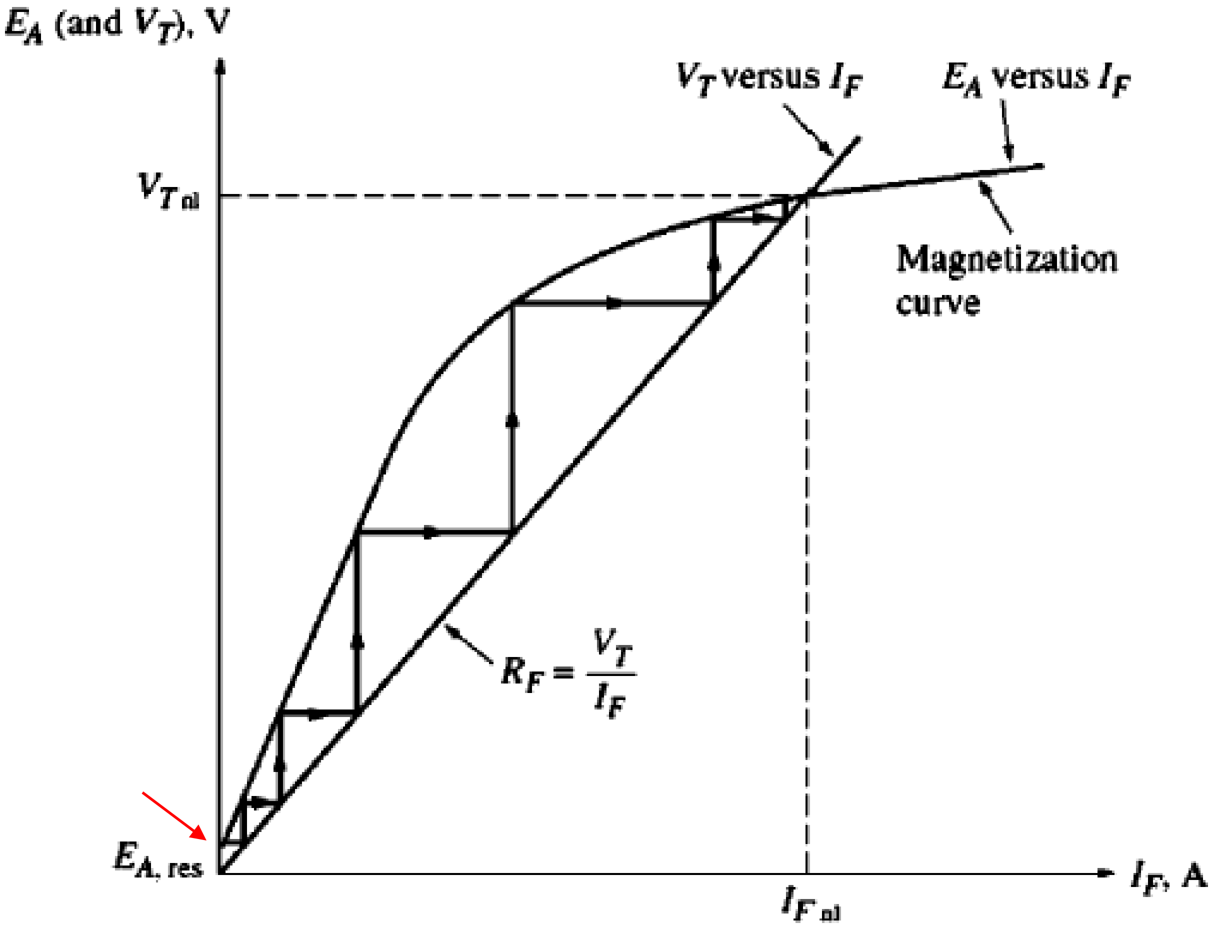
Example

Example: A shunt DC generator, if $E_A = V_T$ = 140 V, the filed current is 5 A, what is the E_A if the filed current decrease BY <u>1 A</u>.

$$\frac{E_A}{E_{A0}} = \frac{\phi}{\phi_0}$$

E_A = 140 * (4/5) = 112 V

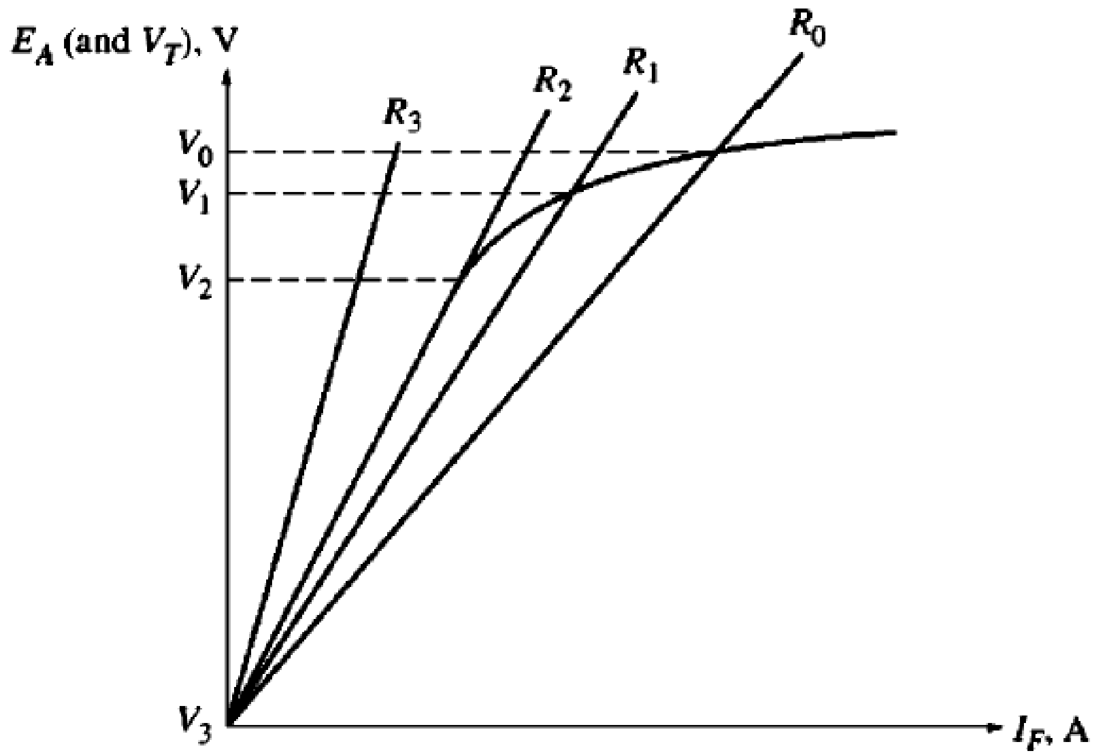
Voltage building in a shunt Generator



Voltage buildup on starting in a shunt dc generator.

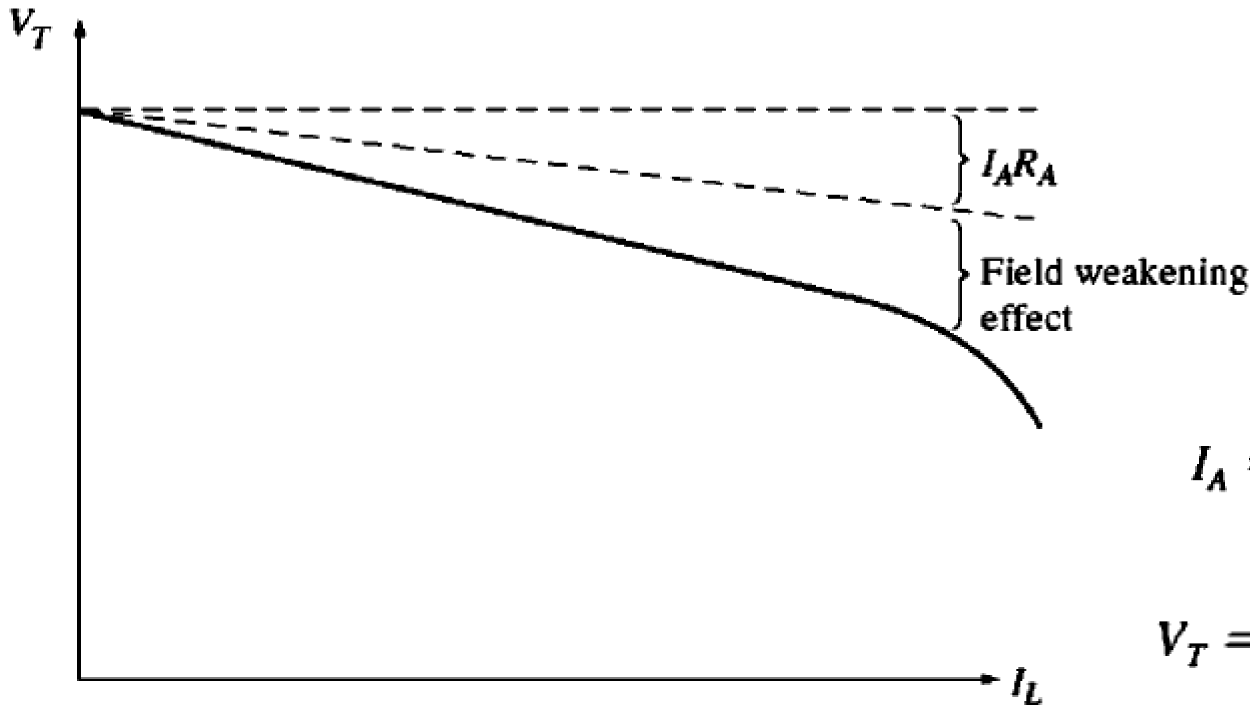
$$E_A = K\phi_{res}\omega$$

What if a shunt generator is started and no voltage builds up? What could be wrong? There are several possible causes for the voltage to fail to build up during starting.



The effect of shunt field resistance on no-load terminal voltage in a dc generator. If $R_F > R_2$ (the critical resistance), then the generator's voltage will never build up.

The terminal characteristic of a shunt DC Generator

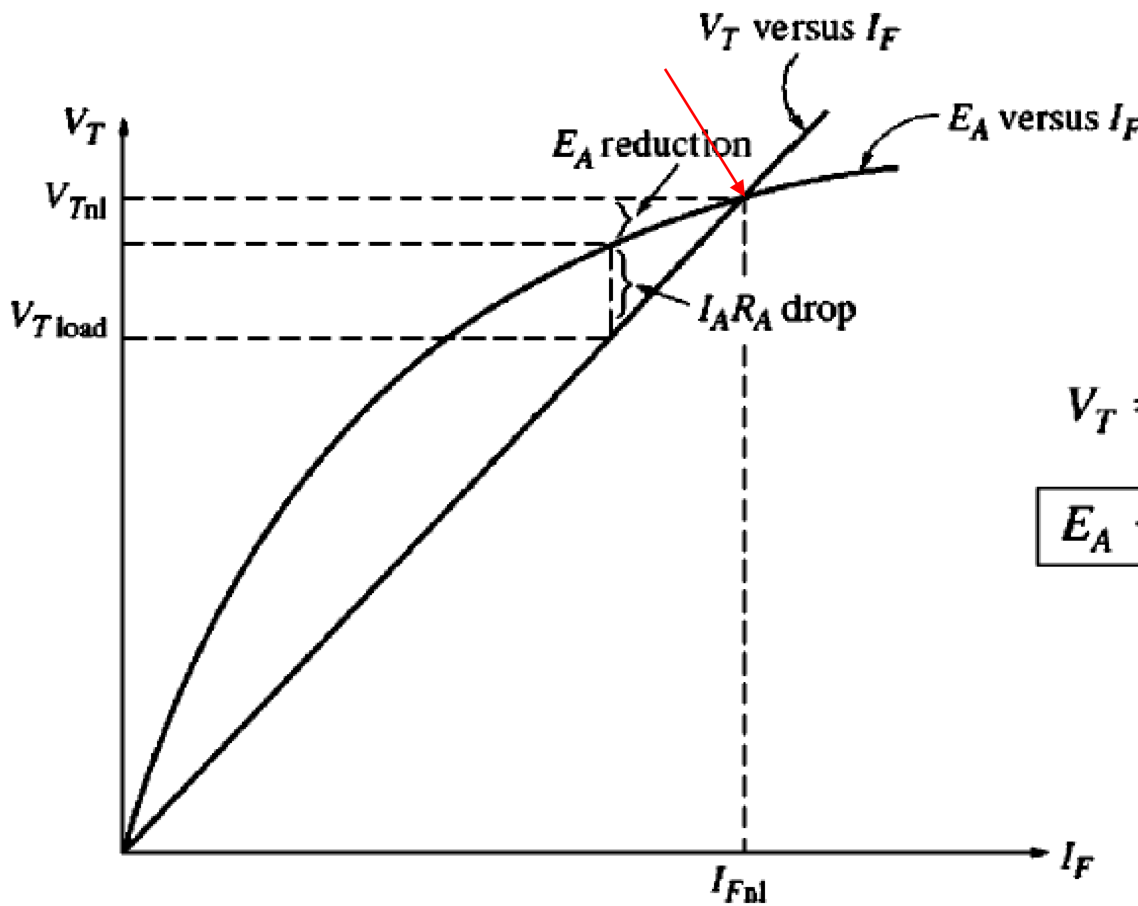


The terminal characteristic of a shunt dc generator.

$$I_A = I_F + I_L \uparrow$$

$$V_T = E_A - I_A \uparrow R_A$$

Analysis of Shunt DC generators



Graphical analysis of a shunt dc generator with compensating windings.

$$I_A = I_F + I_L \uparrow$$

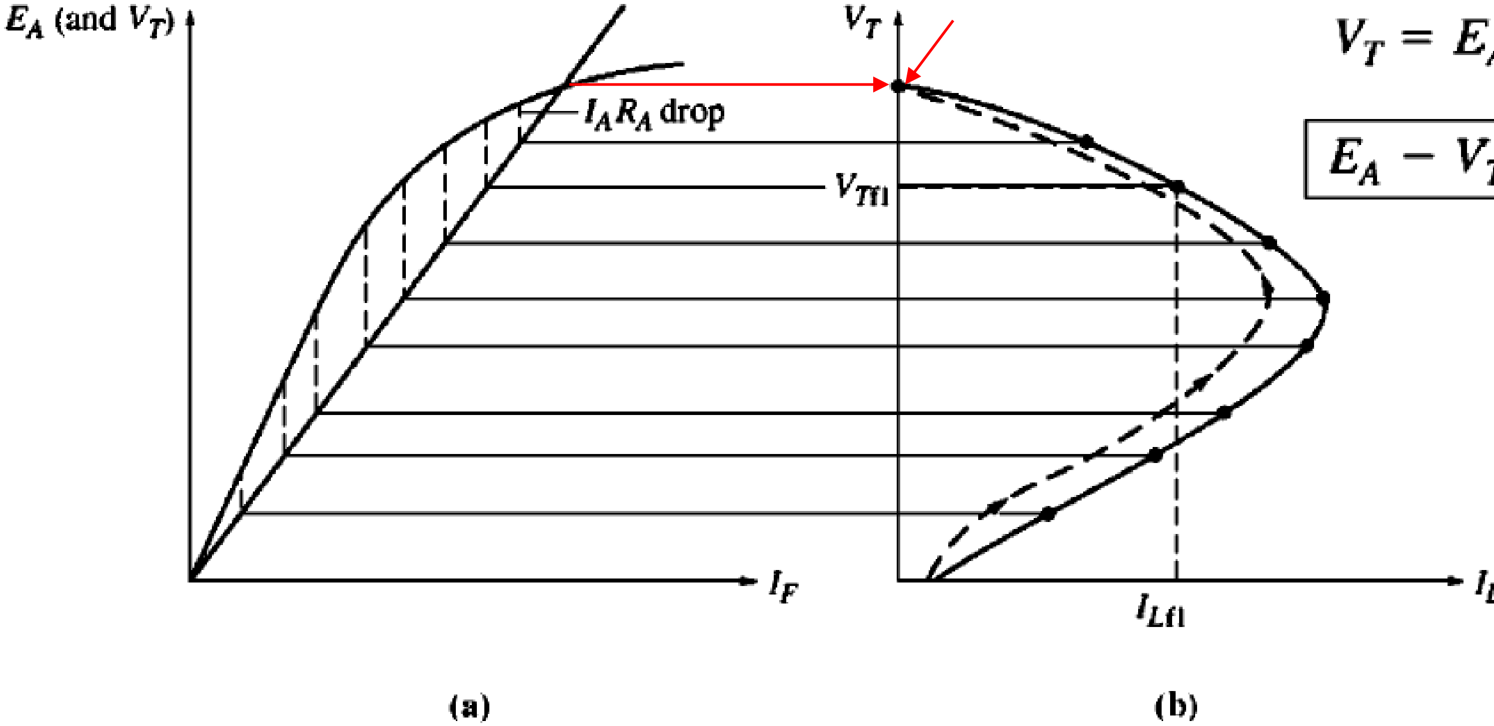
$$V_T = E_A - I_A R_A$$

$$\boxed{E_A - V_T = I_A R_A}$$

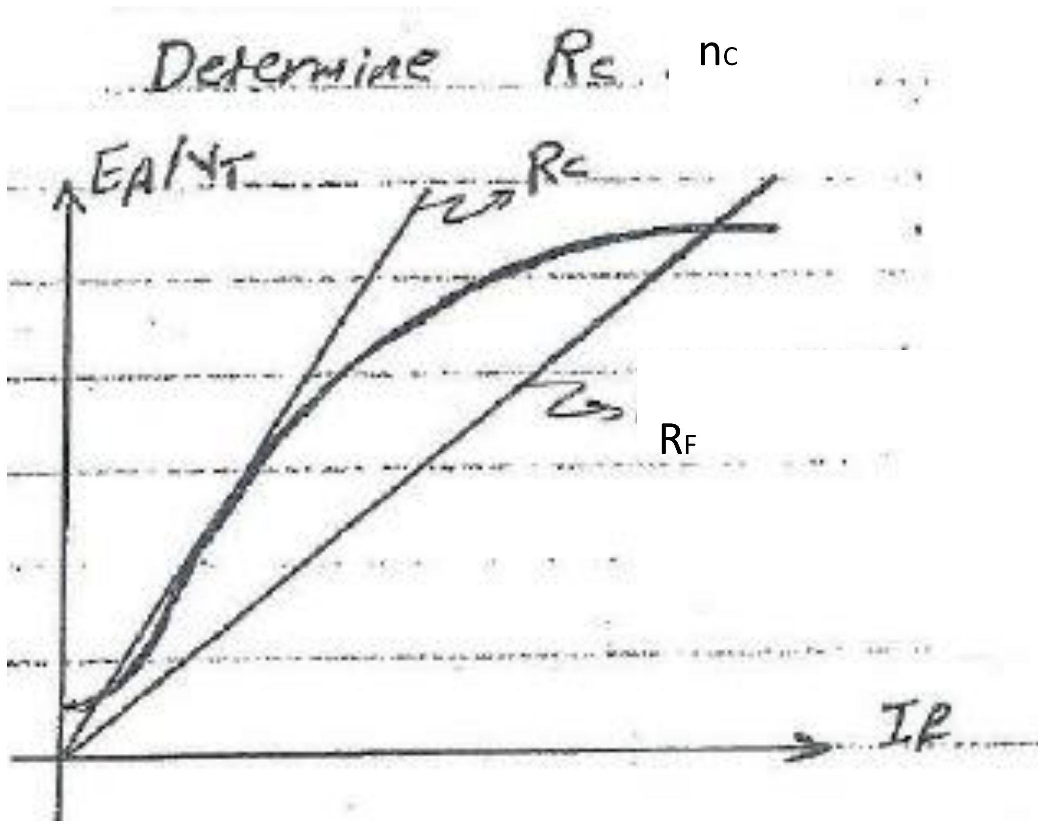
$$I_A = I_F + I_L \uparrow$$

$$V_T = E_A - I_A R_A$$

$$\boxed{E_A - V_T = I_A R_A}$$

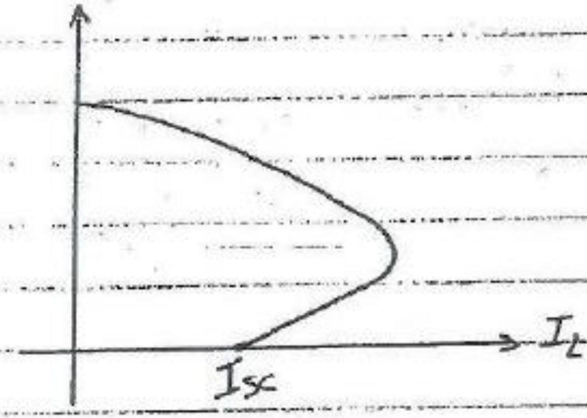


Graphical derivation of the terminal characteristic of a shunt dc generator.


Determine Rc
nC
EA/YT
Rc
RF
IR

Determine I_{SC}

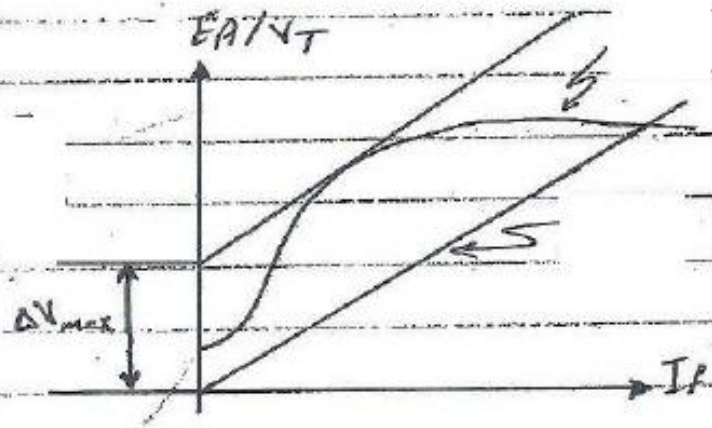
(SSSC)
steady state short curicut current

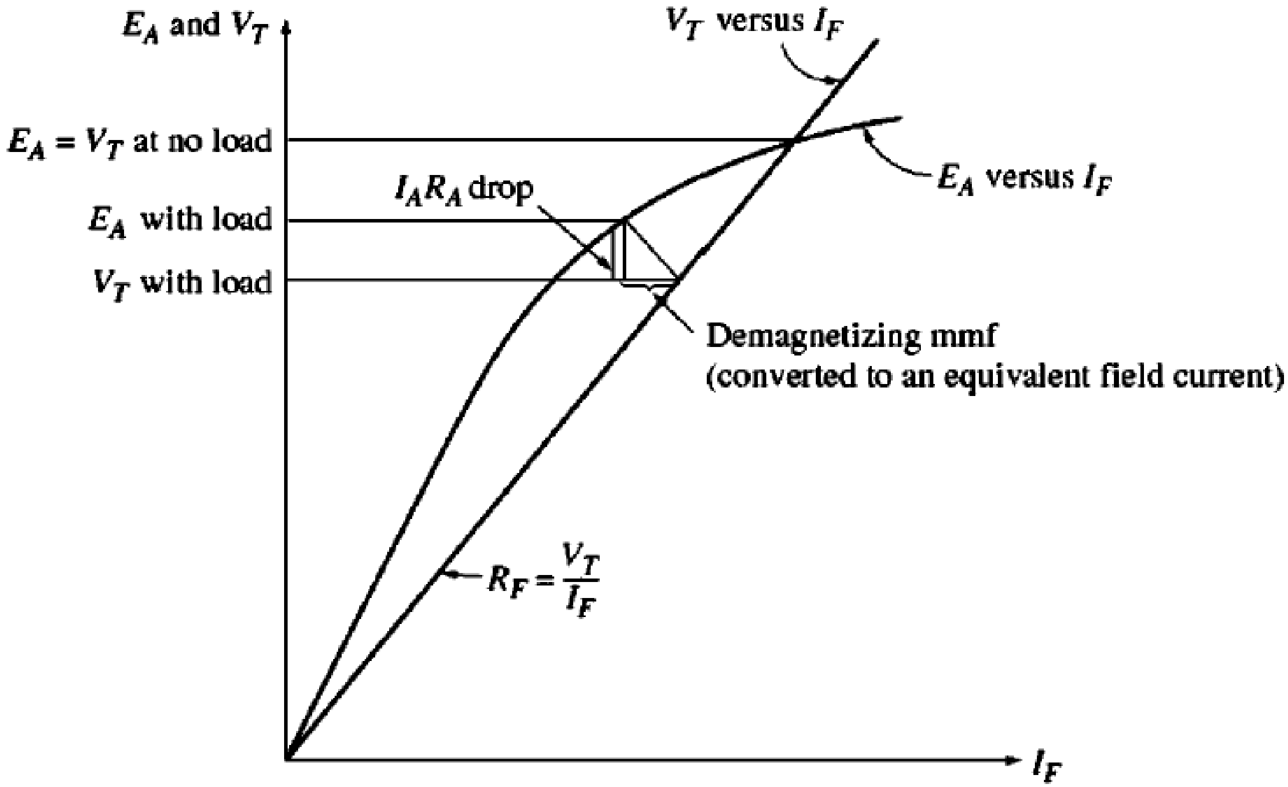


$$I_{SC} = \frac{E_{Rez}}{R_A}$$

Determine I_{max}

$$I_{max} = \frac{\Delta V_{max}}{R_A}$$



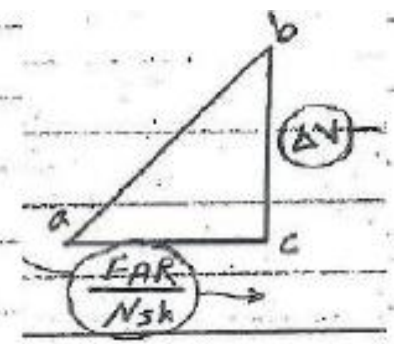


Graphical analysis of a shunt dc generator with armature reaction.

$$F_{net} = F_{sh} - F_{AR}$$

$$I_{net} N_{sh} = I_{sh} N_{sh} - F_{AR}$$

$$I_{net} = I_{sh} - \frac{F_{AR}}{N_{sh}}$$



Induction Motors 7+8

Dr. Feras Alasali

Introduction

- Conversion of electrical power into mechanical power takes place in the rotating part of an electrical motor.
- Machines are called induction machines because the rotor voltage (which produces the rotor current and the rotor magnetic field) is *induced* in the rotor windings rather than being physically connected by wires.
- In AC motors, the rotor does not receive electrical power but conduction by induction in the same way as the secondary of 2-winding transformer receives its power from the primary winding.

Advantages of 3 phase induction motor

- Generally easy to build and cheaper than corresponding dc or synchronous motors
- Induction motor is robust
- The motor is driven by the rotational magnetic field produced by 3 phase currents, hence no commutator or blush is required
- Maintenance is relatively easy and at low cost
- Satisfactory efficiency and reasonable power factor
- A manageable torque-speed curve
- Stable operation under load
- Range in size from few Watts to several MW

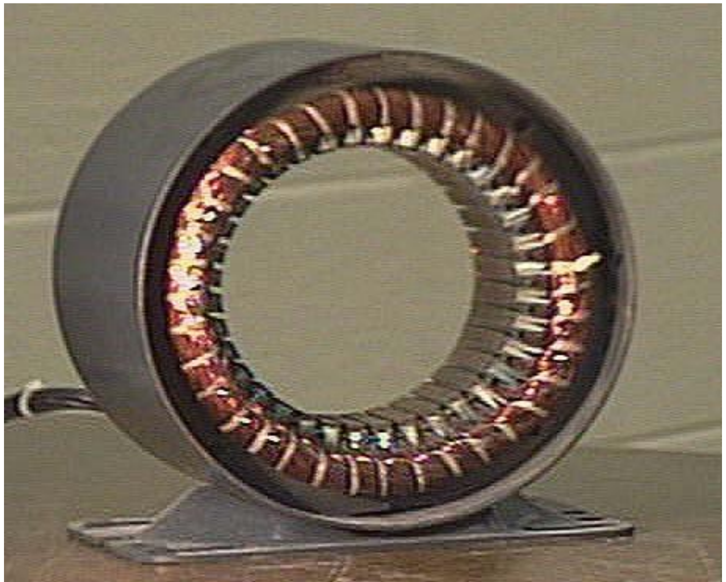
Disadvantages of 3 phase induction motor

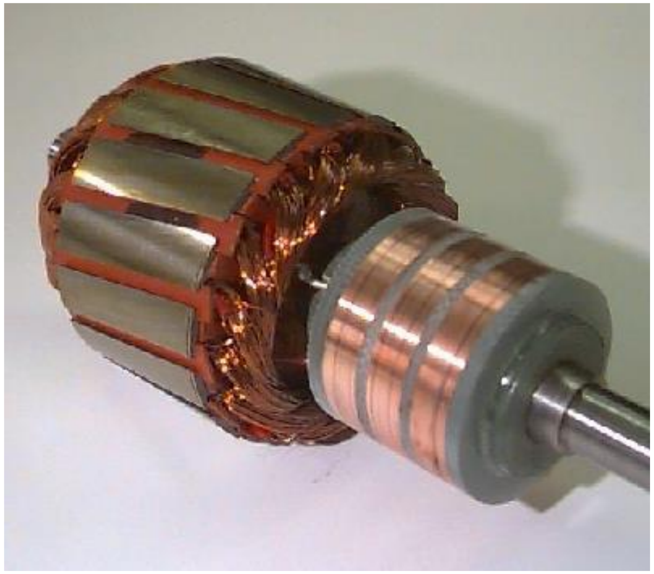
- Induction motor has low inherent starting torque
- Draw large starting currents, typically 6-8 x their full load values
- Speeds not easily controlled as DC motors
- Operate with a poor lagging power factor when lightly loaded

Induction machine construction

An induction motor has two main parts:

- A stator – consisting of a steel frame that supports a hollow, cylindrical core of stacked laminations. Slots on the internal circumference of the stator house the stator winding.
- A rotor – also composed of punched laminations, with rotor slots for the rotor winding.



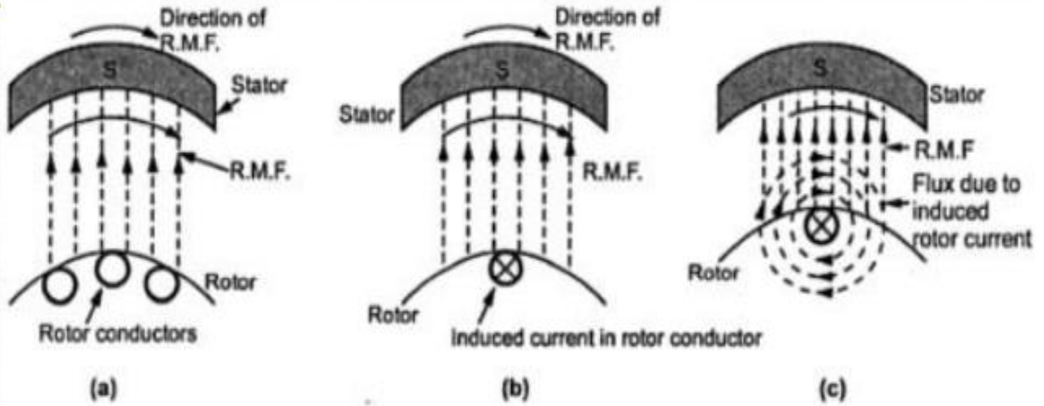


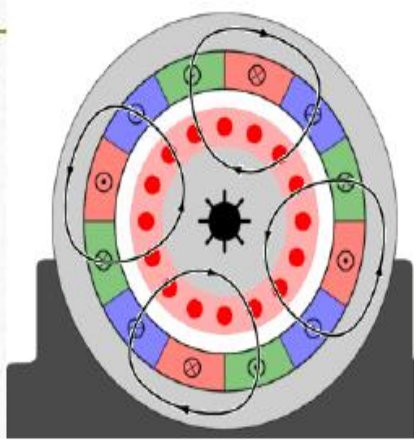
Basic induction motor concepts

(a) When the motor is excited with three-phase supply, three-phase stator winding produce a rotating magnetic field at synchronous speed.

(b) The stator's magnetic field is therefore changing or rotating relative to the rotor. Hence, according to the principle of Faraday's laws of electromagnetic induction, a voltage is induced at the rotor. Thus, when the rotor is short-circuited or closed through an external impedance, a current is induced in the induction motor's rotor.

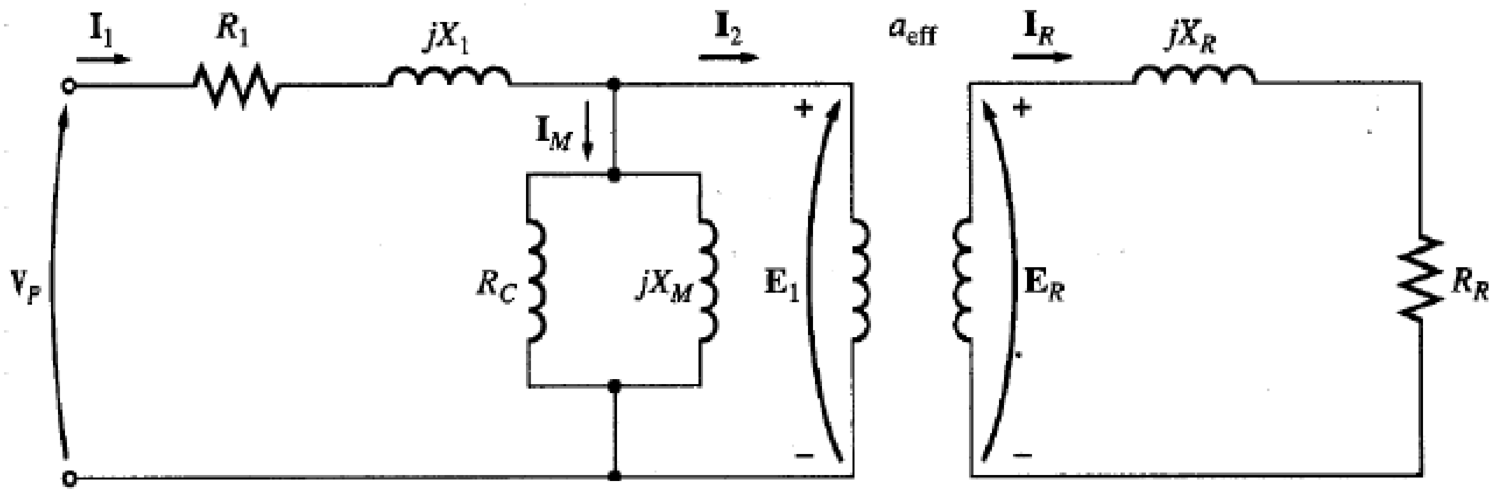
(c) Finally, a flux will be produced in rotor due to induced rotor current forcing the rotor to rotate in the same direction of the stator rotating magnetic field.

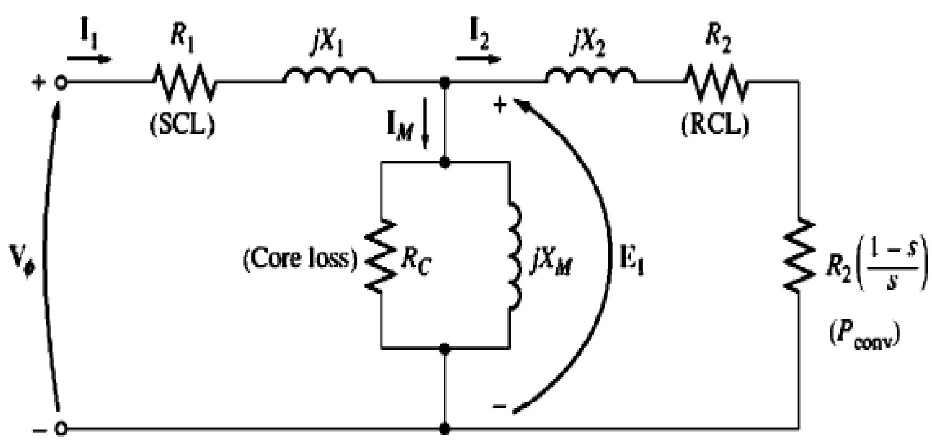




The Transformer Model of an Induction Motor

The transformer model or an induction motor. with rotor and stator connected by an ideal transformer of turns ratio



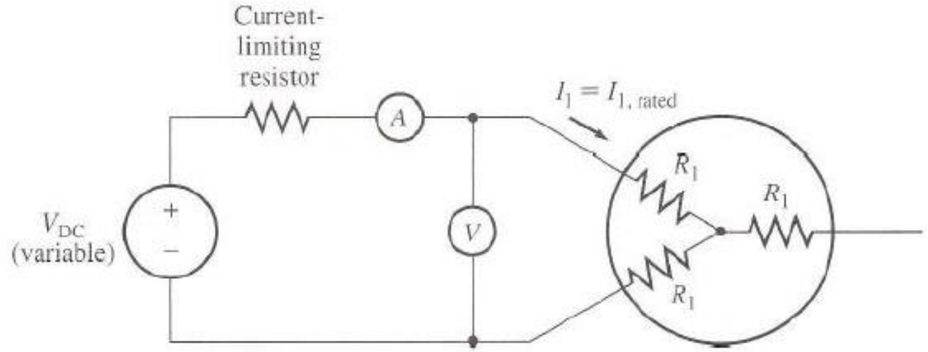


The per-phase equivalent circuit with rotor losses and P_{core} separated.

Determining motor circuit parameters

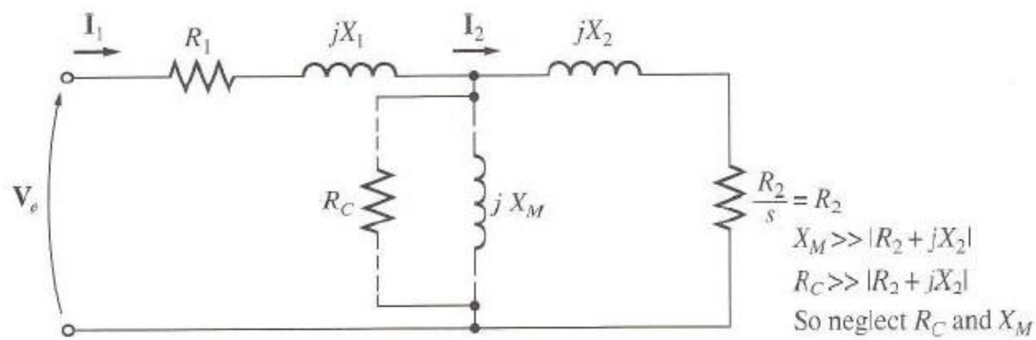
- **DC Test** (or use Ohm-meter)

  Measuring V, I → R1



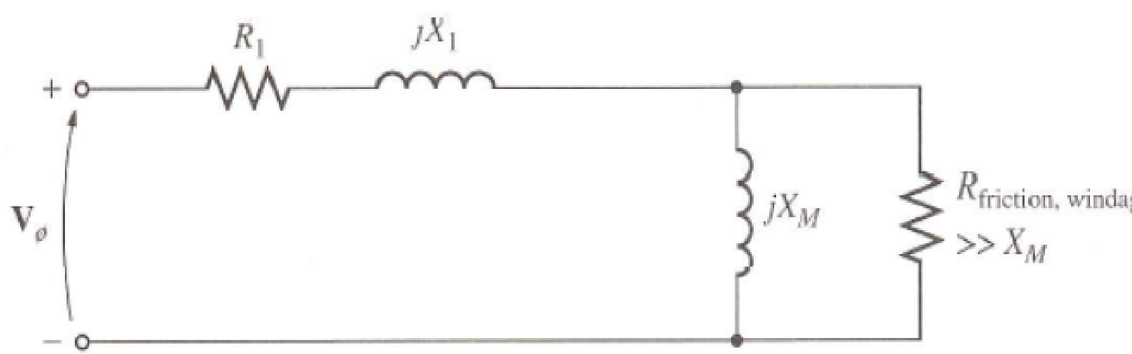
- **Locked-rotor Test**

  Measuring V_ϕ, I1, P, Q
  → R1+R2, X1+X2



- **No-load Test**

  Measuring Vϕ, I1, P, Q
  → R1, X1+Xm



The efficiency of small motors can be determined by directly loading them and by measuring the input and output powers. But in the case of large motors, it is difficult to arrange that much load for them. The power loss will be large if we directly test the load. Therefore indirect methods are used to determine the efficiency of 3-phase induction motors.

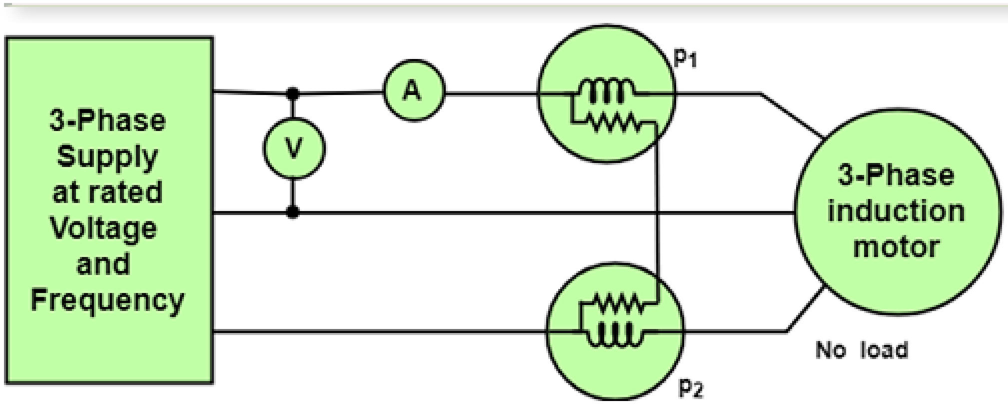
We can perform the following test on the motor to find the efficiency:

- No-Load test.
- Locked Rotor test.
- DC Test

No-Load test

The no-load test of an induction motor is similar to the open-circuit test of a transformer.

- The motor is not connected from its load, and the rated voltage at the rated frequency is applied to the stator to run the motor without a load.
- The voltmeter measures the standard-rated supply voltage and an ammeter measures the no-load current.
- Since the motor is running at no-load, total power is equal to the constant iron loss, friction and winding losses of the motor.
- The no-load test of an induction motor measures the rotational losses of the motor and provides information about its magnetization current.



- In this motor at no- load conditions, the input power measured by the meters must equal the losses in the motor. The rotor copper losses are negligible .

$$\boxed{P_{\text{SCL}} = 3I_1^2 R_1}$$

so the input power must equal

$$P_{\text{in}} = P_{\text{SCL}} + P_{\text{core}} + P_{\text{F\&W}} + P_{\text{misc}}$$
$$= 3I_1^2 R_1 + P_{\text{rot}}$$

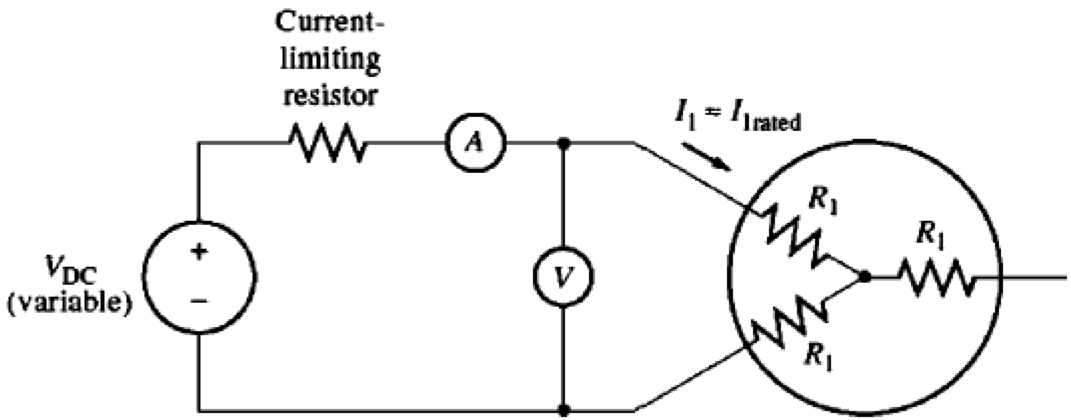
where P_{rot} is the rotational losses of the motor:

$$P_{\text{rot}} = P_{\text{core}} + P_{\text{F\&W}} + P_{\text{misc}}$$

$$\boxed{\left|Z_{\text{eq}}\right| = \frac{V_\phi}{I_{1,\text{nl}}} \approx X_1 + X_M}$$

The DC test for stator resistance

- The rotor resistance *R2* plays an extremely critical role in the operation of an in duction motor. Among other things, R1 determines the shape of the torque-speed curve, determining the speed at which the pullout torque occurs.
- To find the rotor resistance R2 accurately, it is necessary to know R1 so that it can be subtracted from the total.
- This test for R1 independent of R1 , X1 and X2. This test is called the dc test. Basically, a dc volt age is applied to the stator windings of an induction motor. Because the current is dc, there is no induced voltage in the rotor circuit and no resulting rotor current now. Also, the reactance of the motor is zero at direct current.
- Therefore, the only quantity limiting current n ow in the motor is the stator resistance, and that resistance can be determined.

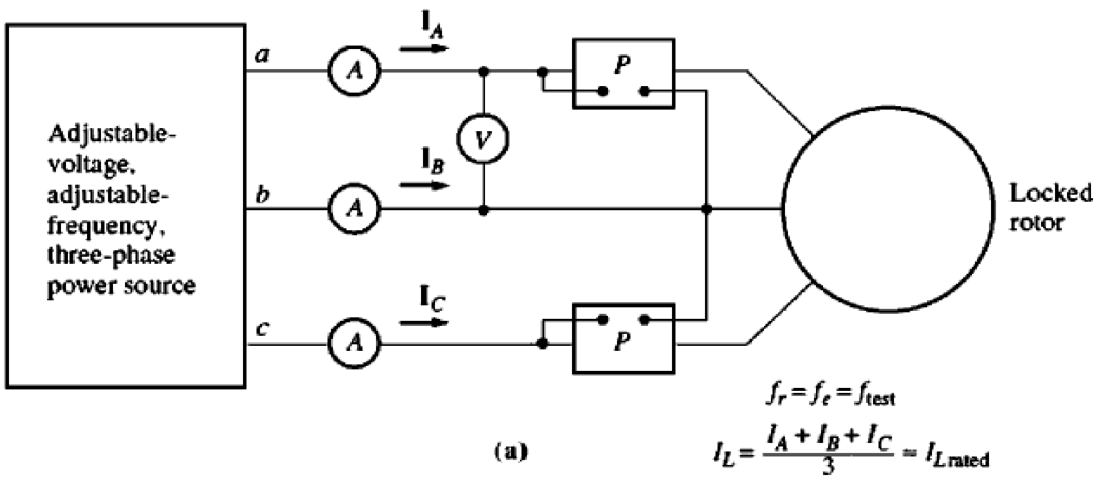


$$2R_1 = \frac{V_{DC}}{I_{DC}}$$

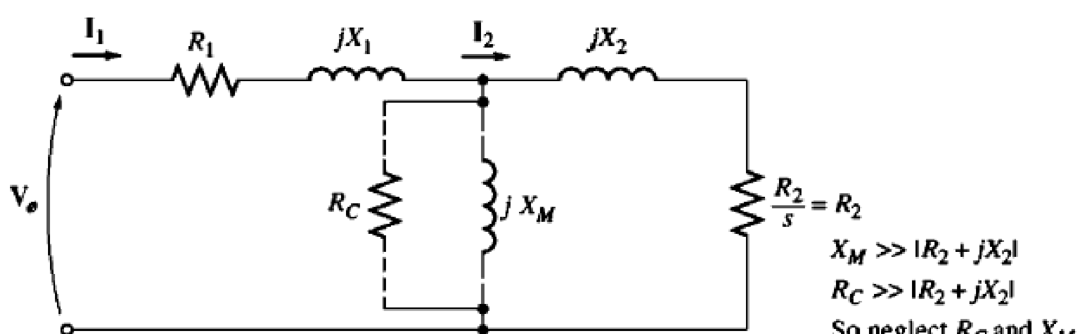
$$\boxed{R_1 = \frac{V_{DC}}{2I_{DC}}}$$

The Locked-Rotor Test

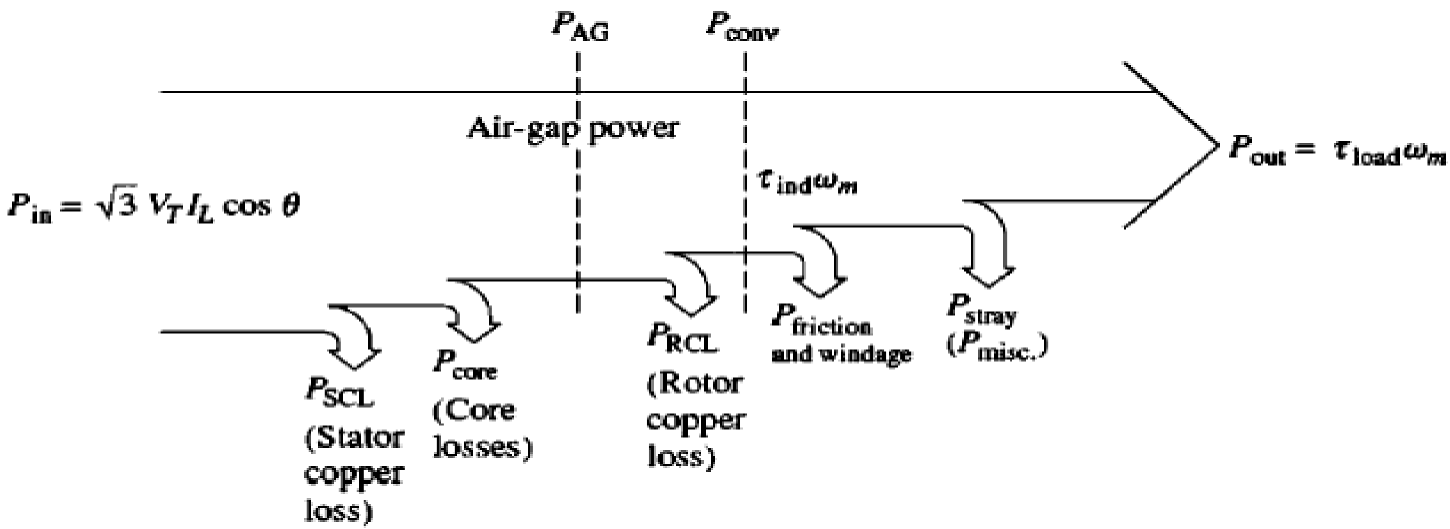
- The third test that can be performed on an induction motor to determine its circuit parameters is called the *locked-rotor test,* or sometimes the *blocked-rotor test.*
- This test corresponds to the short-circuit test on a transformer. **In** this test, the rotor is locked or blocked so that it *cannot* move, a voltage is applied to the motor, and the resulting voltage, current, and power are measured



(a)



(b)

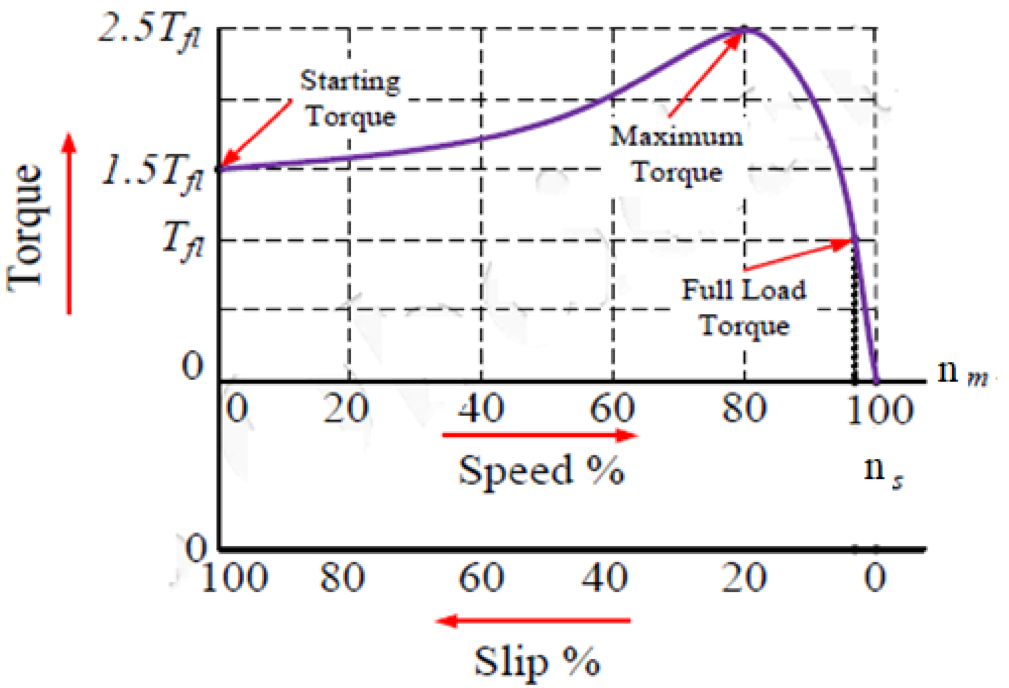


The power-flow diagram of an induction motor.

Torque-Speed Curve

The torque speed (slip) curve for an induction motor gives us the information about the variation of torque with the slip.

When the rotor stationary (at standstill) $n_m = 0$ rpm, the rotor frequency $f_r = f_s$ and the slip $s=1$. At $n_m = n_s$, the rotor frequency $f_r = 0$ Hz, and the slip $s=0$.



- At full load, the motor runs at speed of n_m. When mechanical load increases, motor speed decreases tell the motor torque again becomes equal to the load torque.
- As long as the two torques are in balance, the motor will run at constant (but lower) speed.
- If the load torque exceeds the induction motor maximum torque, the motor will suddenly stop.

Notes: the Induction Motor Torque- Speed Curve

1. The induced torque of the motor is zero at synchronous speed.

2. The torque- speed curve is nearly linear between no load and full load. In this range, the rotor resistance is much larger than the rotor reactance, so the rotor current, the rotor magnetic field, and the induced torque increase linearly with increasing slip.

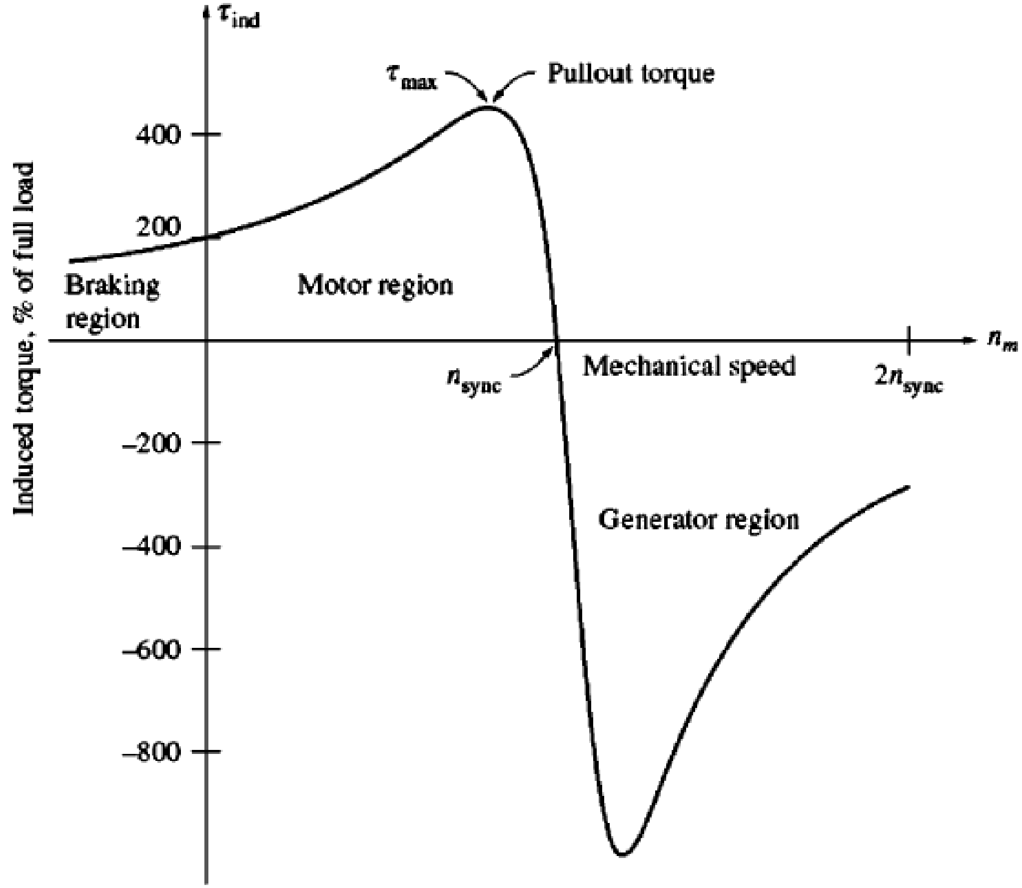
3. There is a maximum possible torque that cannot be exceeded. This torque, called the *pullout torque* or *breakdown torque*, is 2 to 3 times the rated full load torque of the motor.

4. The starting torque on the motor is slightly larger than its full-load torque. So this motor will start carrying any load that it can supply at full power.

5. The torque on the motor for a given slip varies as the square of the applied voltage. This fact is useful in one form of induction motor speed control.

6. If the rotor of the induction motor is driven faster than synchronous speed, then the direction of the induced torque in the machine reverses and the machine becomes a generator, converting mechanical power to electric power.

7. If the motor is turning backward relative to the direction of the magnetic fields, the induced torque in the machine will stop the machine very rapidly and will try to rotate it in the other direction. Since reversing the direction of magnetic field rotation is simply a matter of switching any two stator phases, this fact can be used as a way to very rapidly stop an induction motor. The act of switching two phases in order to stop the motor very rapidly is called plugging.



Induction motor torque–speed characteristic curve, showing the extended operating ranges (braking region and generator region).

- The torque-speed characteristic curve shows that if an induction motor is driven at a speed *greater* than nsync by an external prime mover, the direction of its inducted torque will reverse and it will act as a generator. As the torque applied to its shaft by the prime mover increases, the amount of power produced by the induction generator increases.
- There is a maximum possible induced torque in the generator mode of operation. This torque is known as the *pushover torque* of the generator. If a prime mover applies a torque greater than the pushover torque to the shaft of an induction generator, the generator will overspeed.