

“Moment of Inertia”
Design of Mechatronics Systems

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Moment of inertia → Rotational inertia

How much something going to resist
being angularly accelerated.

Known by the letter “I” through

Newton's second law

$$\alpha = \frac{\sum \tau}{I}$$

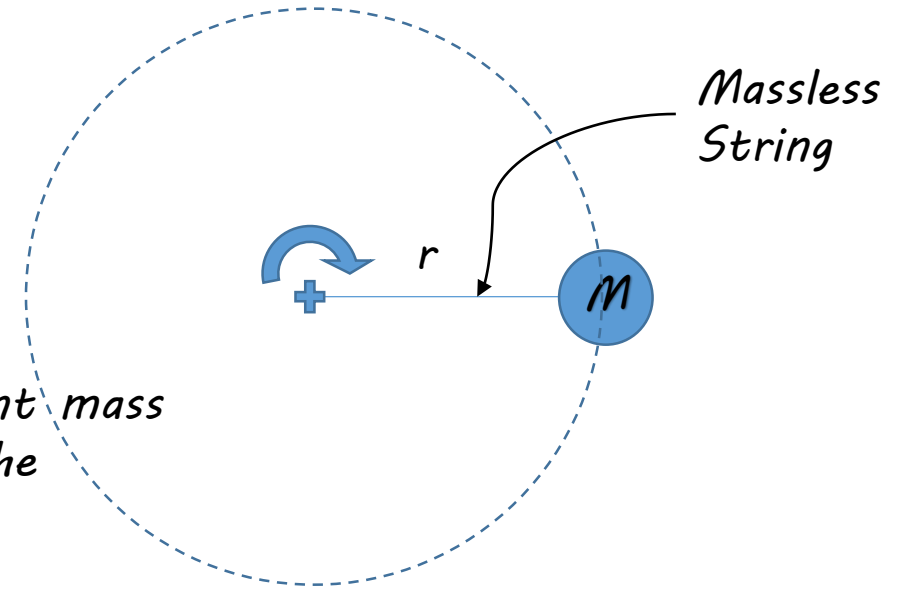
From the equation you see that

- If I is large, then the angular acceleration
Will be a small value
- If I is small, then the angular acceleration
Will be relatively a large value

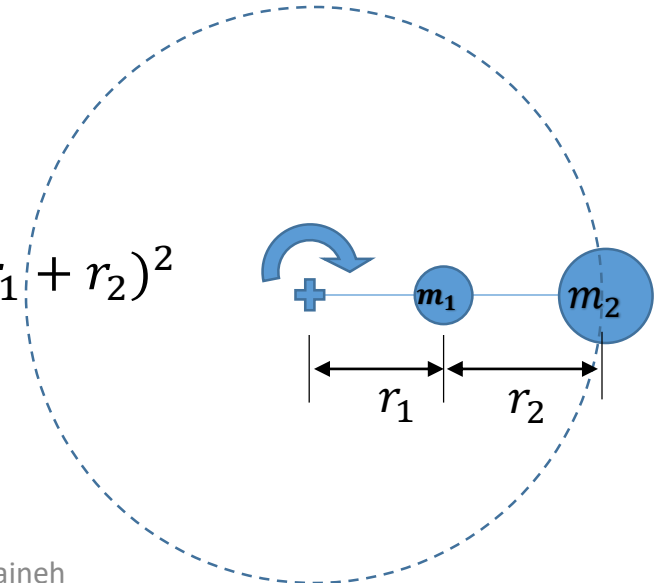
Moment of inertia for a point Mass

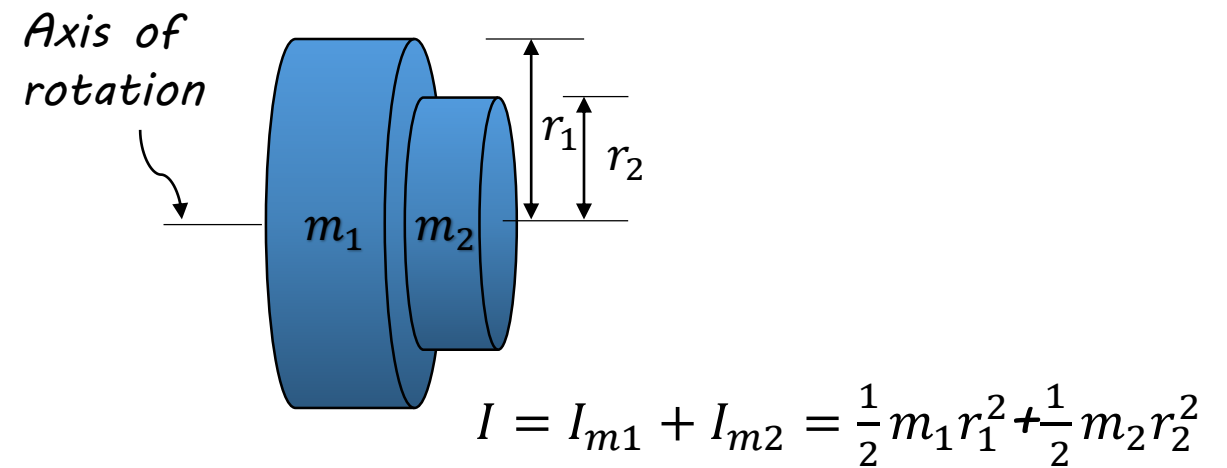
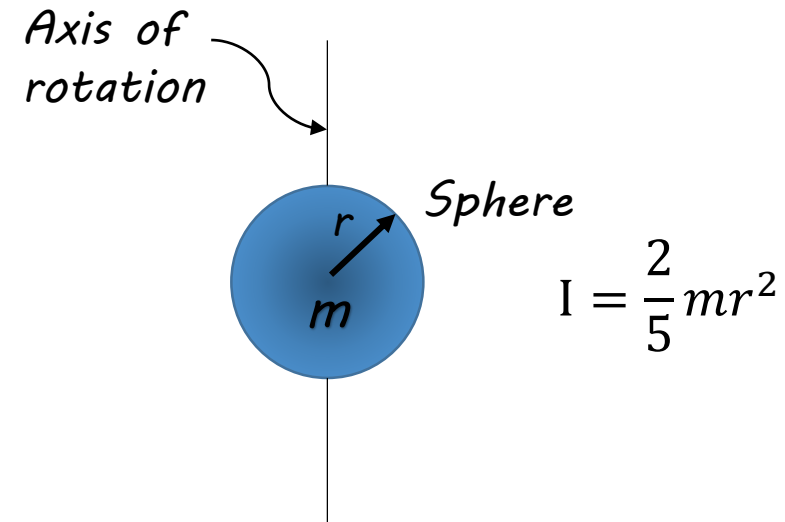
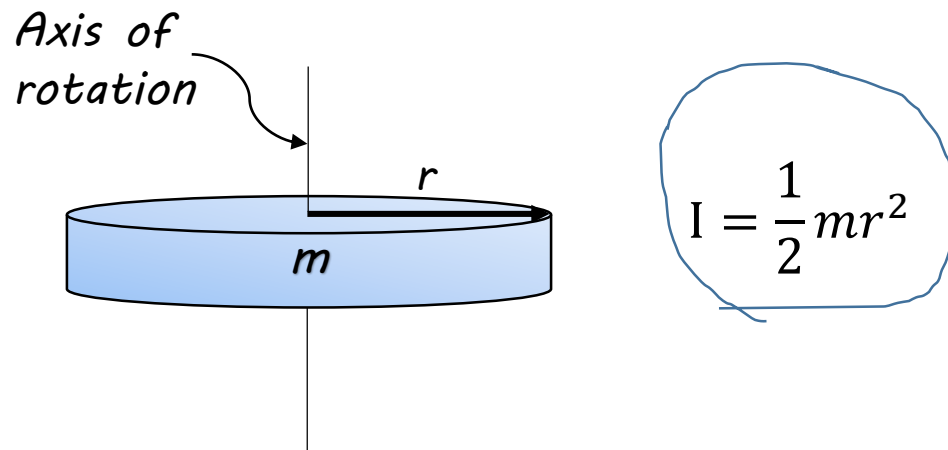
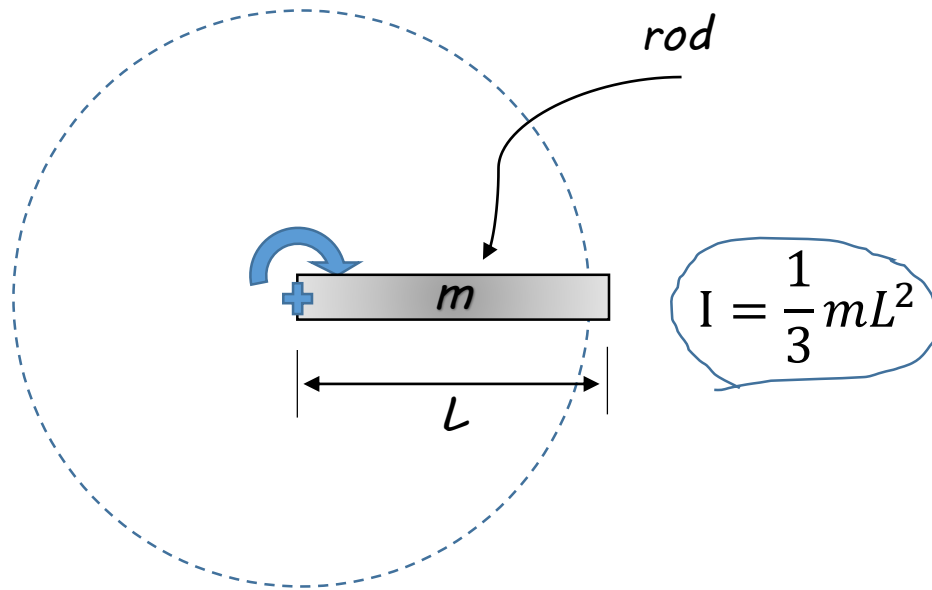
$$I = \underline{m}r^2$$

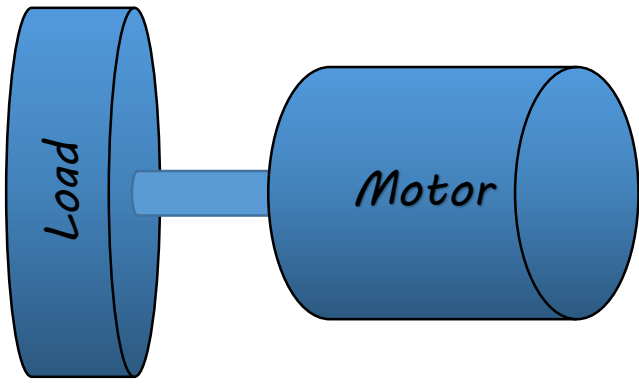
Imagine the point mass
is reaching to the
center
Is “I” larger or
smaller?



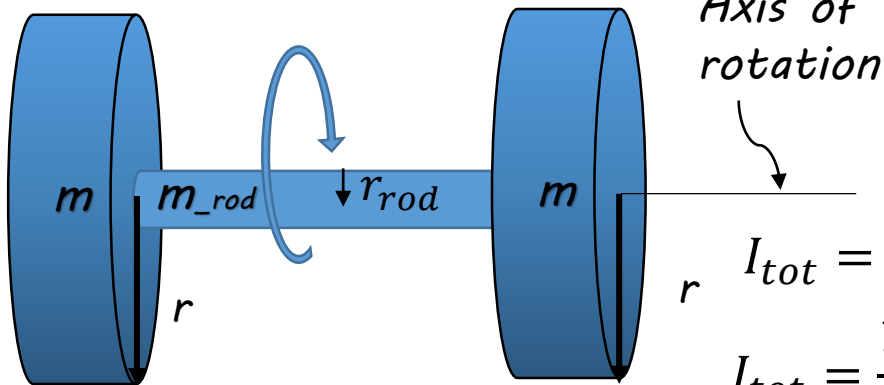
$$I = m_1 r_1^2 + m_2 (r_1 + r_2)^2$$







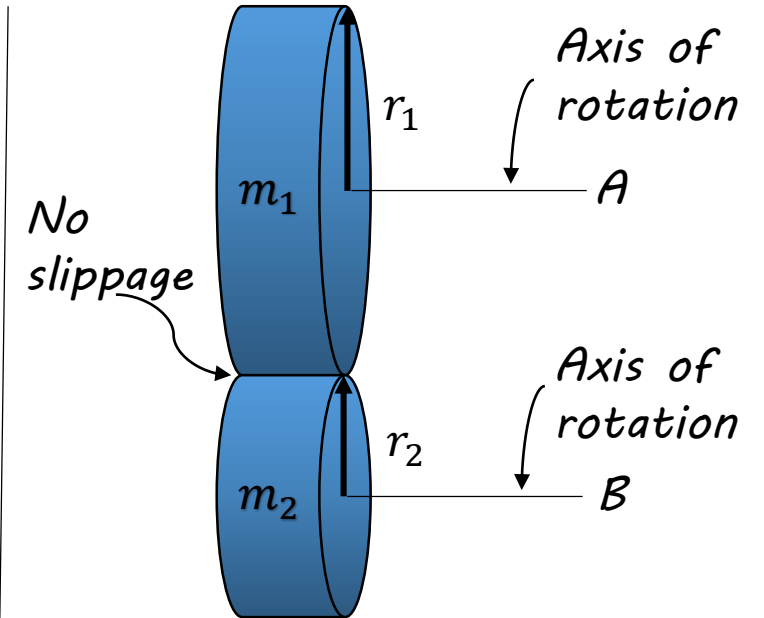
$$I_{tot} = I_{motor} + I_{load}$$



$$\begin{aligned}
 m &= 30 \text{ kg} \\
 m_{rod} &= 10 \text{ kg} \\
 r_{rod} &= 2.5 \text{ cm} \\
 r &= 27 \text{ cm}
 \end{aligned}$$

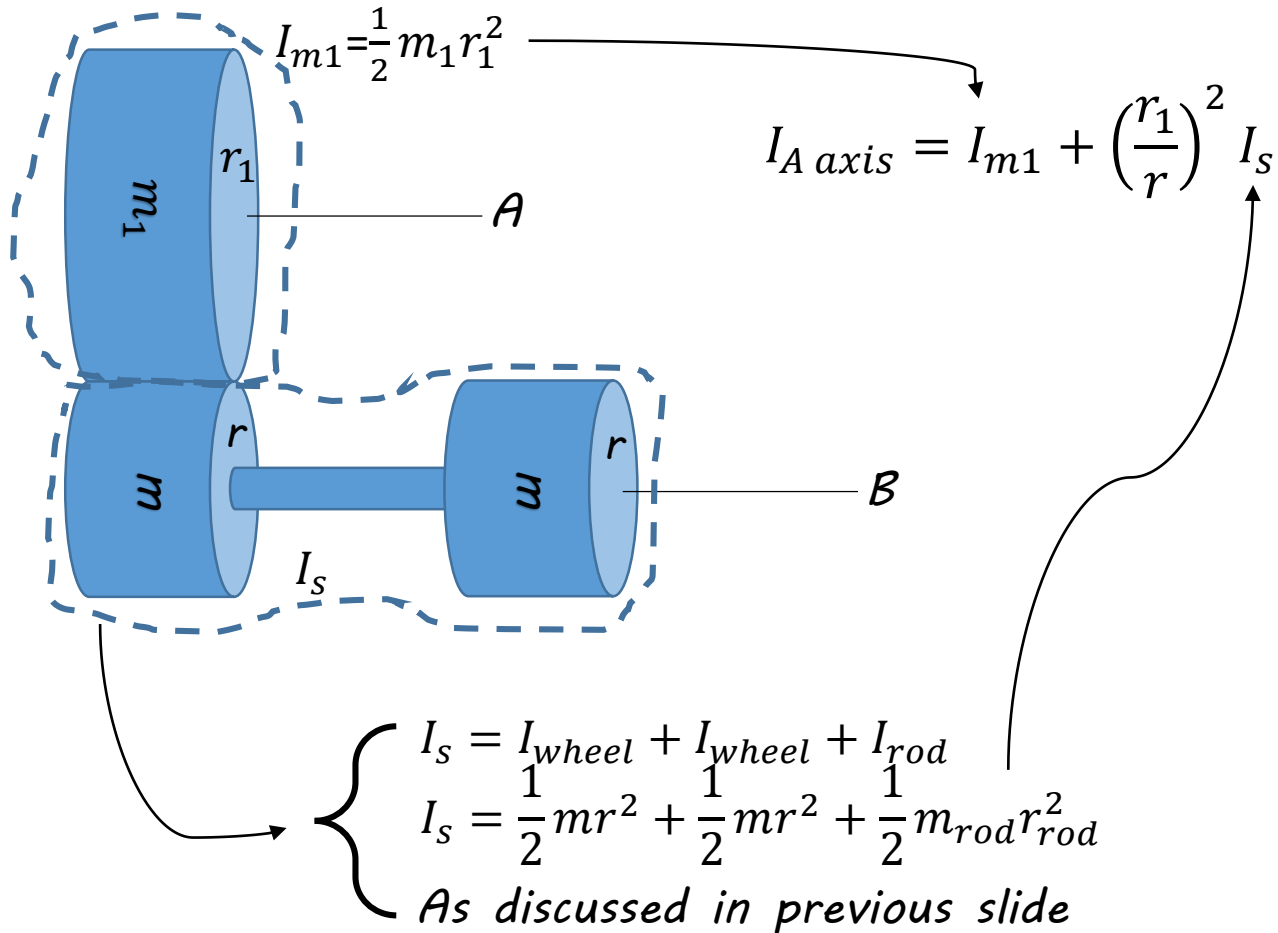
$$\begin{aligned}
 I_{tot} &= I_{wheel} + I_{wheel} + I_{rod} \\
 I_{tot} &= \frac{1}{2}mr^2 + \frac{1}{2}mr^2 + \frac{1}{2}m_{rod}r_{rod}^2 \\
 I_{tot} &= 1.094 + 1.094 + \underline{0.00313}
 \end{aligned}$$

Relatively very small
Why?



In this case the total moment of inertia of a rotation around axis A is:

$$\begin{aligned}
 I_{A \text{ axis}} &= \frac{1}{2}m_1r_1^2 + \frac{1}{2}m_2r_2^2 \quad ? \times \\
 I_{A \text{ axis}} &= \frac{1}{2}m_1r_1^2 + \left(\frac{r_1}{r_2}\right)^2 \frac{1}{2}m_2r_2^2 \quad \checkmark \\
 I_{A \text{ axis}} &= I_{m1} + \underbrace{\left(\frac{r_1}{r_2}\right)^2}_{\text{Gear ratio}} I_{m2}
 \end{aligned}$$



Example: Determine the moment of inertia of the shown system around A axis if you have:

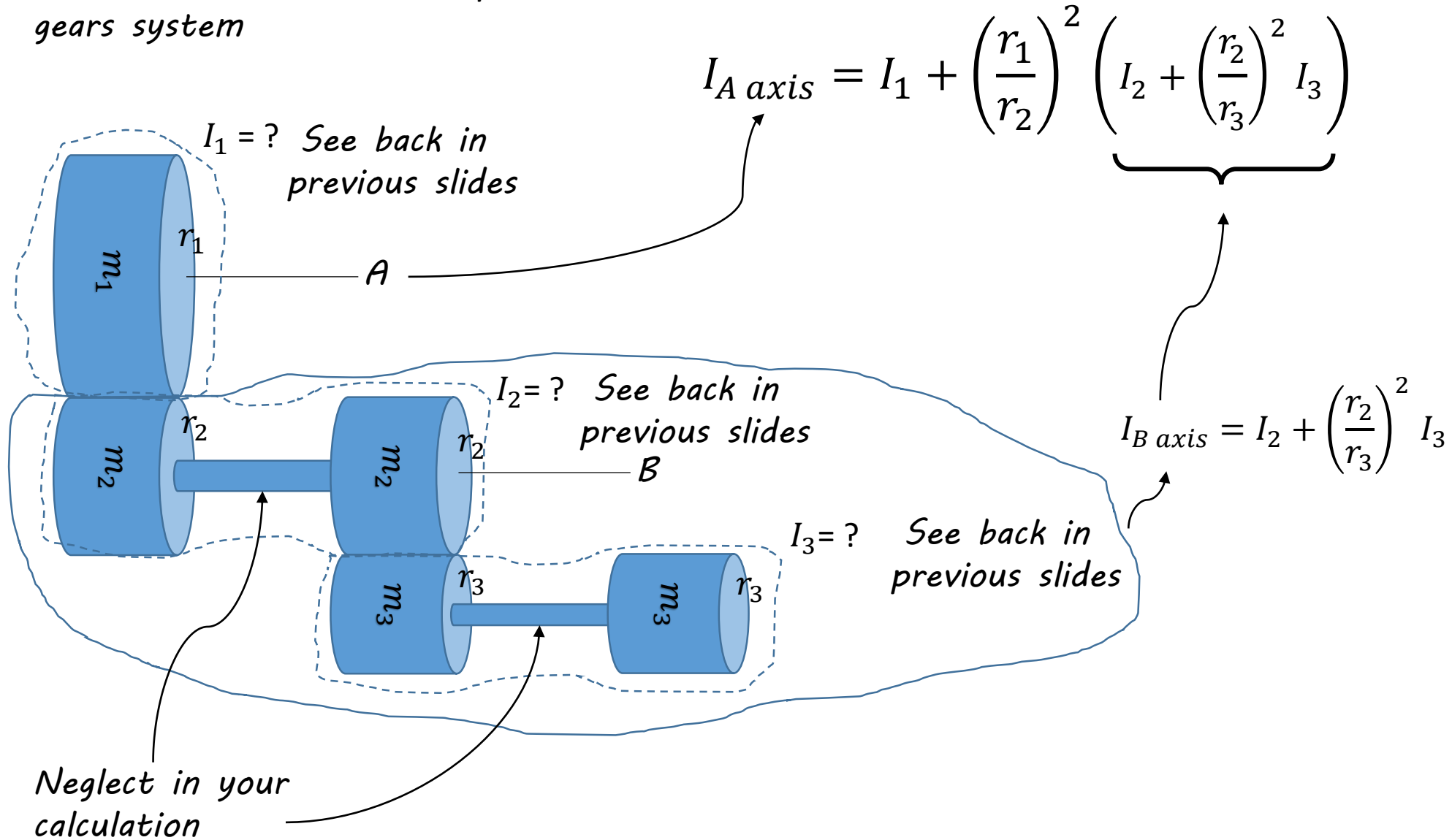
$$m = 30 \text{ kg}, \quad m_{rod} = 10 \text{ kg},$$

$$m_1 = 50 \text{ kg}$$

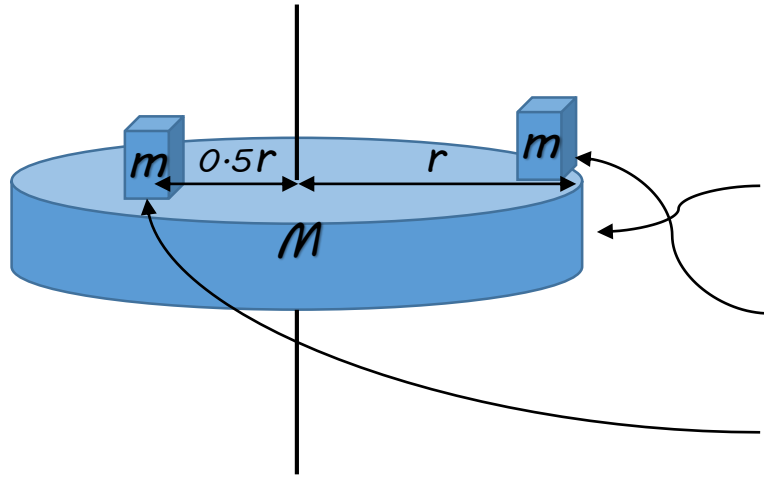
$$r_{rod} = 2.5 \text{ cm}, \quad r = 27 \text{ cm}, \quad r_1 = 35 \text{ cm}$$

Which part of the system can be neglected in moment of inertia calculation?

Moment of inertia of Complex gears system



Rotating solid disc and objects on top



Example: The rotating solid disc has a radius 4 m and mass $M=10$ kg. Two small ($m=2$ kg) objects are placed on top of it. One object is placed half way between the disc's center and its edge. The other object is placed at the edge of the disc. Calculate the system's moment of inertia around the disc rotation axis.

solid disc $I_{disc} = \frac{1}{2}Mr^2$

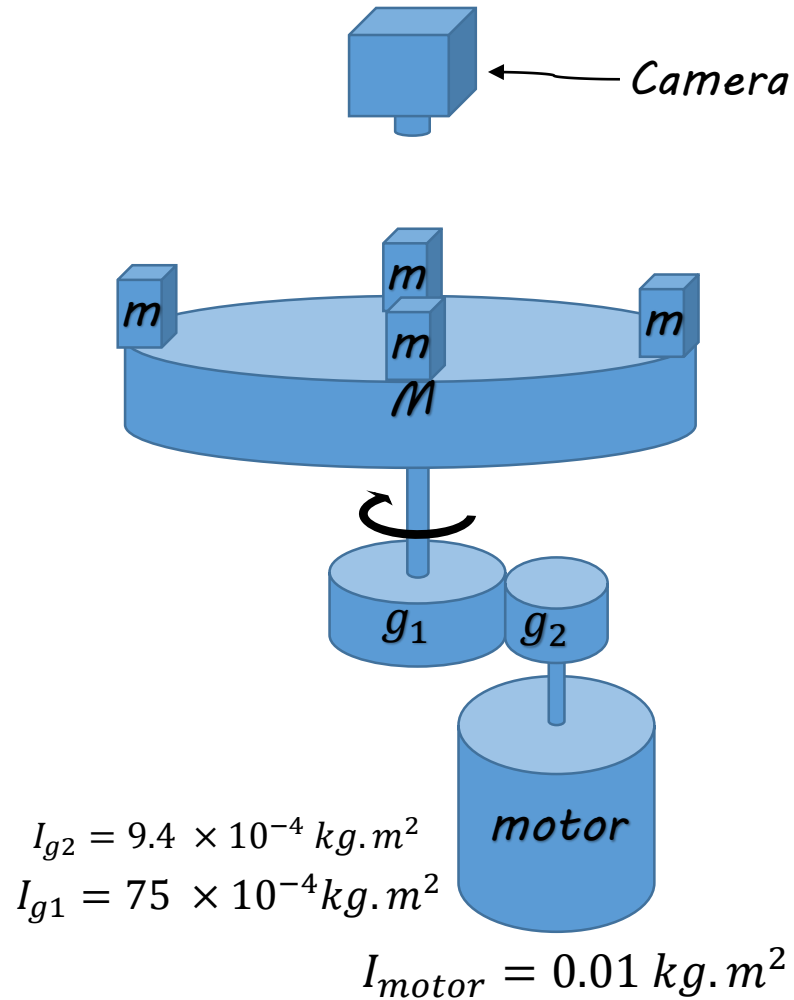
Point mass $I_{obj1} = mr^2$

Point mass $I_{obj2} = m\left(\frac{1}{2}r\right)^2$

$$I_{sys} = I_{disc} + I_{obj1} + I_{obj2} = \frac{1}{2}Mr^2 + mr^2 + m\left(\frac{1}{2}r\right)^2$$
$$= \frac{1}{2}10(4)^2 + 2(4)^2 + 2\left(\frac{1}{2}(4)\right)^2 = ??$$

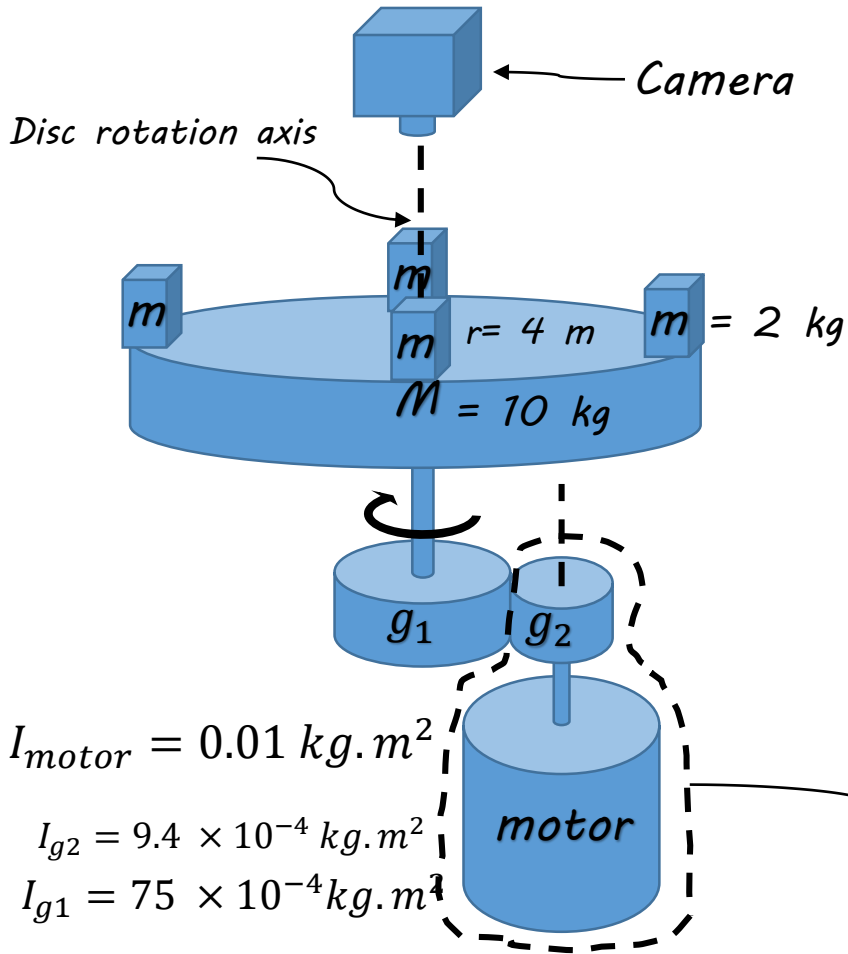
Check your calculator

Calculating motor torque



Example: The rotating solid disc has a radius 4 m and mass $M=10$ kg requires to stop each $\frac{1}{4}$ revolution to place an object m under camera inspection. Four small ($m=2$ kg) objects are placed at the edge of the disc. The system is powered by a servo motor by a gear ratio $\frac{g_2}{g_1} = \frac{1}{2}$. Calculate a) the system's moment of inertia around the motor's rotating shaft, b) the required motor torque if the disc's angular acceleration $\alpha_{disc} = 2 \text{ rad/s}^2$. System efficiency is 100%.

Calculating motor torque



Example: The rotating solid disc has a radius 4 m and mass $M = 10 \text{ kg}$ requires to stop each $\frac{1}{4}$ revolution to place an object m under camera inspection. Four small ($m = 2 \text{ kg}$) objects are placed at the edge of the disc. The system is powered by a servo motor by a gear ratio $\frac{g_2}{g_1} = \frac{1}{2}$. Calculate a) the system's moment of inertia around the motor's rotating shaft, b) the required motor torque if the disc's angular acceleration $\alpha_{\text{disc}} = 2 \text{ rad/s}^2$. System efficiency is 100%.

$$I_1 = I_{\text{disc}} + I_{\text{obj1}} + I_{\text{obj2}} + I_{\text{obj3}} + I_{\text{obj4}}$$

$$I_1 = I_{\text{disc}} + 4 I_{\text{obj}}$$

$$I_1 = \frac{1}{2} M r^2 + 4(m r^2) + I_{g1}$$

$$I_2 = I_{\text{motor}} + I_{g2}$$

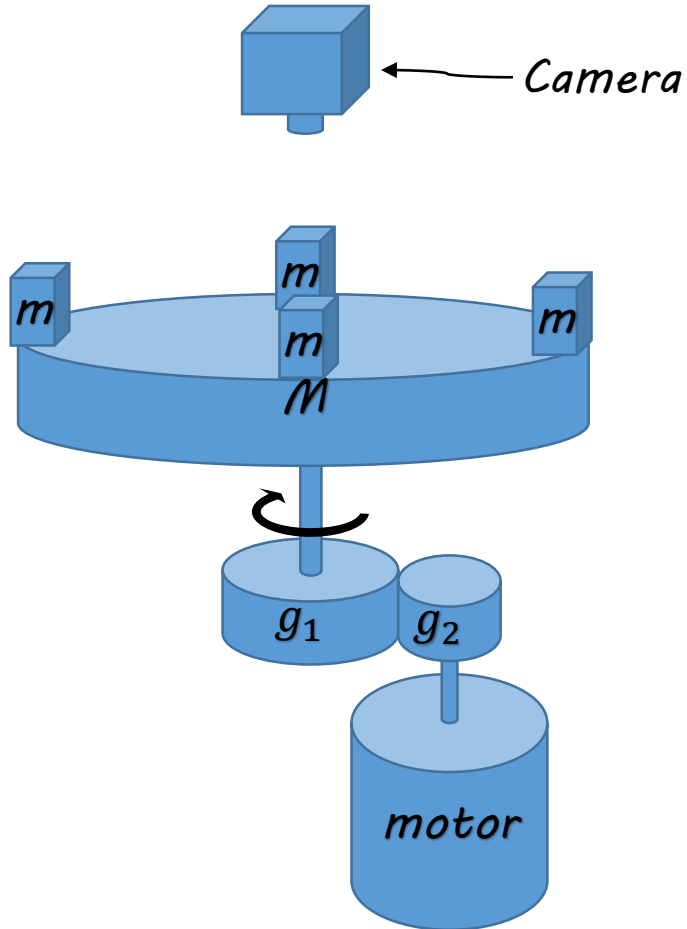
$$I_{\text{sys}} = I_2 + \left(\frac{g_2}{g_1}\right)^2 I_1$$

$$= I_{\text{motor}} + I_{g2} + \left(\frac{g_2}{g_1}\right)^2 \left(\frac{1}{2} M r^2 + 4(m r^2) + I_{g1} \right)$$

$$= 0.01 + 9.4 \times 10^{-4} + \left(\frac{1}{2}\right)^2 \left(\frac{1}{2} 10(4)^2 + 4(2(4)^2) + 75 \times 10^{-4} \right)$$

$$= 52.011 \text{ kg.m}^2$$

Calculating motor torque



Example: The rotating solid disc has a radius 4 m and mass $M = 10\text{ kg}$ requires to stop each $\frac{1}{4}$ revolution to place an object m under camera inspection. Four small ($m = 2\text{ kg}$) objects are placed at the edge of the disc. The system is powered by a servo motor by a gear ratio $\frac{g_2}{g_1} = \frac{1}{2}$. Calculate a) the system's moment of inertia around the motor's rotating shaft, b) the required motor torque if the disc's angular acceleration $\alpha_{\text{disc}} = 2\text{ rad/s}^2$. System efficiency is 100%.

$$\tau_m = I_{\text{sys}} \alpha_m$$

$$\tau_m = I_{\text{sys}} \alpha_m = \frac{52.011 \times 4}{\text{efficiency}}$$

Diagram of the gear train showing two gears, g_1 and g_2 , in mesh. Gear g_1 has radius r_1 and angular acceleration α_{disc} . Gear g_2 has radius r_2 and angular acceleration α_m . The gear ratio is given as $\frac{g_2}{g_1} = \frac{1}{2}$.

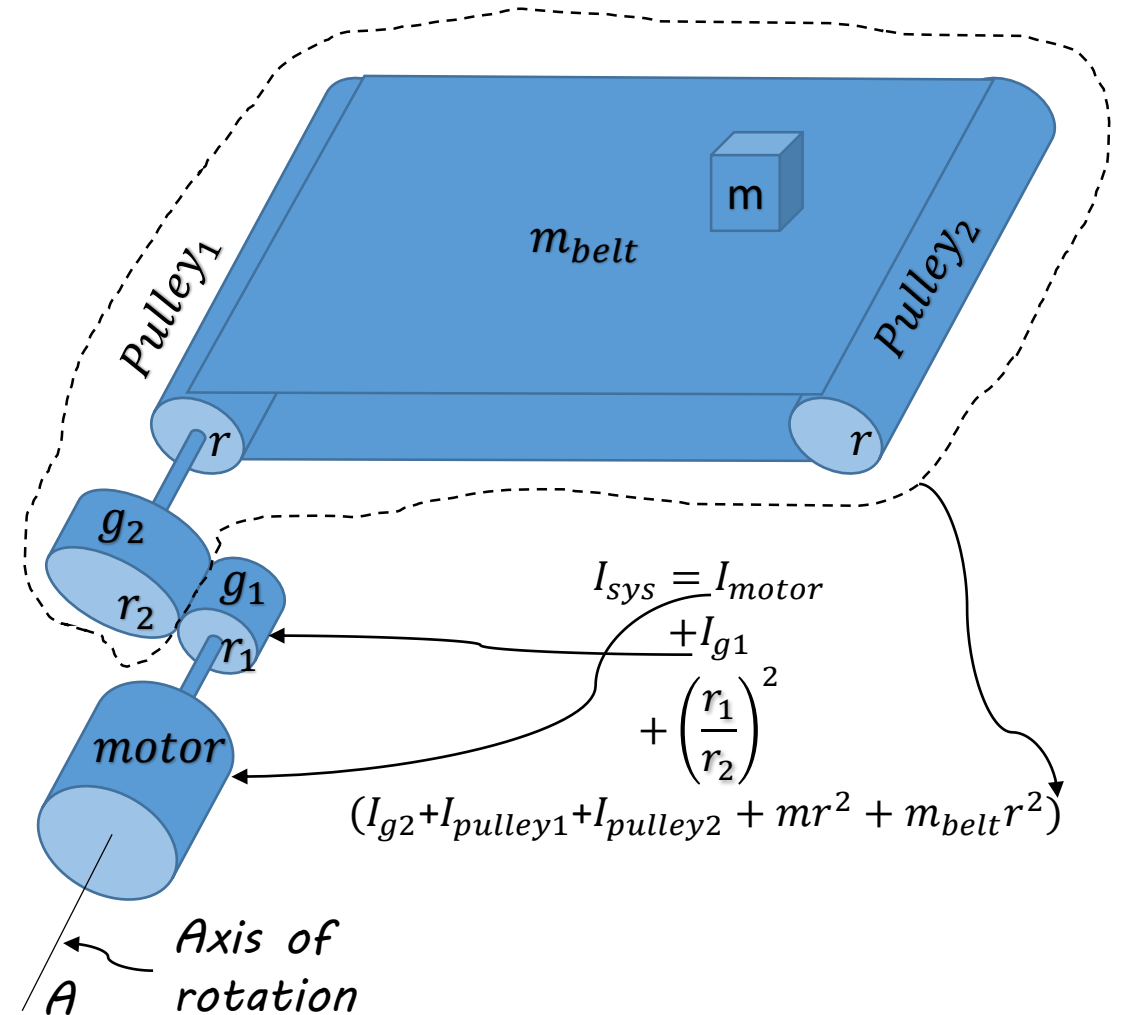
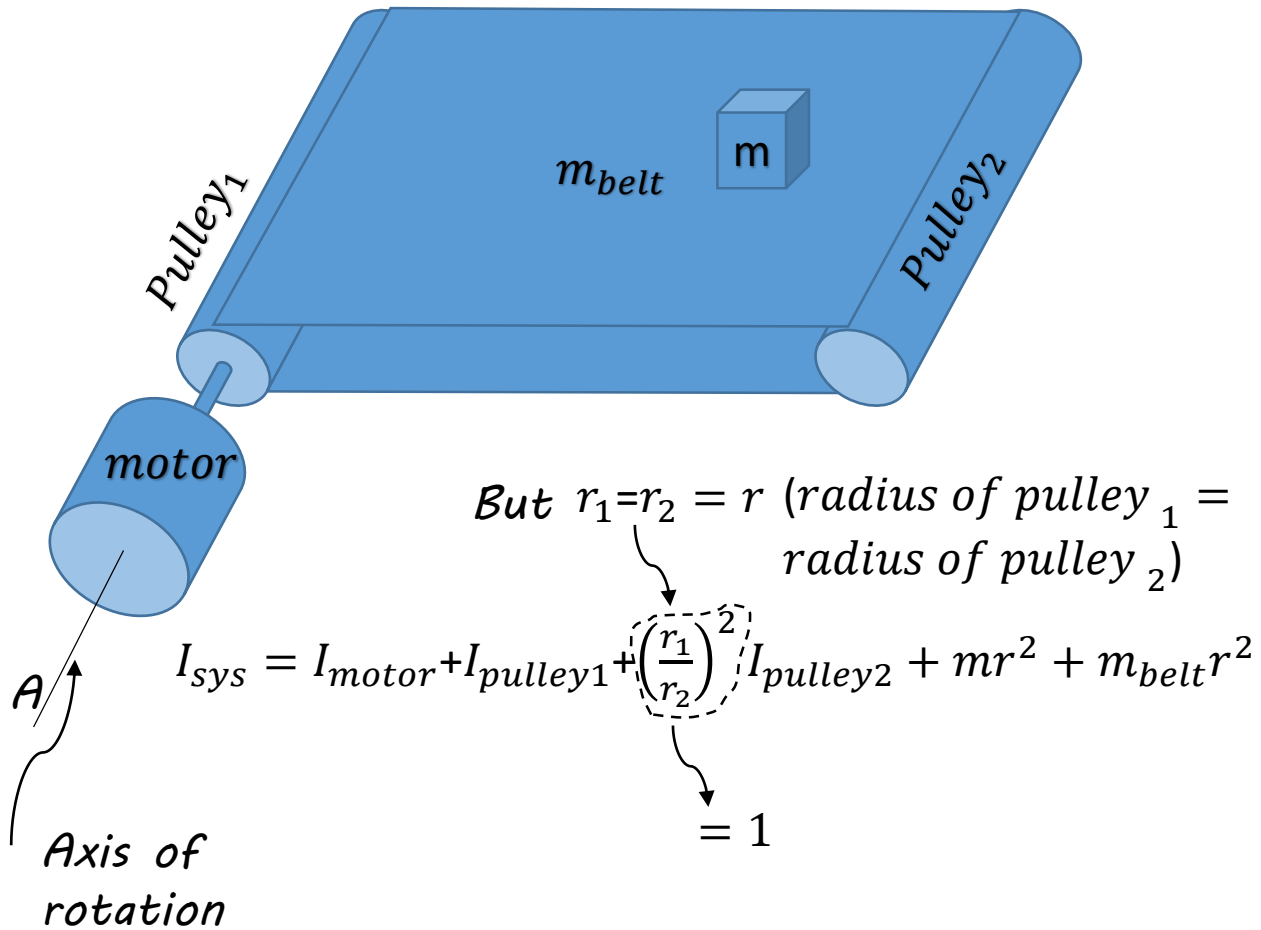
$$r_2 \alpha_m = r_1 \alpha_{\text{disc}}$$

$$\alpha_{\text{disc}} = \left(\frac{r_2}{r_1} \right) \alpha_m$$

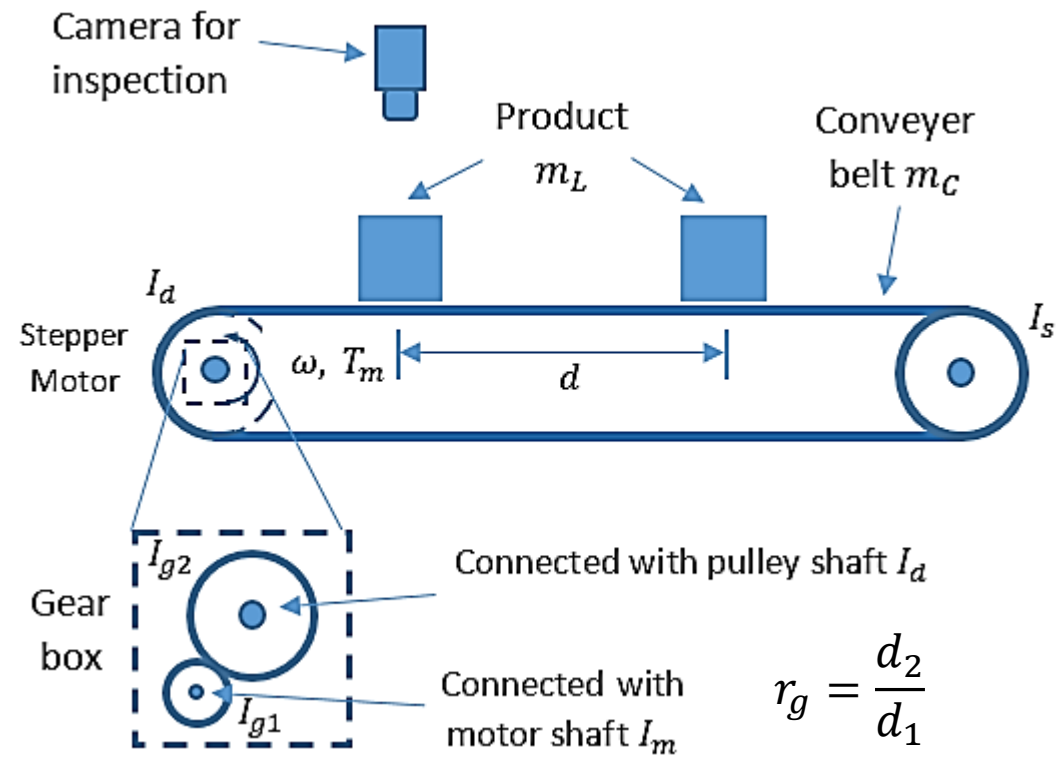
$$\frac{g_2}{g_1} = \frac{1}{2}$$

$$\alpha_m = 2 \alpha_{\text{disc}} = 4\text{ rad/s}^2$$

Tangentially driven load (conveyor belt)



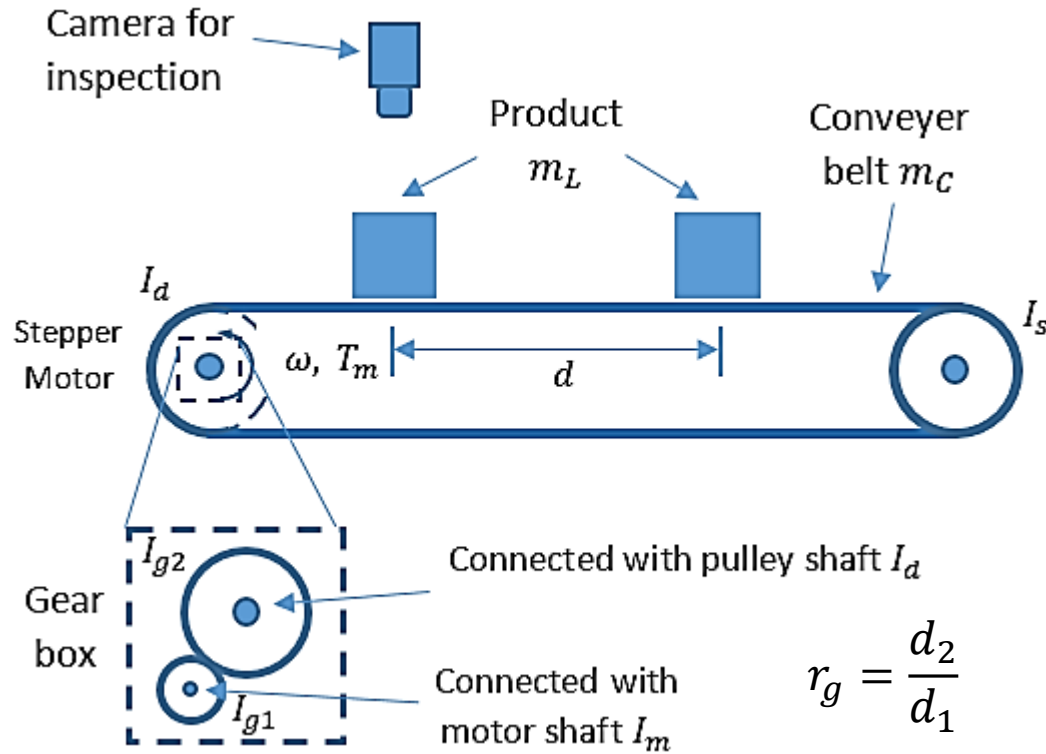
conveyor belt system



$$I_{all} = I_{motor} + I_{g1} + \left(\frac{r_1}{r_2}\right)^2 (I_{g2} + I_d + I_s + m_L r^2 + m_c r^2)$$

$$I_{all} = I_m + I_{g1} + \frac{I_{g2} + I_d + I_s}{r_g^2} + \left(\frac{\frac{d_s}{2}}{r_g}\right)^2 (m_c + m_L) = ??$$

conveyor belt system



Parameter	Value	Unit	Description
I_m	see table below	$\text{kg} \cdot \text{m}^2$	Inertia of the rotor of the stepper motor
I_{g1}	50	$\mu\text{kg} \cdot \text{m}^2$	Inertia of the high speed shaft of the gearbox
I_{g2}	200	$\mu\text{kg} \cdot \text{m}^2$	Inertia of the low speed shaft of the gearbox
I_d	2.0	$\text{mkg} \cdot \text{m}^2$	Inertia of the driving pulley of the conveyor
I_s	2.0	$\text{mkg} \cdot \text{m}^2$	Inertia of the idler pulley of the conveyor
m_C	5.0	kg	Mass of the conveyor belt
m_L	5.0	kg	Mass of the objects on the conveyor
d_s	0.2	m	Diameter of the driving pulley of the conveyor
d	0.1	m	Distance to be advanced
r_g	1:2	-	Gear reduction ratio
T	0.2	s	Time during which to advance
η	80%	-	Overall system efficiency

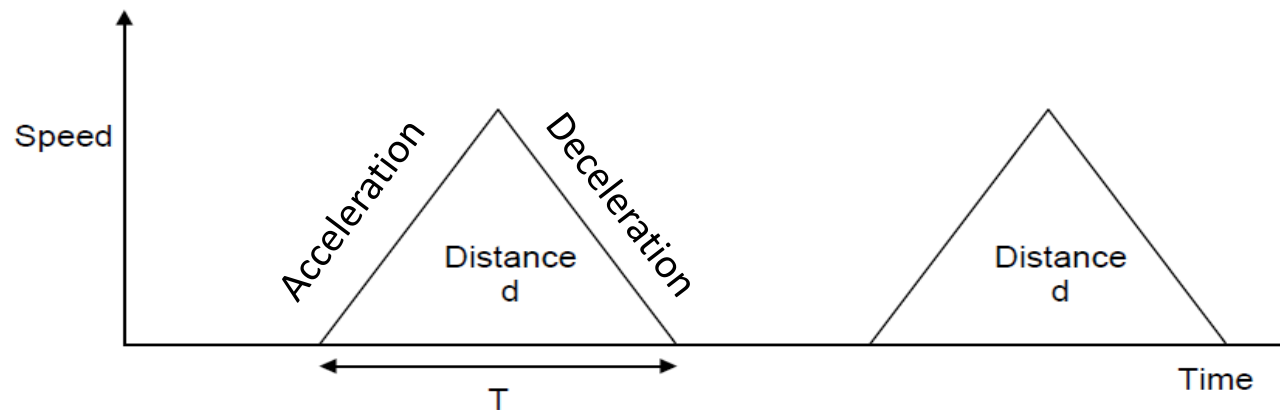
Calculate the system moment of Inertia based on values given in the table above.

$$I_{all} = I_m + I_{g1} + \frac{I_{g2} + I_d + I_s}{r_g^2} + \left(\frac{\frac{d_s}{2}}{r_g}\right)^2 (m_C + m_L) = ??$$

$$I_{all} = I_m + 50 \times 10^{-6} + \frac{200 \times 10^{-6} + 2 \times 10^{-3} + 2 \times 10^{-3}}{2^2} + \left(\frac{\frac{0.2}{2}}{2}\right)^2 (5 + 5) = I_m + 50 \times 10^{-6} + 1050 \times 10^{-6} + 0.025 = I_m + 0.0261 \text{ kg} \cdot \text{m}^2$$

Stepper motor for a conveyor belt

- The products on the conveyor have to stop a while under the camera for inspection separated by a fixed distance d regularly in a specified period of time T .
- In order to achieve this in the specified time, the conveyor has to be accelerated and decelerated to reset at a constant rate until it covers the distance d .
- The values of acceleration and deceleration are equal.



Stepper motor selection for conveyor

- The table below shows four motor models with each motor's available torque (holding torque) and its motor inertia. We need to select a motor suitable to drive the conveyor in

Motor Model	Available Torque (N·m) at ω_{max}	Motor rotor inertia (kg·m ²)
50 SM	0.25	11.8×10^{-6}
101 SM	0.58	35.0×10^{-6}
310 SM	2.63	187.0×10^{-6}
1010 SM	7.41	805.0×10^{-6}

- We need first to calculate the required acceleration.
- The maximum speed based on the operation is given by $\frac{v_{max}-0}{2} = \frac{\text{distance } (d) / 2}{\text{time } (T) / 2} = \frac{0.1/2}{0.2/2} \rightarrow v_{max} = 1 \text{ m/s}$
- The acceleration is $a = v_{max} / (\frac{T}{2}) = 1/0.1 = 10 \text{ m/s}^2$
- The stepper motor will be selected to overcome the accelerating conveyor with its load.
- The required value of angular acceleration at the motor shaft is $\alpha = r_g \times \frac{a}{\frac{d_s}{2}} = 2 \times \frac{10}{0.1} = 200 \text{ rad/s}^2$

Torque and Inertia calculations

- $\sum T = I_{all} \cdot \alpha \rightarrow T_m = I_{all} \cdot \alpha$
- *We next need to calculate the equivalent inertia of the whole system at the high speed shaft (motor shaft).*
- $I_{all} = I_m + I_{g1} + \frac{I_{g2} + I_d + I_s}{r_g^2} + \left(\frac{\frac{d_s}{2}}{r_g}\right)^2 (m_C + m_L) = (I_m + 0.0261) \text{ kg.m}^2$
- *Which motor (from the table) will be fit for this system ?*
- $T_m = I_{all} \cdot \alpha = (I_m + 0.0261) \times 200$
- *But because of frictions and losses we consider system efficiency equals to 80% so:*

$$T_m = (I_m + 0.0261) \times 200 / 0.8 = 250 \times I_m + 6.525 = 6.72 \text{ N.m}$$