

Accelerometer

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Acceleration Fundamentals

- *What is Acceleration?*

- *Definition: the time rate of change of velocity*

- $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

What are the units?—Acceleration is measured in (ft/s²) or (m/s²)

What is a “g”?—A “g” is a unit of acceleration equal to Earth’s gravity at sea level 9.81m/s².

The table below gives an indication of some “g” reference point

Description	“g” level
Earth’s gravity	1g
Passenger car in corner	2g
Bumps in road	2g
Space shuttle	10g

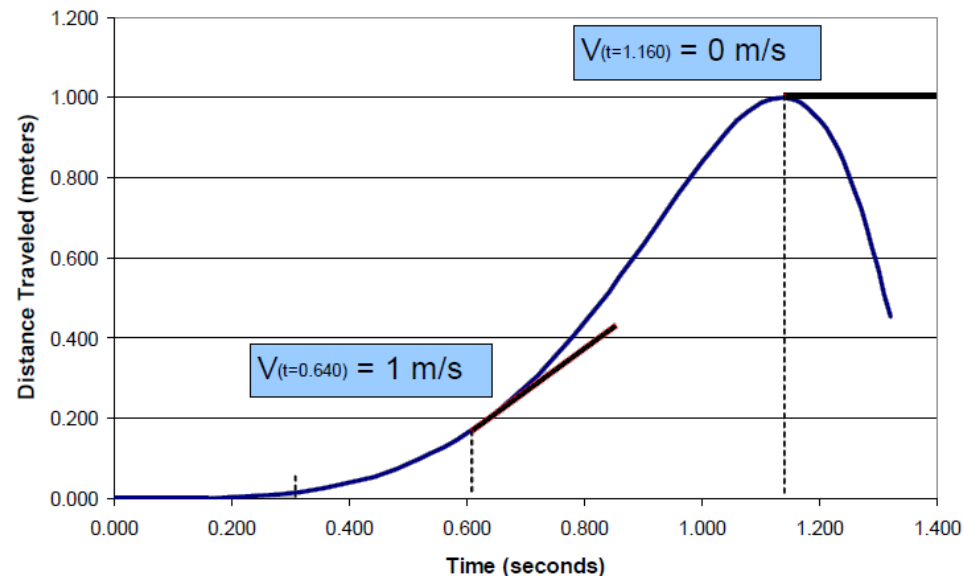
Distance, Velocity, and Acceleration

What is the time rate of change of velocity?

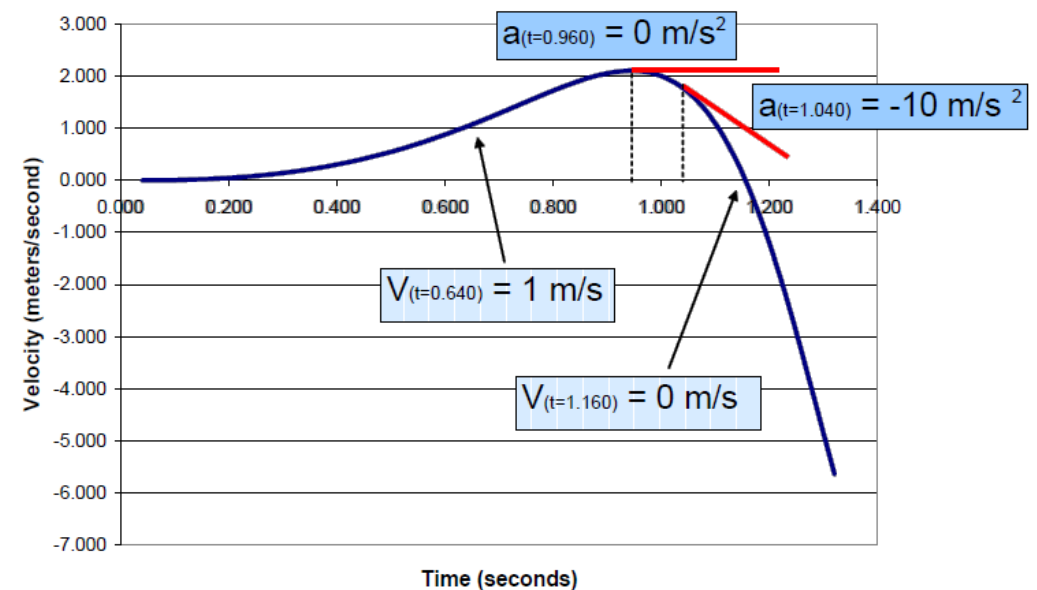
*When plotted on a graph, velocity is the **slope** of distance versus time.*

*Acceleration is the **slope** of velocity versus time.*

How to find velocity from distance traveled



How to find acceleration from velocity



Type of accelerometers

Sensor Category	Key Technologies
Capacitive	Metal beam or micro machined feature produces capacitance; change in capacitance related to acceleration
Piezoelectric	Piezoelectric crystal mounted to mass -voltage output converted to acceleration
Piezoresistive	Beam or micro machined feature whose resistance changes with acceleration
Hall Effect	Motion converted to electrical signal by sensing of changing magnetic fields
Magnetoresistive	Material resistivity changes in presence of magnetic field
Heat Transfer	Location of heated mass tracked during acceleration by sensing temperature

- The most common produced acceleration sensor based on the magnitude of cost and performance is the **Capacitive Acceleration Sensor (CAS)**.

MEMS Capacitive accelerometer

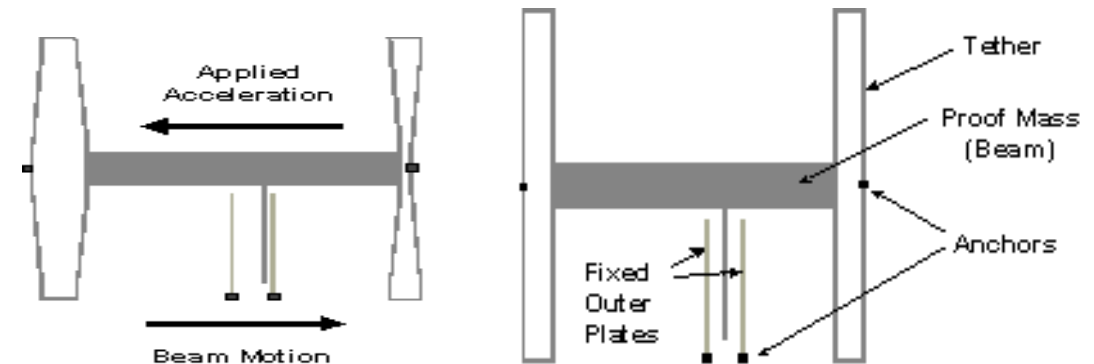
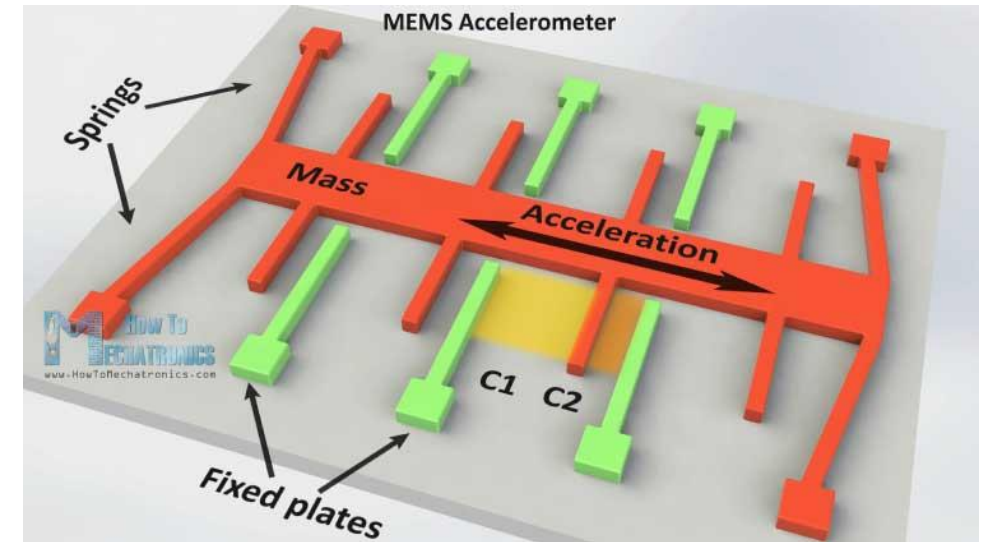
*The basic principle of operation behind the MEMS accelerometer is the displacement of a small **proof mass**.*

When acceleration is applied to the device, a force develops which moves the mass.

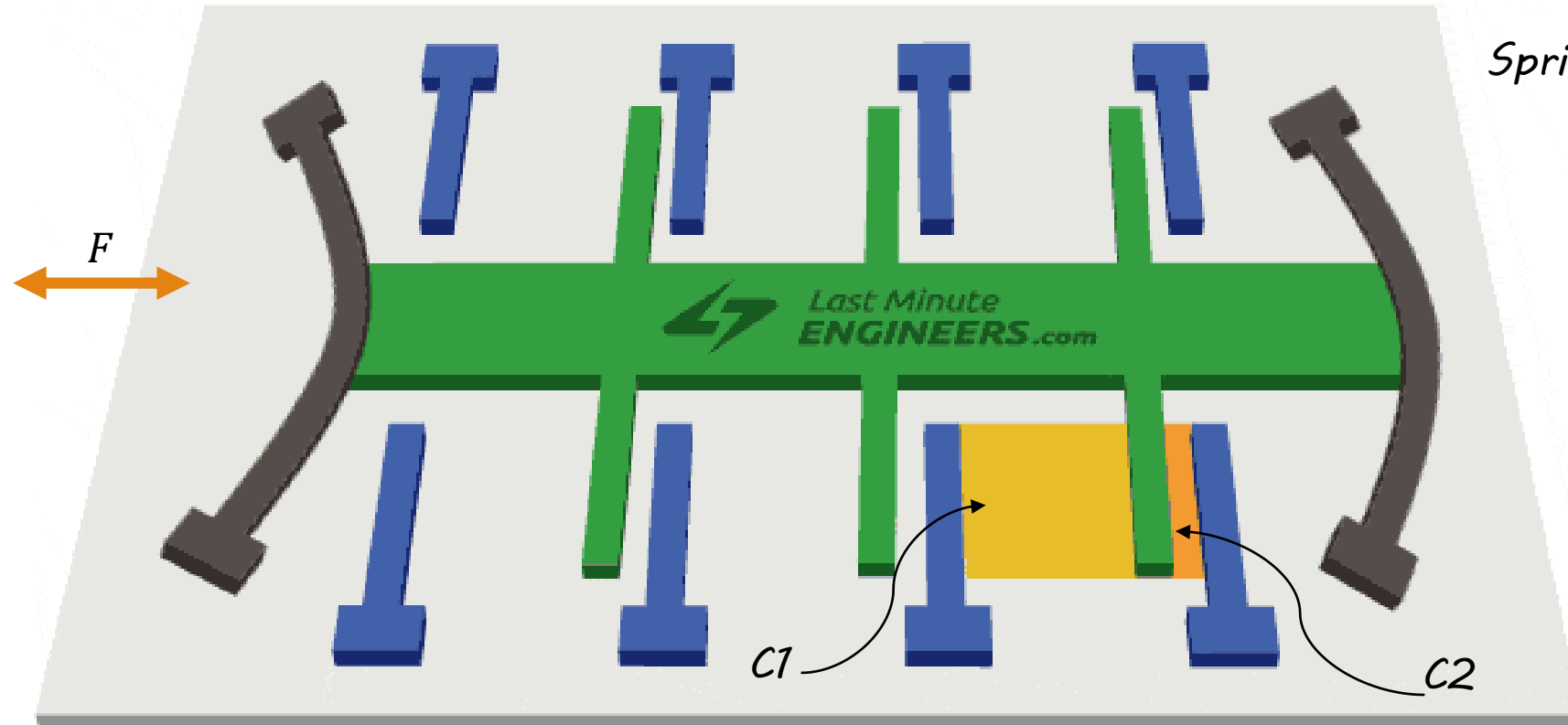
The support beams act as a spring.

The fluid (usually air) inside the IC acts as a damper.

The plates are charged with an electrical current. Changing the gap between the plates changes the electrical capacity of the system, which can be measured as a voltage output. The value is processed and it will correspond to a particular acceleration value



MEMS Capacitive accelerometer



Spring Mass damper system

$$F = ma + kx + b\dot{x}$$

Piezo-electric accelerometer

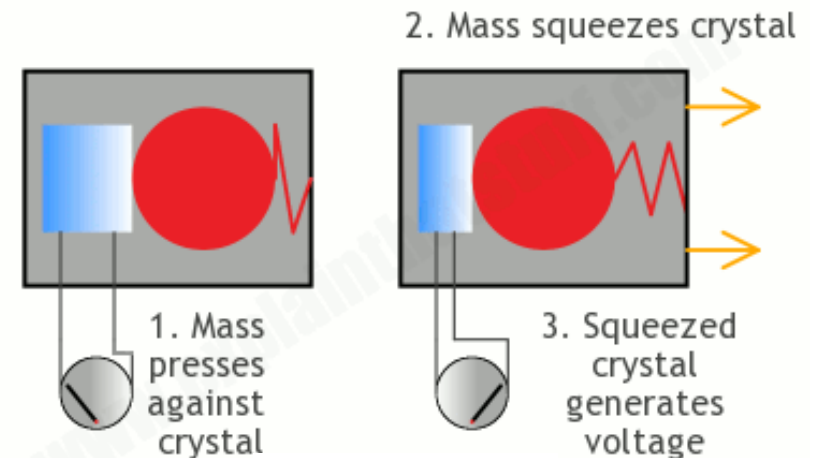
When certain types of crystal are compressed, charges of opposite polarity accumulate on opposite sides of the crystal.

The charge accumulates on the crystal and is translated and amplified into either an output current or voltage.

Piezoelectric accelerometers only respond vibration or shock.

They have a wide dynamic range, but can be **expensive**.

It measures the **Force**, then calculates the acceleration by $a = F/m$



$$F = Ma$$

$$Q = dF = dMa$$

$$V = \frac{dF}{C} = \frac{dMa}{C}$$

Q: a charge developed by the crystal

C: The capacitance of the piezoelectric material

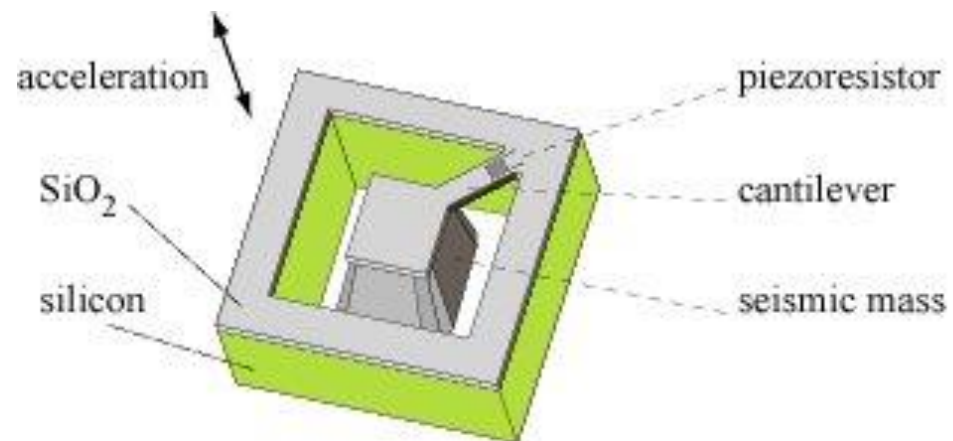
Piezoresistive accelerometers

Known as Strain gauge accelerometers.

Work by measuring the electrical resistance of a material when mechanical stress is applied.

They are preferred in high shock applications and they can measure acceleration down to 0 Hz.

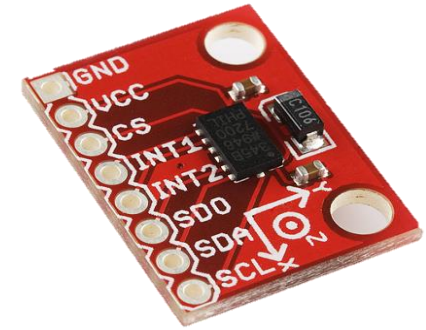
They have a limited high frequency response.



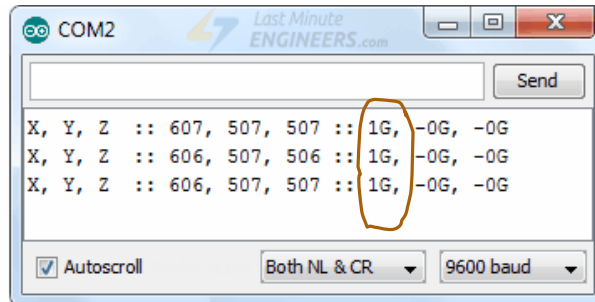
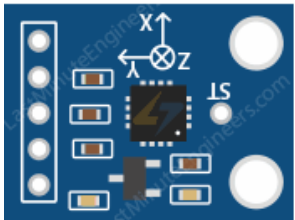
Acceleration Sensor Terminology

The accelerometer gives values according to its direction.

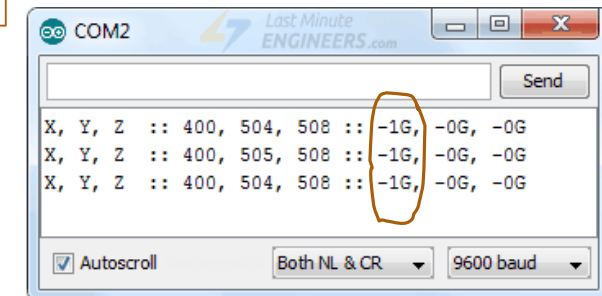
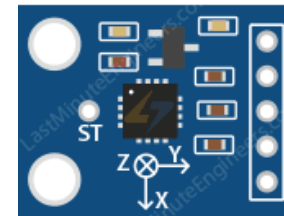
- **+1g**: Output of the sensor with the base connector pointed up
- **0g**: Output of the sensor with the base connector horizontal
- **-1g**: Output of the sensor with the base connector pointed down



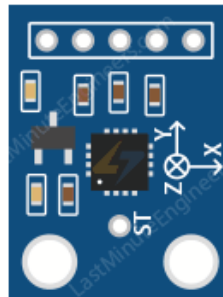
Accelerometer x up



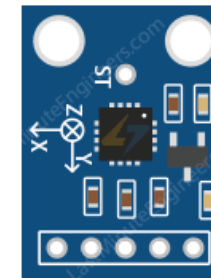
Accelerometer x Down



Accelerometer y up



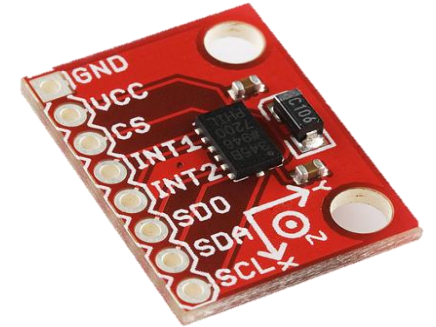
Accelerometer y Down



Acceleration Sensor Terminology

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- **+1g:** Output of the sensor with the base connector pointed up
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Linearity: The maximum deviation of the calibration curve from a straight line.

$$\text{Linearity} = V_{out,0g} - \frac{1}{2}(V_{out,+1g} + V_{out,-1g})$$

Sensitivity: A measure of how much the output of a sensor changes as the input acceleration changes. Measured in Volts/g.

$$\text{Sensitivity} = \frac{\Delta V_{out}}{\Delta g} = \frac{V_{out,+1g} - V_{out,-1g}}{2g}$$

Acceleration Sensor Terminology

V_{cc}: The voltage supplied to the input of the sensor

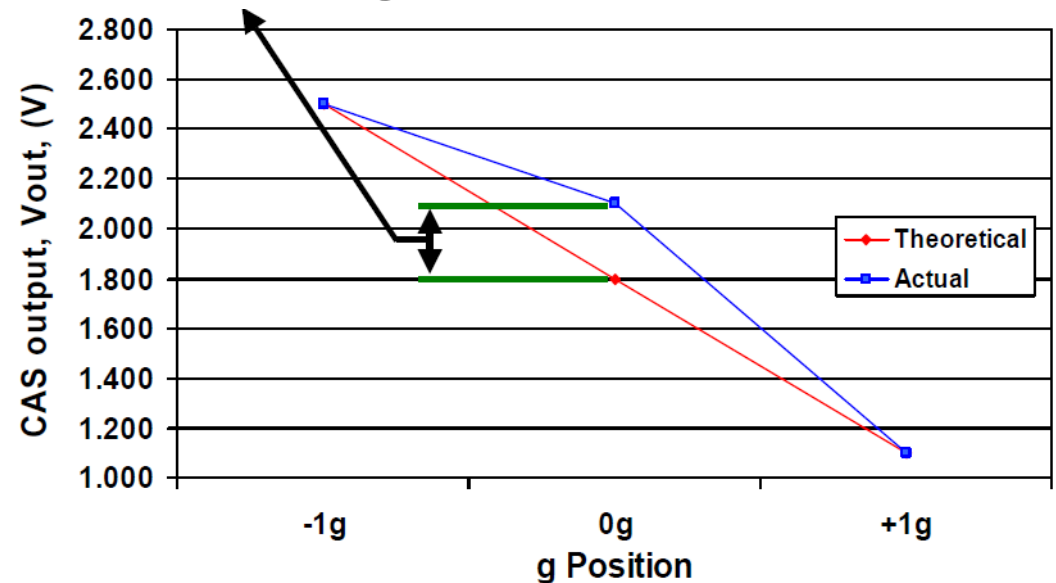
−5.000 ± 0.005V for CAS device

% V_{cc}: Readings are often represented as a % of the supply voltage. This allows for correction due to supply voltage variances between readings.

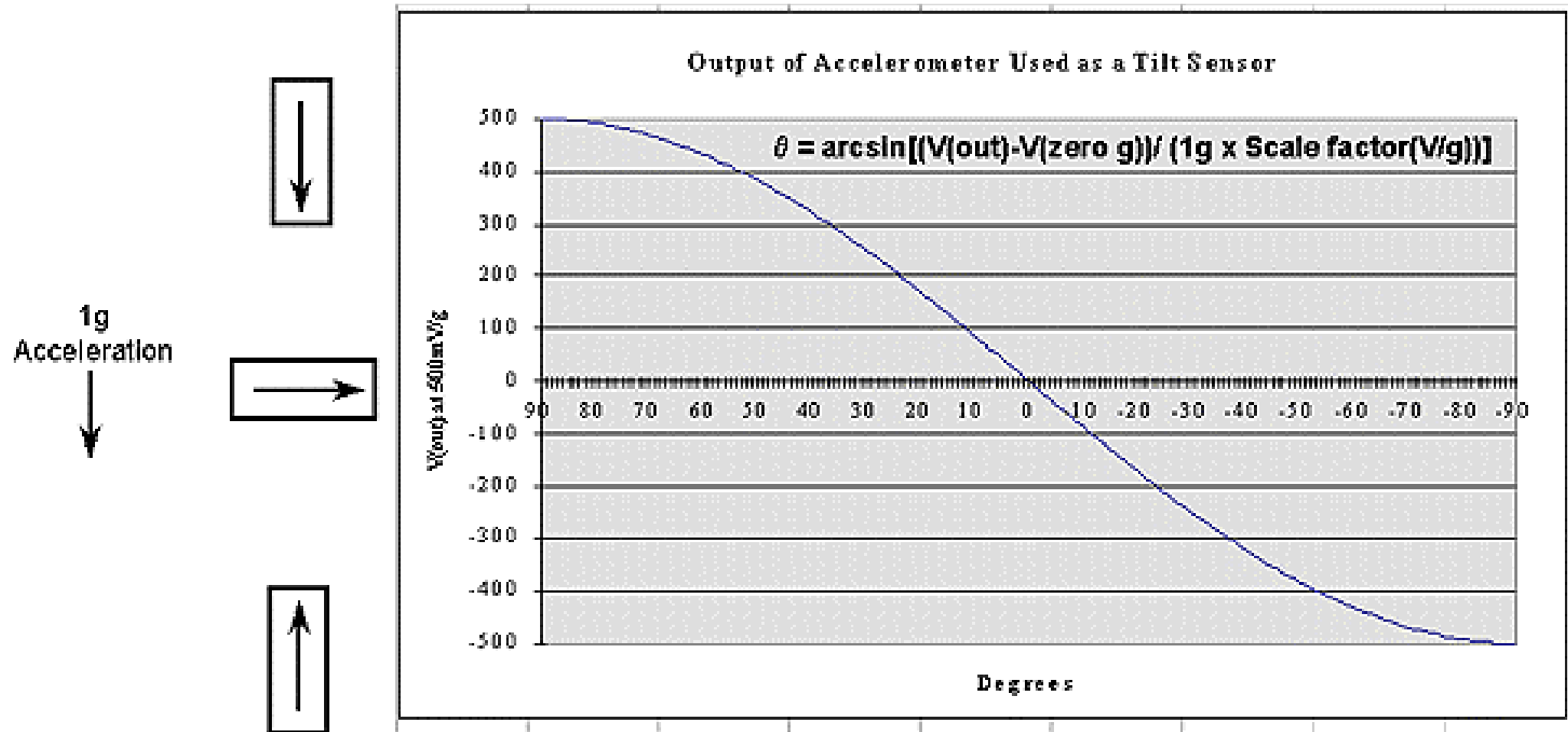
Ex: Sensitivity & Linearity

$$\text{Sensitivity} = \frac{\Delta V_{out}}{\Delta g} = \frac{V_{out,+1g} - V_{out,-1g}}{2g} = \frac{1.1V - 2.5V}{2g} = \frac{1.1V - 2.5V}{2g} = -0.7 \frac{\text{Volts}}{g}$$

$$\text{Linearity} = V_{out,0g} - \frac{1}{2}(V_{out,+1g} + V_{out,-1g}) = 2.1 - \frac{1}{2}(1.1 + 2.5) = 0.3 \text{Volts}$$



Effect of Tilt on DC Accelerometer



Typical Accelerometer Applications

Tilt / Roll

Vibration / “Rough-road” detection

Can be used to isolate vibration of mechanical system from outside sources

Vehicle skid detection

Often used with systems that deploy “smart” braking to regain control of vehicle

Impact detection

To determine the severity of impact, or to log when an impact has occurred

Input / feedback for active suspension control systems

Keeps vehicle level

Tilt angle derivation

- The orientation of the smartphone can be defined by its roll, pitch and yaw rotations from an initial position
- The roll, pitch and yaw rotation matrices, which transform a vector (such as the earth's gravitational field vector g) under a rotation of the coordinate system of Figure by angles ϕ in roll, θ in pitch and ψ in yaw about the x , y and z axes respectively, are:

$$R_x(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{pmatrix} \quad R_y(\theta) = \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix}$$

$$R_z(\psi) = \begin{pmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



Tilt angles by accelerometer measurements

The accelerometer output G_p (measured in the native accelerometer units of g) is:

$$G_p = \begin{pmatrix} G_{px} \\ G_{py} \\ G_{pz} \end{pmatrix} = Rg = R \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} g$$

- Where G_p are the accelerometer measurements.
- R is the rotation matrix and can be expressed by: $R = R_{yxz} = R_x(\phi)R_y(\theta)R_z(\psi)$

$$\frac{G_p}{\|G_p\|} = \frac{1}{\sqrt{G_{px}^2 + G_{py}^2 + G_{pz}^2}} \begin{pmatrix} G_{px} \\ G_{py} \\ G_{pz} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix} \begin{pmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Solving for the Pitch and Roll Angles

$$= \begin{pmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{pmatrix}$$

$$\frac{\mathbf{G}_p}{\|\mathbf{G}_p\|} = \begin{pmatrix} -\sin \theta \\ \cos \theta \sin \phi \\ \cos \theta \cos \phi \end{pmatrix} \Rightarrow \frac{1}{\sqrt{G_{px}^2 + G_{py}^2 + G_{pz}^2}} \begin{pmatrix} G_{px} \\ G_{py} \\ G_{pz} \end{pmatrix} = \begin{pmatrix} -\sin \theta \\ \cos \theta \sin \phi \\ \cos \theta \cos \phi \end{pmatrix}$$

$$\frac{\cos \theta \sin \phi}{\cos \theta \cos \phi} = \tan \phi = \frac{G_{py}}{G_{pz}}$$

$$\frac{-\sin \theta}{\cos \theta \sin \phi} = \frac{G_{px}}{G_{py}} \Rightarrow \tan \theta = \frac{-G_{px} \sin \phi}{G_{py}}$$

Note: The yaw angle ψ represents the smartphone rotation from north around z axis but its determination requires the addition of a magnetometer sensor.

Working Example

Determine the roll and pitch angles defining the orientation of the smartphone relative to the starting position with the smartphone flat on the table assuming the aerospace rotation sequence R_{xyz} for the smartphone accelerometer reading :

$$G_p = \begin{bmatrix} -0.3243 \\ 0.5010 \\ 9.5609 \end{bmatrix}$$

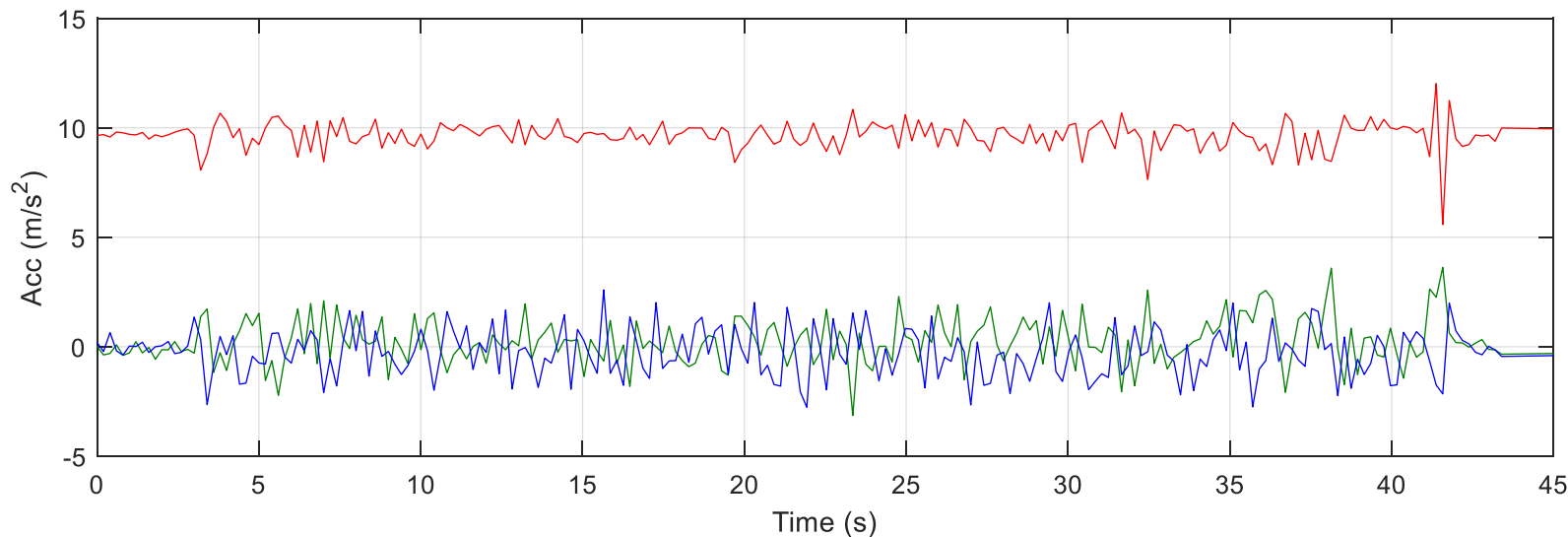
Solution:

$$\tan \phi = \frac{G_{py}}{G_{pz}} = \frac{0.501}{9.5609} = 0.052353 \text{ rad} \Rightarrow \phi = 3^\circ$$

$$\tan \theta = \frac{-G_{px} \sin \phi}{G_{py}} = \frac{0.3243 \sin 0.052353}{9.5609} = 0.001775 \text{ rad} \Rightarrow \theta = 0.102^\circ$$

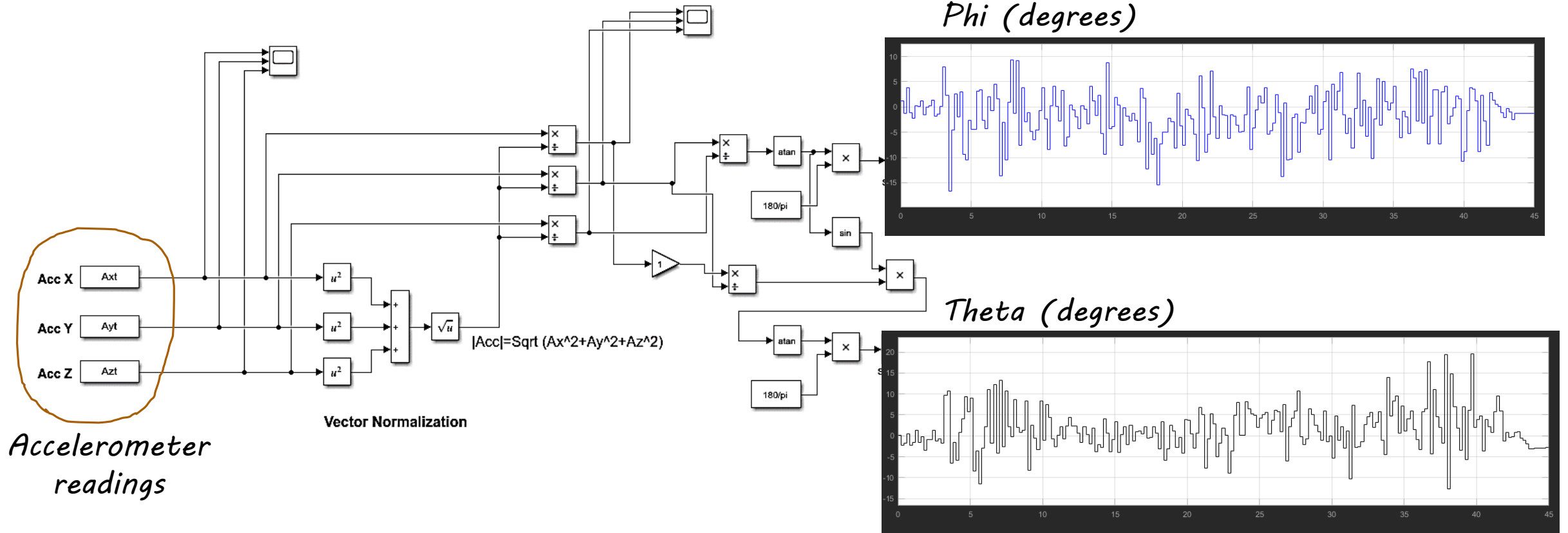
Real-time tilt angles in matlab

- In this example a real-time 3-axis accelerometer data are used in a quadcopter for rotation angles calculation.
- 3-axis acceleration readings are provided with time.
- The accelerometer readings are updated at 25 Hz (25 readings every 1 second).



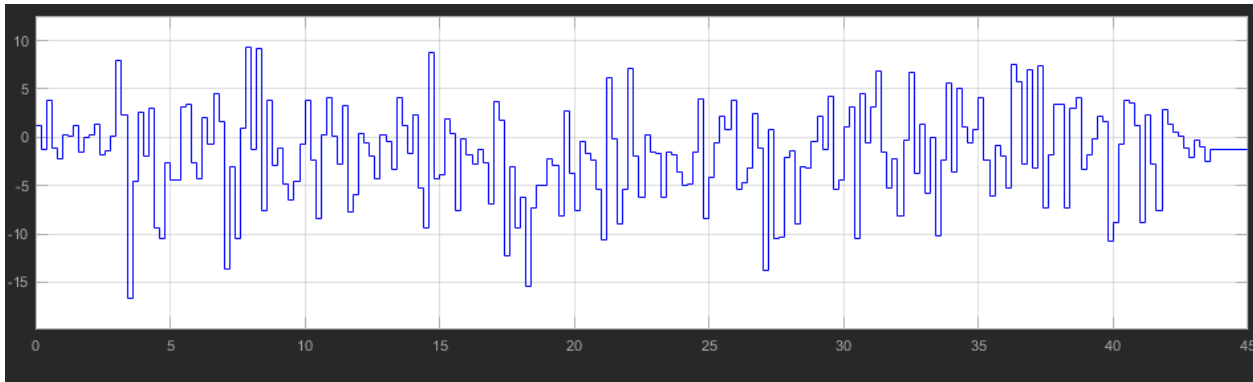
	Time	Acc X	Acc Y	Acc Z
1	0	0.0143	0.1936	9.6454
2	0.0405	-0.1024	-0.0892	9.8447
3	0.0801	-0.1569	-0.1237	10.0629
4	0.1205	-0.2826	-0.1979	10.0894
5	0.1599	-0.3390	-0.2851	9.8578
6	0.1999	-0.3753	-0.2165	9.6895
7	0.2404	-0.4139	-0.0736	9.5458
8	0.2800	-0.4026	0.1242	9.5592
9	0.3203	-0.4137	0.2931	9.5474
10	0.3599	-0.3243	0.5010	9.5609
11	0.4001	-0.3067	0.6469	9.5790
12	0.4405	-0.2246	0.5448	9.5481
13	0.4800	-0.0671	0.4658	9.6097
14	0.5204	0.0051	0.1948	9.7069
15	0.5599	0.0559	-0.0162	9.8033
16	0.6000	0.0860	-0.1906	9.8046
17	0.6407	0.0634	-0.3507	9.7851
18	0.6799	-0.0377	-0.3115	9.7961
19	0.7202	-0.0928	-0.3631	9.8100
20	0.7602	-0.2556	-0.4510	9.7905
21	0.8002	-0.3966	-0.3845	9.7688
22	0.8403	-0.4181	-0.2563	9.7083
23	0.8800	-0.5482	0.0236	9.6294

Matlab model to find tilt angles

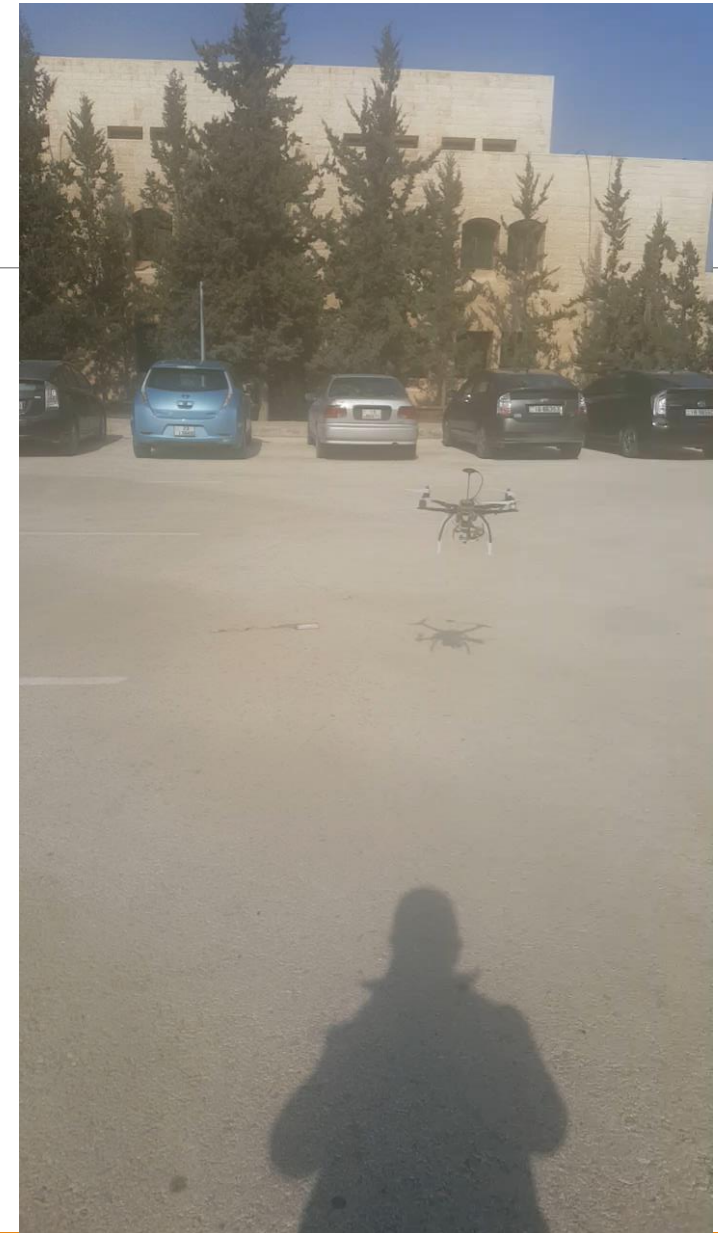
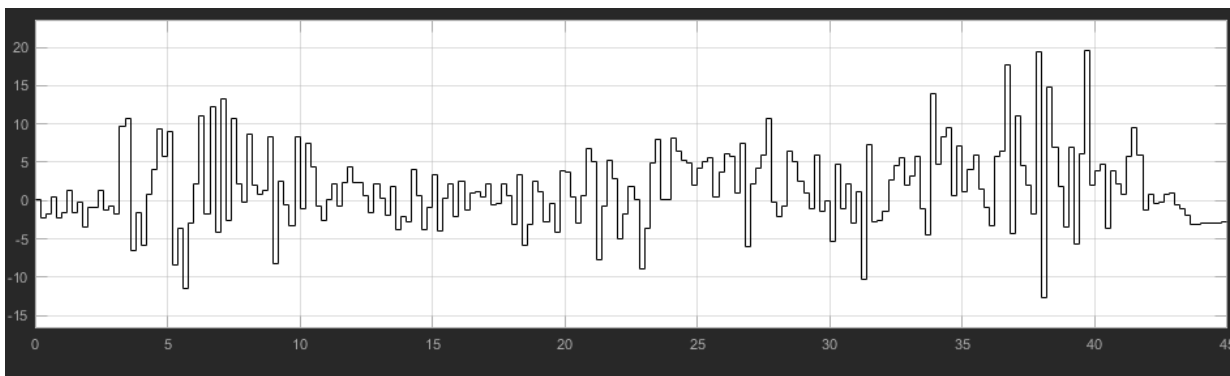


Tilt angles in drone

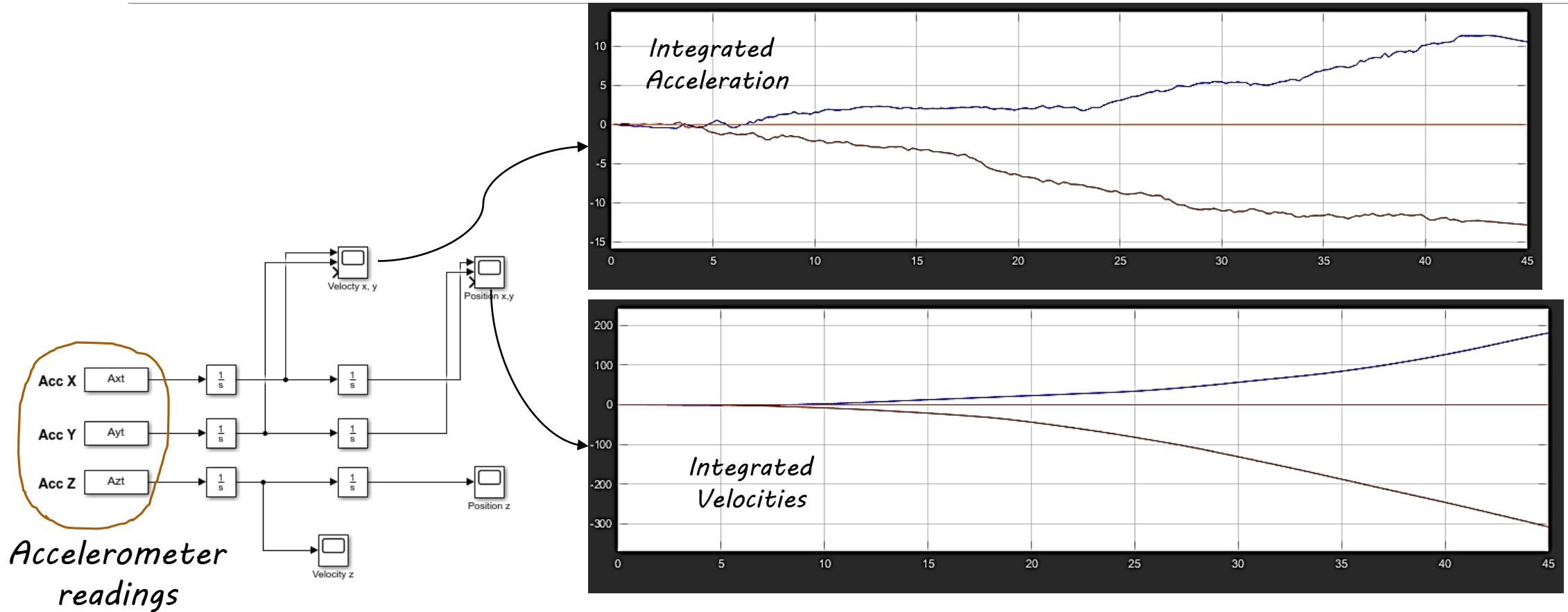
Phi (degrees)



Theta (degrees)

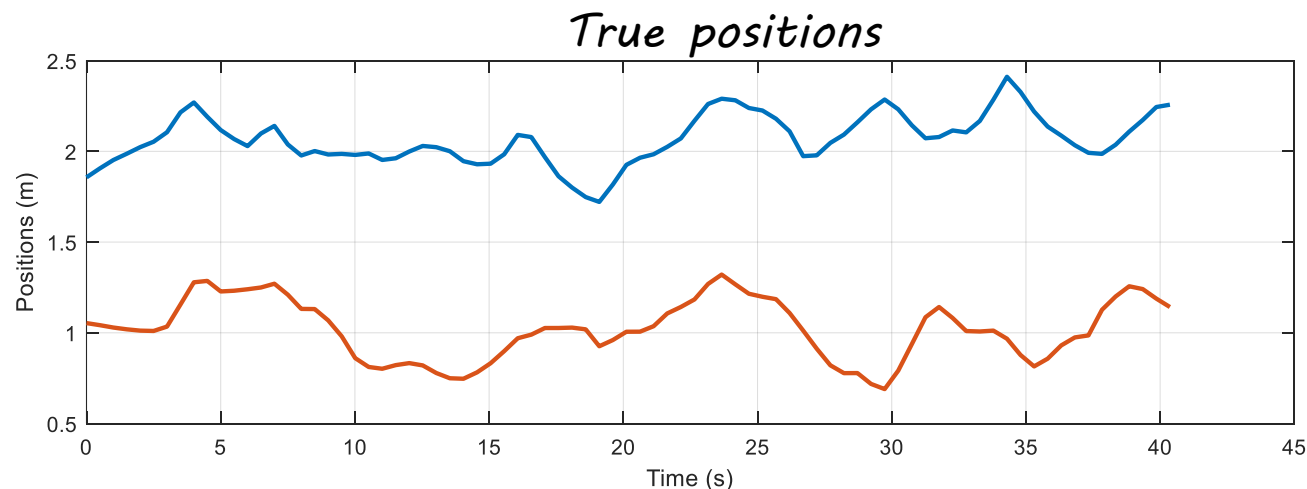


Accelerometer to calculate positions



Accelerometer to calculate positions

- From previous figures, the first and second integrations of the accelerometer readings show divergence in velocities and positions calculations.
- Figure below shows the true positions (xy) of the drone during the flight.
- Using Accelerometer data without filtration process is incorrect solution in position estimation.
- Accelerometer needs calibration process for better positions and rotations angles estimations.



Sensor calibration

In some cases calibration is not necessary, such as

- *Screen portrait/landscape rotation.*
- *Laptop lid open/close detection.*

Accelerometer calibration is recommended for applications that require better tilt-measurement accuracy, such as

- *Automobile alert systems.*
- *Tilt-compensated electronic compasses.*
- *Level monitoring systems.*

Accelerometer model

In practice, accelerometers are influenced by bias, drift and noises in measurements. Thus the measurement signals of a 3-axis accelerometer can be modeled as:

$$a_m = (IS)a + b + \mu$$

$$\begin{bmatrix} a_{mx} \\ a_{my} \\ a_{mz} \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} + \begin{bmatrix} \mu_x \\ \mu_y \\ \mu_z \end{bmatrix}$$

Where:

a_m : Raw measurements of the accelerometer

a : True values of acceleration in each axis.

b : Bias error (offset)

S : Scaling error

μ : Zero-mean Gaussian white noise

Calibration procedure

The bias errors and the scaling factors in inertial sensor can be eliminated with standard static calibration process such as six-position static test.

This method requires static data collection of long period at each axis of the sensor pointing alternately up and down.

$$\begin{cases} b_{axis} = \frac{\bar{M}^{up} + \bar{M}^{down}}{2} \\ S_{axis} = 1 - \frac{\bar{M}^{up} + \bar{M}^{down}}{2K} \end{cases} \quad axis = x, y, z$$

where \bar{M} is average value of the measurement obtained by an accelerometer or a gyroscope by pointing each sensitive axis alternately up and down

$K = g$ refers to the gravitational constant.

Calibration procedure

- 1) Orient Accelerometer +ve x axis toward up and hold to read the axis readings
- 2) Read and save samples of 2 seconds (let us say 50 samples).
- 3) Find the average of them ($\bar{M}^{up} = \frac{\sum_{i=1}^{50} Ax(i)}{50} = -9.6890$).
- 4) Repeat 1 to 3 for -ve x axis to find $\bar{M}^{Down} = 9.9390$.
- 5) Calculate the axis bias $b_x = \frac{-9.689+9.939}{2} = 0.125$.
- 6) Calculate the slope $S_x = 1 - \frac{-9.689+9.939}{2 (9.82)} = 0.9873$.
- 7) The new calibrated readings are then calculated $a_x = \frac{Ax-0.125}{0.9873}$.
- 8) Repeat from 1 to 7 for y axis and z axis.

$$\begin{cases} b_{axis} = \frac{\bar{M}^{up} + \bar{M}^{down}}{2} \\ S_{axis} = 1 + \frac{\bar{M}^{up} + \bar{M}^{down}}{2K} \end{cases}$$