تقدم لجنة ElCoM الاكاديمية

تلخيص لمادة:

معالجة اشارات

جزيل الشكر للطالبة:

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⇒ Euler's Relation :

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}, \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2}, \quad |e^{j\theta}| = \sqrt{\cos^2(\theta) + \sin^2(\theta)}$$

$$\arg(\theta) = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = \theta$$

* Three Cases for exponential function :

$$x(t) = Ce^{at}$$

① C and a are real numbers

- $a > 0$ the function called growing (inc)

unstable

- $a < 0$ the function called decaying (dec)

stable

- $a = 0$ the func. called marginal

بين ال stable و unstable

② C complex and a imaginary

$$\begin{bmatrix} C = Ae^{j\theta} \\ a = jW \end{bmatrix} \rightarrow x(t) = Ae^{j\theta} e^{jwt}$$

$$\boxed{x(t) = A\cos(\theta + wt) + jA\sin(\theta + wt)}$$

③ Both C and a are complex

$$\begin{bmatrix} C = Ae^{j\theta} \\ a = \sigma + jW \end{bmatrix} \rightarrow x(t) = Ae^{j\theta} e^{(\sigma + jw)t}$$

$$\boxed{x(t) = Ae^{\sigma t}[\cos(wt + \theta) + j\sin(wt + \theta)]}$$

Signals Transformation →

Time Transformation (التغير بيصير على ال x-axis)

- Scalling: $x(\tau) = x(at)$
- reversing: $x(\tau) = x(-t)$
- shifting: $x(\tau) = x(t \pm t_0)$

Amplitude Transformation (التغير بيصير على ال y-axis)

- Scalling: $x(\tau) = a\,x(t)$
- Reversing: $x(\tau) = -x(t)$
- shifting: $x(\tau) = x(t) \pm B$

← إذا كان السؤال بيجمع ال (Amp+time transfor.) ، ببلش بال time أولاً بعدين حق رسمها بغير على ال amplitude ، داضر إشي ..

بيرسلم فوق بعض

الأصلية → التغير على x → التغير على y

time. amp.

Invarient

7

varient with time

Ch.3 | Continuos time Linear time invarient system

$x_1(t \pm t_0) \longrightarrow \boxed{LTI} \longrightarrow y_1(t \pm t_0)$

$x_2(t \pm t_0) \longrightarrow \boxed{LTI} \longrightarrow y_2(t \pm t_0)$

$x_1(t \pm t_0) \pm x_2(t \pm t_0) \longrightarrow \boxed{LTI} \longrightarrow y_1(t \pm t_0) \pm y_2(t \pm t_0)$

* Impulse representation of continuos time signal :

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

* To find $h(t)$: $\delta(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$

impulse response

$$\boxed{y(t) = \int_{-\infty}^{\infty} \delta(t)\,h(t-t_0)\,dt}$$

* Convolution Properties ⇒

convolve sign

① $y(t) = h(t) * \delta(t) = \delta(t) * h(t) = h(t)$

② $y(t) = \int_{-\infty}^{\infty} \delta(t)\,h(t-t_0)\,dt = \int_{-\infty}^{\infty} \delta(t-t_0)\,h(t)\,dt = h(\tau)$ ⇐ [Convolution Integral]

③ $x(t) * \delta(t) = \delta(t) * x(t) = x(t)$, where $\boxed{x(t) : \text{impulse response}}$

④ $\delta(t \pm t_0) * x(t) = x(t \pm t_0)$, $\delta(t) * x(t \pm t_0) = x(t \pm t_0)$

*** ⑤ $\delta(t-t_0) * x(t-t_1) = x(t-t_0-t_1)$

Differential Equation model method of undetermined Coefficient

Ex Let $\frac{dy(t)}{dt} + 2y(t) = 2$, $y_0 = 4$, find $y(t)$?

$Y(t) = Y_c(t) + Y_p(t)$

natural response — forced response

تعويض من ③

$\boxed{Y_c(t) = ce^{st}}$

$\boxed{Y_p(t) = k}$

1. assume $\frac{dy_c(t)}{dt} + 2y_c(t) = 0$ ← ce^{st}

$sce^{st} + 2ce^{st} = 0 \rightarrow ce^{st}[2+s] = 0$

$\boxed{s=-2}$

$y_c(t) = ce^{-2t}$

2. $Y_p(t) = k$, [y كل لا تعويض]

$0 + 2k = 2$

$\boxed{k=1}$

$\therefore Y(t) = 3e^{-2t} + 1$ #

3. $Y(t) = ce^{-2t} + 1$

$4 = ce^{-2t} + 1 \rightarrow \boxed{c=3}$

⑧

Properties of fourier transform →

1. Time shifting / frequency shifting

$F[x(t-t_0)] = x(\omega)e^{-j\omega t_0}$

$F[x(t+t_0)] = x(\omega)e^{j\omega t_0}$

$x(\omega-\omega_0) = F[x(t)e^{j\omega_0 t}]$

$x(\omega+\omega_0) = F[x(t)e^{-j\omega_0 t}]$

بالعكس

6. Time shifting :

$$Z\left[f[n-n_o]\,u[n-n_o]\right] = \sum_{n=n_o}^{\infty} f[n-n_o]\, z^{-n_o-n}$$

$$= f[0]\,z^{-n_o-0} + f[1]\,z^{-n_o-1} + f[2]\,z^{-n_o-2} + \cdots$$

$$= z^{-n_o}\,\underbrace{\left[f[0]\,z^{0} + f[1]\,z^{-1} + f[2]\,z^{-2} + \cdots\right]}_{f(z) \rightarrow \frac{1}{1-z}}$$

$$= z^{-n_o}\, f(z)$$

Digital Filter Example :-

The difference equation for the α filter is →

It describes a low pass digital filter "α - filter" ⇐ $y[n] - (1-\alpha)\,y[n-1] = \alpha\, x[n]$

- α : constant
تحدد قيمتها من ميزات الفلتر نفسه

$$y(z) - (1-\alpha)\,z^{-1} y(z) = \alpha\, X(z)$$

$$y(z)\left[1 - (1-\alpha)\,z^{-1}\right] = \alpha\, X(z)$$

$$Y(z) = \frac{\alpha\, X(z)}{\frac{z}{z} - (1-\alpha)\frac{1}{z}} = \frac{\alpha z}{z-(1-\alpha)}\, X(z)$$

the system transfer function : $\dfrac{Y(z)}{X(z)} = H(z) = \dfrac{\alpha z}{z-(1-\alpha)}$

* If $X(z)$ is a unit step input (to find the unitstep response of the α - filter)

$$X(z) = \frac{z}{z-1}$$

$$\boxed{Y(z) = \frac{\alpha z}{z-(1-\alpha)} \cdot \frac{z}{z-1}}$$

→ to find $y[n]$, we will use a partial fraction expansion, where $\alpha = 0.1$ to simplify the calculation

$$F[x(t)] = \frac{\pi}{2}\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right] + \frac{j\omega}{\omega_0^2-\omega^2}$$

* switched sine :

$x(t) = u(t)\sin(\omega_0 t)$, find $F[x(t)]$?

$$x(t) = u(t)\left[\frac{e^{j\omega_0 t}-e^{-j\omega_0 t}}{2j}\right] = \frac{1}{2j}\left[u(t)e^{j\omega_0 t} - u(t)e^{-j\omega_0 t}\right] = \frac{1}{2j}\left[\pi\delta(\omega-\omega_0)+\frac{1}{j(\omega-\omega_0)} - \pi(\delta(\omega+\omega_0)) - \frac{1}{j(\omega+\omega_0)}\right]$$

$$= \frac{\pi}{2j}\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right] + \frac{\omega_0}{\omega_0^2-\omega^2}$$

Ex Let $x_1(t) = 2\cos(200\pi t)$

$x_2(t) = 5\cos(1000\pi t)$

$x_3(t) = x_1(t)\,x_2(t) = 10\cos(200\pi t)\cos(1000\pi t)$, Find $X_3(\omega)$?

→ using euler's formula :

$$x_3(t) = \cancel{10}^{\,5}\cos(200\pi t)\left[\frac{e^{j1000\pi t}+e^{-j1000\pi t}}{\cancel{2}}\right]$$

$$= 5\cos(200\pi t)e^{j1000\pi t} + 5\cos(200\pi t)e^{-j1000\pi t}$$

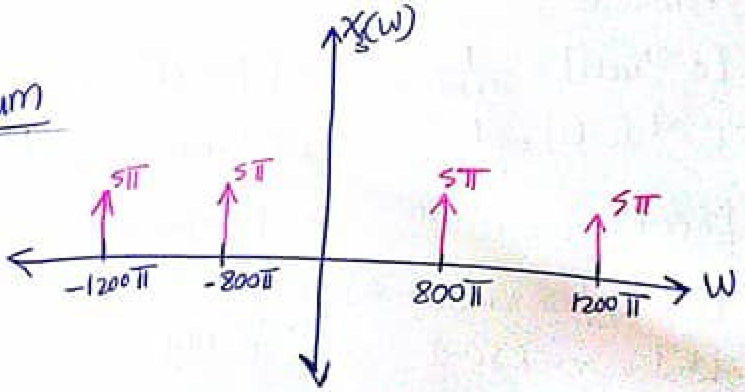
$$X_3(\omega) = F\left[5\cos(200\pi t)e^{j1000\pi t}\right] + F\left[5\cos(200\pi t)e^{-j1000\pi t}\right]$$ shifting (ω-domain)

$$= 5\pi\left[\delta(\omega-200\pi-1000\pi)+\delta(\omega+200\pi-1000\pi)\right]$$

$$+ 5\pi\left[\delta(\omega-200\pi+1000\pi)+\delta(\omega+200\pi+1000\pi)\right]$$

$$X_3(\omega) = 5\pi\left[\delta(\omega-1200\pi)+\delta(\omega-800\pi)\right] + 5\pi\left[\delta(\omega+1200\pi)+\delta(\omega+800\pi)\right]$$

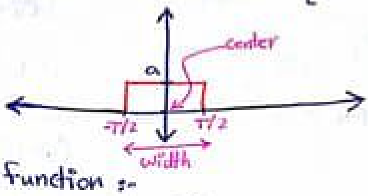
Frego spectrum



Scanned by CamScanner

③ Rectangular pulse function →

$$a\,\mathrm{rec}\left(\frac{t-\text{center}}{\text{width}}\right)=\mathrm{rec}\left(\frac{t}{T}\right)=\begin{cases}1(a), & -\frac{T}{2}<t<\frac{T}{2}\\ 0, & \text{otherwise}\end{cases}$$

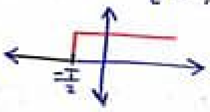


أي شي تعتمد ال center
center = $\frac{\text{width}}{2}$ *
$T/2$ = center + $\frac{\text{width}}{2}$ *
$-T/2$ = center − $\frac{\text{width}}{2}$

* By unit step function :-

1 $\mathrm{rec}\left(\frac{t}{T}\right)=u\left(t+\frac{T}{2}\right)-u\left(t-\frac{T}{2}\right)$

$u\left(t+\frac{T}{2}\right)=\begin{cases}1, & t>-\frac{T}{2}\\ 0, & t<-\frac{T}{2}\end{cases}$ $\qquad u\left(t-\frac{T}{2}\right)=\begin{cases}1, & t>\frac{T}{2}\\ 0, & t<\frac{T}{2}\end{cases}$



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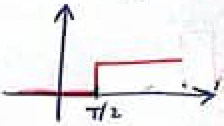
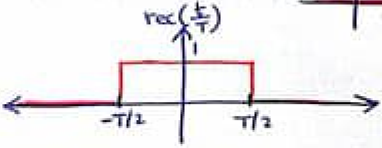


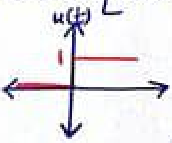


2 $\mathrm{rec}\left(\frac{t}{T}\right)=u\left(\frac{T}{2}-t\right)-u\left(\frac{-T}{2}-t\right)$

3 $\mathrm{rec}\left(\frac{t}{T}\right)=u\left(t+\frac{T}{2}\right)u\left(\frac{T}{2}-t\right)$

Ex represent $x(t)=\cos(t)\left[u(t)-u(t-2T)\right]$?

$t=2T$
$=\begin{cases}1, & t>2T\\ 0, & t<2T\end{cases}$



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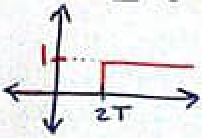

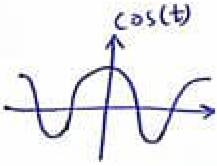

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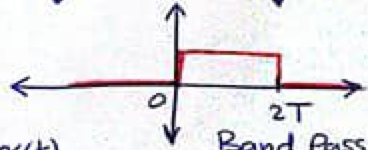

Band Pass filter Represantation

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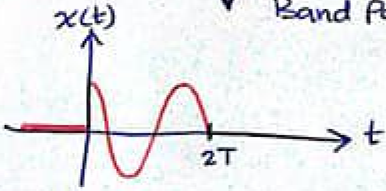


B. Continuos System :-

* How to connect systems →

① Summing point ② Product point ③ Simple system

$$\rightarrow \frac{Y(z)}{z} = \frac{0.1\,z}{(z-1)(z-0.9)}$$

نحل z عن طريق تجزئة الكسور (partial fraction method)

$$\frac{0.1\,z}{(z-1)(z-0.9)} = \frac{k_1\,?}{z-0.9} + \frac{k_2\,?}{z-1}$$

$$k_1 = \cancel{(z-0.9)}\frac{0.1\,z}{(z-1)\cancel{(z-0.9)}}\Bigg|_{z=0.9} = \frac{0.1(0.9)}{(0.9-1)} = -0.9$$

$$k_2 = \cancel{(z-1)}\frac{0.1\,z}{\cancel{(z-1)}(z-0.9)}\Bigg|_{z=1} = \frac{0.1}{1-0.9} = 1$$

$$\therefore \left[\frac{Y(z)}{z} = \frac{-0.9}{z-0.9} + \frac{1}{z-1}\right] * z$$

* Recall that :
$u[n] \xrightarrow{z} \frac{z}{z-1}$
$a^n \xrightarrow{z} \frac{z}{z-a}$

$$Y(z) = \frac{-0.9\,z}{z-0.9} + \frac{z}{z-1}$$

$0.9 \rightarrow a$

z^{-1} بناءً على الصيغ السابقة

$$y[n] = -0.9\,(0.9)^n + u[n]$$

$$\boxed{y[n] = -(0.9)^{n+1} + 1}\ ,\ n \geq 0$$

③ **Ex** Find $Z[\cos(bn)]$?

$\rightarrow \cos(bn) = \dfrac{e^{jbn} + e^{-jbn}}{2}$

$$Z[\cos(bn)] = \tfrac{1}{2} Z[e^{jbn}] + \tfrac{1}{2} Z[e^{-jbn}]$$

$$= \tfrac{1}{2}\left[\frac{z}{z-e^{jb}}\right] + \tfrac{1}{2}\left[\frac{z}{z-e^{-jb}}\right]$$

$$= \tfrac{1}{2}\left[\frac{z}{z-e^{jb}} + \frac{z}{z-e^{-jb}}\right]$$

نوحد مقامات

$$= \frac{z}{2}\left[\frac{(z-e^{-jb})+(z-e^{jb})}{(z-e^{jb})(z-e^{-jb})}\right]$$

radian constant

$$= \frac{z}{2}\left[\frac{2z-(e^{jb}+e^{-jb})}{z^2-(e^{jb}+e^{-jb})z+1}\right] = \frac{z(z-\cos b)}{z^2-2z\cos b+1}$$

* Recall that :-

$a^n = (e^{\beta})^n = e^{\beta n}$, $\beta = \ln a$

$$\left[e^{\beta n} \xrightarrow{Z} \frac{z}{z-e^{\beta}}\right]$$

Z transform properties ⇒

1. Linear property :- $Z[a_1 f_1[n] + a_2 f_2[n]] = a_1 f_1(z) + a_2 f_2(z)$

2. frequency scaling property :- $Z[e^{-an} f[n]] = f(z)\big|_{z = ze^{a}}$

3. Time scalling :- $Z[f[k\,n]] = f(z^{k})$ → +ve no.

4. Convolution in time :-

$$x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]\, y[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]\, y[k]$$

أو τ

$\rightarrow Z[x[n] * y[n]] = x(z)\, y(z)$

5. Initial value & steady state (final value) property :-

$$f[0] = \lim_{z\to\infty} F(z) \quad , \quad f[\infty] = \lim_{n\to\infty} f[n] = \lim_{z\to 1} (z-1) f(z)$$

example : consider the unit step function $u[n]$, Find $f[0]$ & $f[\infty]$?

$$f[0] = \lim_{z\to\infty} \frac{z}{z-1} = \lim_{z\to\infty} \frac{1}{1-\frac{1}{z}} = 1$$, the initial va

$$f[\infty] = \lim_{z\to 1} \cancel{(z-1)} \frac{z}{\cancel{z-1}} = \lim_{z\to 1} z = 1$$, the final val

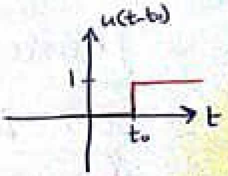
← إذا كان السؤال بيجمع ال (Amp+time transfor) ، ببلش بال time أولًا
بعدين حتى رسمها بغير على ال amplitude داخل إشي ..

برسم فوق بعض
الأصلية ← التغير على x ← التغير على y
time . amp .

Singularity Functions →

① Unit step function :

$u(t)\,,\,1(t) \Rightarrow u(t-t_0) = \begin{cases} 1 & , t > t_0 \\ 0 & , t < t_0 \end{cases}$



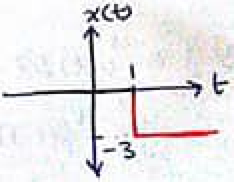
بتنعكس اذا كنا نحكي عن $u(-t)$

Properties

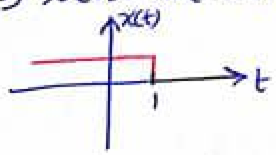
* $[u(t-t_0)]^K = u(t-t_0)$
* $u(at-t_0) = u(t-\frac{t_0}{a})$ → بشيل ال a نهائيًا من ال $u(t)$
* $\int_{-\infty}^{\infty} x(t)u(t)dt = \int_{0}^{\infty} x(t)dt$

Ex Plot : a) $x(t) = -3u(t-1)$

$x(t) = \begin{cases} -3 & , t > 1 \\ 0 & , t < 1 \end{cases}$



b) $x(t) = u(-t+1)$

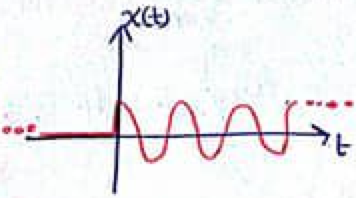


$x(t) = \begin{cases} 1 & , t < 1 \\ 0 & , t > 1 \end{cases}$

لأنو $-t$

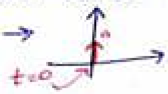
c) $x(t) = \cos(wt).u(t)$

$= \begin{cases} \cos(wt) & , t > 0 \\ 0 & , t < 0 \end{cases}$



② Unit impulse function :

$a\delta(t) \rightarrow$



Properties

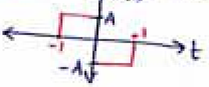
* $\int_{-\infty}^{\infty} \delta(t)dt = 1 \quad , \quad \delta(t) = \frac{du(t)}{dt} \quad , \quad \delta(t-t_0) = \frac{du(t-t_0)}{dt}$
* $\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)\int_{-\infty}^{\infty}\delta(t)dt = f(0)$

$$\ldots \frac{1}{1+jw} \quad \frac{1}{1-jw} = \frac{\ldots}{1+w^2}$$

$$F\left(\frac{d\, e^{-|t|}}{dt}\right) = \frac{jw}{1+w^2} \quad \#$$

7- Integration → $F\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right] = \frac{1}{jw} X(w) + \pi X(0)\, \delta(w)$

Ex Find fourier for $\int_{-\infty}^{t} x(t)\, dt$:-



$$x(t) = \begin{cases} A\ rec(t+0.5) & , -1 < t < 0 \\ -A\ rec(t-0.5) & , 0 < t < 1 \end{cases}$$

$$F[x(t)] = A\,\text{sinc}\left(\tfrac{w}{2}\right) e^{j0.5w} - A\,\text{sinc}\left(\tfrac{w}{2}\right) e^{-0.5jw}$$

$$= 2jA\,\text{sinc}\left(\tfrac{w}{2}\right)\left[\frac{e^{j0.5w} - e^{-j0.5w}}{2j}\right] = 2Aj\,\text{sinc}\left(\tfrac{w}{2}\right)\sin(0.5w) \cdot \frac{w/2}{w/2}$$

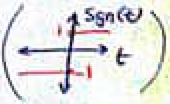
$$= Ajw\,\text{sinc}^2\left(\tfrac{w}{2}\right)$$

$$F\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right] = A\,\text{sinc}^2\left(\tfrac{w}{2}\right) + 0$$

~~Property~~

Formulas

* $F[e^{-\alpha t} u(t)] = \frac{1}{\alpha + jw}$
* $F[e^{\alpha t} u(-t)] = \frac{1}{\alpha - jw}$
* $F[\delta(t \pm t_0)] = e^{\pm jwt_0}$
* $F[e^{jw_0 t}] = 2\pi\, \delta(w - w_0)$
  $F[K] = 2\pi K\, \delta(w)$ → from duality
* $F[a\, rec(\frac{t}{\alpha T})] = a\alpha T\, \text{sinc}(w \times \frac{\alpha T}{2})$ — width
* $F[a\, rec(\frac{t-\beta}{\alpha T})] = a\alpha T\, \text{sinc}(w \times \frac{\alpha T}{2})\, e^{-jw\beta}$
* $F[sgn(t)] = \frac{2}{jw}$



* $F[\cos(w_0 t)] = \pi[\delta(w - w_0) + \delta(w + w_0)]$
  $F[\sin(w_0 t)] = \frac{\pi}{j}[\delta(w - w_0) - \delta(w + w_0)]$

* Switched cosine :-
  $x(t) = u(t)\cos(wt)$, find $F[x(t)]$?

→ $x(t) = u(t) \times \left(\frac{e^{jwt} + e^{-jwt}}{2}\right) = \frac{1}{2}\left[u(t)e^{jwt} + u(t)e^{-jwt}\right]$

$$F[x(t)] = \frac{\pi}{2}\left[\delta(w - w_0) + \delta(w + w_0)\right] + \frac{jw}{w_0^2 - w^2}$$

* switched sine :-
  $x(t) = u(t)\sin(wt)$, find $F[x(t)]$?

$$x(t) = u(t)\left[\frac{e^{jwt} - e^{-jwt}}{2j}\right] = \frac{1}{2j}\left[u(t)e^{jwt} - u(t)e^{-jwt}\right] = \frac{1}{2j}\left[\pi\delta(w - w_0) + \frac{1}{j(w - w_0)} - \pi(\delta(w + w_0)) - \frac{1}{j(w + w_0)}\right]$$

$$= \frac{\pi}{2j}\left[\delta(w - w_0) - \delta(w + w_0)\right] + \frac{w_0}{w_0^2 - w^2}$$

(12)

Ex Let $x_1(t) = 2\cos(200\pi t)$
$x_2(t) = 5\cos(1000\pi t)$
... $\cos(\ldots\pi t)$, Find $X_3(w)$?

Ch.11 Z Transform "In linear System"

Definition ⇒

① Bilateral "two sides" :-

$$Z_b[f[n]] = \sum_{n=-\infty}^{\infty} f[n] z^{-n} \quad , \quad F_b(z) = \dots + f[-2]z^{2} + f[-1]z^{1} + f[0] + f[1]z^{-1} + \dots$$

⇒ The reverse z transform Z_b^{-1} of :-

$$F_b^{-1}[z] = f[n] = \frac{1}{2\pi j} \int_{\Gamma} F_b(z) z^{n-1} . dz$$

↳ Particular ccw closed path Γ

المطلوب حل عليه ⇒ ② Unilateral "One side" :-

$$Z[f[n]] = F[z] = \sum_{n=0}^{\infty} f[n] z^{-n} \quad , \quad F_b(z) = f[0] + f[1]z^{-1} + f[2]z^{-2} + \dots$$

"In closed form" ↵

"Series Expansion" $\sum_{n=0}^{\infty} a^0 + a^1 + a^2 + \dots = \frac{1}{1-a}$, $|a| < 1$

Ex Find Z transform for :-

① $u[n] \rightarrow$

$$Z[u[n]] = \sum_{n=0}^{\infty} u[n] z^{-n}$$

$$= u[0]z^{0} + u[1]z^{-1} + u[2]z^{-2} + \dots$$

$$= 1.1 + 1.z^{-1} + 1.z^{-2} + \dots$$

$$= \frac{1}{1-z^{-1}} \quad , \quad |z^{-1}| < 1$$

series Expansion

$$\boxed{u[n] \xrightarrow{Z} \frac{z}{z-1}}$$

② $a^n \rightarrow \quad Z[a^n] = \sum_{n=0}^{\infty} a^n . z^{-n}$

$$= a^0 z^0 + a z^{-1} + a^2 z^{-2} + \dots$$

$$= \frac{1}{1-az^{-1}} = \frac{z}{z-a} \quad , \quad |az^{-1}| < 1$$

$$\boxed{a^n \xrightarrow{Z} \frac{z}{z-a}}$$

③ $\delta[n-n_0] = \begin{cases} 1 & , n = n_0 \\ 0 & , n \neq n_0 \end{cases} \rightarrow Z[\delta[n-n_0]] = \sum_{n=0}^{\infty} \delta[n-n_0] z^{-n} = z^{-n_0}$

$$\boxed{\delta[n-n_0] \xrightarrow{Z} z^{-n_0}}$$

④ $\boxed{\delta[n] \xrightarrow{Z} 1}$

LTI System Applications

→ Transfer function $H(z)$ is the ratio between :-

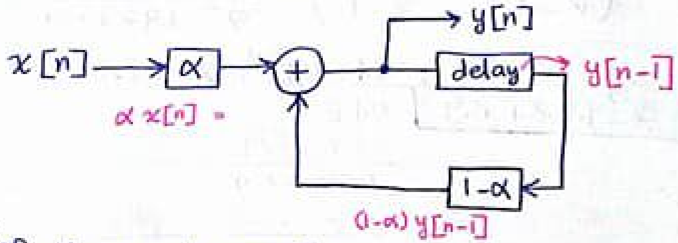
$$\frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_{N-1} z^{-N+1} + b_N z^{-N}}{a_0 + a_1 z^{-1} + \dots + a_{N-1} z^{-N+1} + a_N z^{-N}} * \frac{z^N}{z^N}$$

$$= \frac{b_0 z^N + b_1 z^{N-1} + \dots + b_{N-1} z + b_N}{a_0 z^N + a_1 z^{N-1} + \dots + a_{N-1} z + a_N}$$

Ex Represent the difference equation for α-filter by assuming $\alpha = 0.1$?

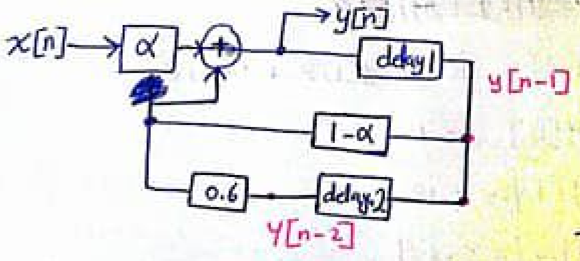
$$y[n] - (1-\alpha)\, y[n-1] = \alpha x[n]$$

$$y[n] - 0.9\, y[n-1] = \alpha x[n]$$



* if the equation is for example :-

$$y[n] - 0.6\, y[n-2] = \alpha x[n]$$



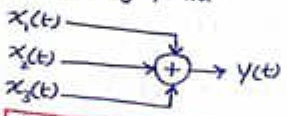
By taking the z transform :-

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.1\, z}{z - 0.9}$$

B. Continuos System :

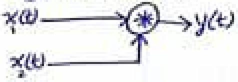
* How to connect systems →

① Summing point



$\boxed{y(t) = x_1(t) + x_2(t) + x_3(t)}$

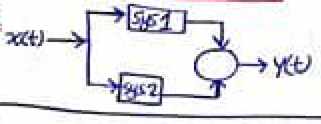
② Product point



$\boxed{y(t) = x_1(t) \cdot x_2(t)}$

③ Simple system

Ex



$y(t) = T_1[x_1(t)] + T_2[x_2(t)]$

Properties of continuos system →

① Varying with time
- Variance: $x(t) \to y(t-t_0)$ — اختلاف الزمن
- Invariance: $x(t-t_0) \to y(t-t_0)$ — phase shift / شفت الزمن

② Invertibility
- invertible: if $y = 3x$ so $\boxed{x = y/3}$ (just one i/p for one o/p)
- non invertible: if $y = x^2$ so $\boxed{x = \pm\sqrt{y}}$ (two inputs for one output)

③ Causality
- causal (ما بتعتمد على future ، حفظ القيمة الحالية والسابقة)
- non causal (تعتمد على القيمة التالية)

④ Stability
- $x(t)$ bounded i/p
  - bounded o/p → stable
  - unbounded o/p → unstable

⑤ Memory
- dynamic → memory $\left[\text{time i/p} \neq \text{time o/p}\right]$
- static → memoryless $\left[\text{time i/p} = \text{time o/p}\right]$

⑥ Linearity
- $\left[\begin{array}{l} y(\alpha x_1 + \beta x_2) = y(\alpha x_1) + y(\beta x_2) \\ y(x_1 + x_2) = y(x_1) + y(x_2) \end{array}\right]$ → Linear system

Ex ① Check for memory :

$y(t) = 5x(t)$, $y(t) = x(t-3)$

memoryless   memory

② Check for Causality :

$y(t) = x(t-3)$, $y(t) = x(t+3)$

$0 > -3$ pre → Causal $0 < 3$ next → non Causal

$y(t-3) = x(n-2)$

$-3 < -2$ future → non Causal

$\boxed{\text{memory كل (non causal) مو}}$

③ Check for Linearity :

$\boxed{y(t) = 2x(t) + 3}$

Let $\alpha = 2$, $\beta = 3$, $x_1 = 3$, $x_2 = 0.5$

$y(x_1 + x_2) \stackrel{?}{=} y(x_1) + y(x_2)$

$y(3.5) \stackrel{?}{=} y(3) + y(0.5)$

$2(3.5) + 3 \stackrel{?}{=} 2(3) + 3 + 2(\frac{1}{2}) + 3$

$10 \neq 9 + 4$ — non linear

④ Check for stability :

$y(t) = x(t)$, $y(t) = k\int_0^t x(\tau)d\tau$

BIBO   Isn't BIBO

⑤ Check for varying w/ time :

$y(t) = \sin(x t)$, $y(t) = x(t+1) + 3x(t-1)$

Invarient   varient with time

Ch.3 | Continuos time Linear time invarient system

4. Duality → (لا يكون معي تعبير متعودة عليه بال time-domain ويطلع بال (w)) حل ⇐

Ex
① $F\left[\text{sinc}\left(\frac{t}{2}\right)\right] = 2\pi\, rec(\omega)$

$k\delta(t) \xrightarrow{F} k$

$k \xrightarrow{F} 2\pi\,\delta(-\omega)$

② $F\left[5\,\text{sinc}\left(\frac{3t}{2}\right)\right] = 10\pi\, rec\left(\frac{-\omega}{3}\right)$

$\frac{3}{2}, 2$

③ $F\left[x(t) = 5\tau\,\text{sinc}\left(\frac{\tau t}{2}\right)\right] \Rightarrow \left(rec\left(\frac{t}{\tau}\right) = \tau\,\text{sinc}\left(\frac{\omega}{2}\tau\right)\right)$

$x(\omega) = 10\pi\, rec\left(\frac{-\omega}{\tau}\right)$, even function

$\therefore \left(\begin{array}{l} F[x(t)] = x(\omega) \\ F[x(\omega)] \underset{t=-\omega}{\neq} 2\pi\, x(t) \end{array} \right)$

5. Convolution → $F[x_1(t) * x_2(t)] = x_1(\omega)\, x_2(\omega)$

$F[x_1(t)\, x_2(t)] = \frac{1}{2\pi}[x_1(\omega) * x_2(\omega)]$

Ex $h(t) = \cos(3t)$
$x(t) = rec(t)$, find the o/p of LTI sys. ? $[y(\omega)]$

→ $y(\omega) = h(\omega)\, x(\omega)$

$= [\delta(\omega-3) + \delta(\omega+3)]\left[\pi\,\text{sinc}\left(\frac{\omega}{2}\right)\right] \xrightarrow{F^{-1}} = \frac{1}{2\pi}\left[\pi e^{-3jt} rec(t) + \pi e^{3jt} rec\right.$

6. Differentiation → $F\left[\frac{d^n x(t)}{dt}\right] = (j\omega)^n x(\omega)$

Ex find $F \frac{d e^{-|t|}}{dt}$?

$F\left[e^{-|t|}\right] = \begin{cases} e^{-t}, & t > 0 \\ e^{t}, & t < 0 \end{cases}$

$\boxed{e^{-|t|} = e^{-t}u(t) + e^{t}u(-t)}$

$F\{e^{-|t|}] = \frac{1}{1+j\omega} + \frac{1}{1-j\omega} = \frac{1}{1+\omega^2}$

$F\left(\frac{d e^{-|t|}}{dt}\right) = \frac{j\omega}{1+\omega^2}$ #

7. Integration → $F\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right] = \frac{1}{j\omega} x(\omega) + \pi x(0)\,\delta(\omega)$

Ex Find fourier for $\int_{-\infty}^{t} x(t)\, dt$:-

$x(t) = \begin{cases} A\, rec(t+0.5) & ,-1 < t < 0 \\ -A\, rec(t-0.5) & ,0 < t < 1 \end{cases}$

$y = ce^{...} + 1 \rightarrow \boxed{c = 3}$ ⑧

Properties of fourier transform →

1. Time shifting / frequency shifting

$F[x(t-t_0)] = x(w)e^{-jwt_0}$

$F[x(t+t_0)] = x(w)e^{jwt_0}$

$x(w-w_0) = F[x(t)e^{jw_0t}]$

$x(w+w_0) = F[x(t)e^{-jw_0t}]$

بالعكس

Ex Find fourier transform for :

① $\delta(t-5) = e^{-jw5}$ ② $\delta(3t+6) = \frac{1}{3}\delta(t+2) = \frac{1}{3}e^{jw2}$

من خصائص $\delta(t)$

③ $rec(t-3) = sinc(\frac{w}{2})e^{-3jw}$

④ $rec(\frac{t-3}{2}) = 2\,sinc(w)e^{-jw3}$

$\boxed{F[A\,rec(\frac{t}{2T})] = 2AT\,sinc(\frac{2T}{2})}$ width, half width

2. Time Scalling / Time transformation

$F[x(at)] = \frac{1}{|a|}x(\frac{w}{a})$

$F[x(at-t_0)] = \frac{1}{|a|}x(\frac{w}{a})e^{-j(\frac{w}{a})t_0}$

عامل shifting

3. Linearity →

$x_1(t) \xrightarrow{F} x_1(w)$

$x_2(t) \xrightarrow{F} x_2(w)$

$x_1(t) + x_2(t) \xrightarrow{F} x_1(w) + x_2(w)$

Ex Find fourier :

① $x(t) = b\cos(wt)$

→ $x(t) = b\left(\frac{e^{jwt} + e^{-jwt}}{2}\right) = \frac{b}{2}[e^{jwt} + e^{-jwt}]$

$F[x(t)] = \frac{b}{2}F[e^{jwt}] + F[e^{-jwt}]$

$= \frac{b}{2}*(2\pi)[\delta(w-w_0) + \delta(w+w_0)] = \pi b[\delta(w-w_0) + \delta(w+w_0)]$

② $\delta(t) + e^{-3t}u(t) = x(t)$

→ $x(w) = 1 + \frac{1}{3+jw}$

لما يكون $x(t)$ Linear
بوزع ال F على عناصر $x(t)$

③ $x(t) = b\sin(wt)$

$= \frac{b}{2j}[e^{jwt} - e^{-jwt}]$

→ $x(w) = \frac{b}{2j}(2\pi)[\delta(w-w_0) + \delta(w+w_0)]$

4. Duality → (لما يكون معي تعبير متعودة عليه بال time-domain وبطلب بال (w))

حل ⇐

Ex ① $F[sinc(\frac{t}{2})] = 2\pi\,rec(w)$

② $F[5\,sinc(\frac{3t}{2})] = 10\pi\,rec(\frac{-w}{3})$

$k\delta(t) \xrightarrow{F} k$

$k \xrightarrow{F} 2\pi\,\delta(-w)$

Ch.2

A. Continuos time signal :

Classification of Signals

- **Continuos & discrete**
  - Continuos: - Sinusoidal waves; - infinite values
  - discrete: - $x[n]$; - Finite values
- **Periodic & Aperiodic**
  - من خلال رسمة الاقتران بعرف اذا السيغنال تكرر نفسها بعد كل T_0 → fundamental period
  - $f = \frac{1}{T_P}$, $\omega = 2\pi f$
  - او باستخدام الشرط: $\boxed{x(t) \overset{?}{=} x(t+T)}$
- **Even & odd**
  - $\boxed{x_e(t) = x(-t)}$ $\boxed{x_o(t) = -x(-t)}$
  - * to find $x_e(t)/x_o(t)$ from $x(t)$ →

$$x_e(t) = \frac{x(t)+x(-t)}{2}$$

$$x_o(t) = \frac{x(t)-x(-t)}{2}$$

- **Analog & Digital**
  - Analog: smooth, continuos
  - Digital: stepping, square

* Notes about Even & odd continuos signals →

X	X
O	O

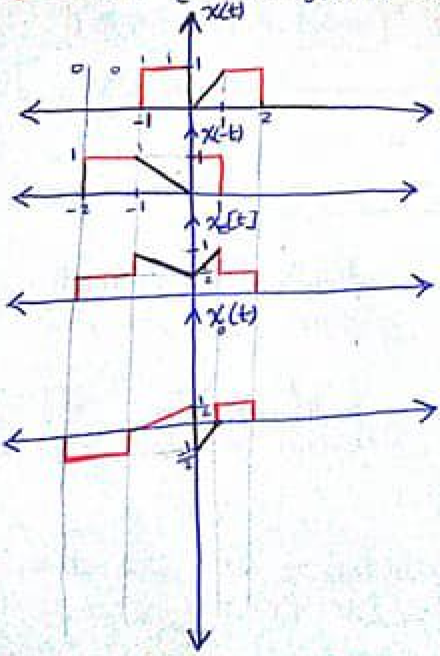
Even

-O	X
-X	O

odd

Even * Even = even , odd + odd = odd
Even + Even = even , odd * even = odd
odd * odd = even , even + odd = neither

Ex Plot $x_e(t)$ and $x_o(t)$ for $x(t)$:



1. برسم $x(-t)$ [فوق بعض برسمهم أحسن]
2. بطبق على قانون $\left[x_e(t) = \frac{x(t)+x(-t)}{2}\right]$ عند كل قيمة لـ t

← اذا كان عندي [inc. or dec. linear equ.] بطبق زيها على $x_o(t)/x_e(t)$

3. بطبق على $\left[x_o(t) = \frac{x(t)-x(-t)}{2}\right]$ عند كل قيمة لـ t

(1)

⇒ Euler's Relation :

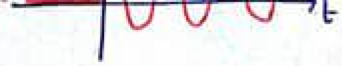
$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}, \quad \sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2}, \quad |e^{j\theta}| = \sqrt{\cos^2(\theta)+\sin^2(\theta)}$$

$$\arg(\theta) = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = \theta$$

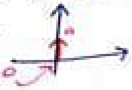
* Three Cases for exponential function :

$\boxed{x(t) = Ce^{at}}$



② Unit impulse function :

$a\,\delta(t) \rightarrow$



Properties

* $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$, $\delta(t) = \frac{du(t)}{dt}$, $\delta(t-t_0) = \frac{du(t-t_0)}{dt}$ #

* $\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = f(0)\int_{-\infty}^{\infty}\delta(t)\,dt = f(0)$ ($t=0$)

ما بتروح إلا بالتكامل $\int_{-\infty}^{\infty}$

* $\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$, $f(t)\,\delta(t-t_0) = f(t_0)\,\delta(t-t_0)$

t_0 لازم تكون ضمن حدود التكامل

* $\delta(t) = \delta(-t) \rightarrow$ even function

* $\int_{-\infty}^{\infty} \delta(at-t_0)\,dt = \frac{1}{|a|}\int_{-\infty}^{\infty}\delta(t-\frac{t_0}{a})\,dt$

* $\int_{-\infty}^{\infty} f(t\mp t_0)\,\delta(t)\,dt = f(\mp t_0)$ ($t=0$) ، $\int_{-\infty}^{\infty} f(t-t_0)\,\delta(t\pm t_1)\,dt = f(\pm t_1 - t_0)$ ($t=\mp t_1$)

* $u(t-t_0) = \int_{-\infty}^{\infty} \delta(\tau-\tau_0)\,d\tau = \begin{cases} 1 , & t > t_0 \\ 0 , & t < t_0 \end{cases}$

$\boxed{\text{Ex}}$ Find :

a) $\int_{-2}^{3} e^{-2t}\,u(t)\,\delta(t-1)\,dt$ أولاً u(t) بتأثر على حدود التكامل

$= \int_{0}^{3} e^{-2t}\,\delta(t-1)\,dt$ $t=1$ (ضمن حدود التكامل)

$= e^{-2}$ #

b) $\int_{-2}^{3} e^{-2t}\,u(-t)\,\delta(t-1)\,dt = \int_{-2}^{0} e^{-2t}\,\delta(t-1)\,dt = \phi$ ($t=1$)

مو من ضمن حدود التكامل بالمرة

c) $\int_{-2/\pi}^{2/\pi} \cos(2\pi t)\,\delta(4\pi t)\,dt = \frac{1}{4\pi}\int_{-2/\pi}^{2/\pi} \cos(2\pi t)\,\delta(t)\,dt = \frac{1}{4\pi}$ ($t=0$)

d) $\int_{-\infty}^{t} e^{2t}\,\delta(t-1)\,dt = e^{2(1)}\int_{-\infty}^{t}\delta(t-1)\,dt = e^{2}u(t-1)$

(طالما حدود التكامل متغيرة رح يكون الناتج u(t) لا δ(t))

③ Rectangular pulse function →

$a\,\text{rec}\left(\frac{t-\text{center}}{\text{width}}\right) = \text{rec}\left(\frac{t}{T}\right) = \begin{cases} 1(a), & -\frac{T}{2} < t < \frac{T}{2} \\ 0 , & \text{otherwise} \end{cases}$

Inverse Z transform ، by using :-

Z^{-1}

- Long Division
- Complex inversion integral
  $f[n] = \frac{1}{2\pi j} \oint F(z) z^{n-1} \, dz$
  ⇓
  [بس ما مستخدمها لأنها صعبة]
- Partial fraction expansion
  ⇓
  حل معادلة ك
  α-filter
  لما حسبنا [illegible]
  y[n]

Ex Find z^{-1} for $y(z)$ using long division :-

$$y(z) = \left(\frac{0.1\,z}{z-0.9} * \frac{z}{z-1} \right) = \frac{0.1\,z^2}{z^2 - 1.9z + 0.9}$$

$$\begin{array}{r|l}
 & 0.1 + 0.19z^{-1} + 0.271z^{-2} + \dots \\
\hline
z^2 - 1.9z + 0.9 & 0.1z^2 \\
 & -\left(\cancel{0.1z^2} - 0.19z + 0.09\right) \\
\cline{2-2}
 & 0.19z - 0.09 \\
 & -\left(\cancel{0.19z} - 0.361 + 0.171z^{-1}\right) \\
\cline{2-2}
 & 0.271 - 0.171z^{-1} \\
 & \vdots
\end{array}$$

$$\Rightarrow \quad y(z) = y(0) + y(1)z^{-1} + y(2)z^{-2} + \dots$$

$y[0] = 0.1$

$y[1] = 0.19$

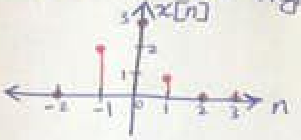
$y[2] = 0.271$

Ch.10 Discrete LTI Systems

* Impulse representation of discrete time signals :

$$x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$$

Ex Consider the signal $x[n]$ shown below :



$$x[n] = x_{-1}[n] + x_0[n] + x_1[n]$$

عوض $x[n]$ [illegible]

$$= x[-1]\,\delta[n+1] + x[0]\,\delta[n] + x[1]\,\delta[n-1]$$

$$\therefore \boxed{x[n] = \sum_{k=-1}^{1} x[k]\,\delta[n-k]} \rightarrow = \begin{cases} x[k], & n=k \\ 0, & n\neq k \end{cases}$$

Convolution for discrete time systems →

$$\delta[n] \rightarrow \boxed{\text{Sys.}} \rightarrow h[n]$$

For LTI sys : $x[k]\,\delta[n-k] \rightarrow x[k]\,h[n-k]$

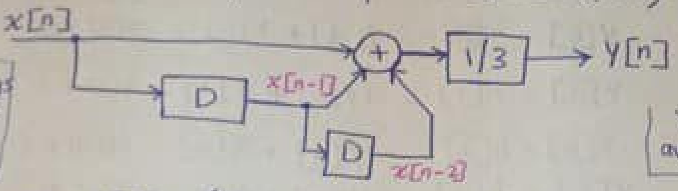
* the convolution sum :

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] \longrightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

(Convolve sign) الـ
بالديسكريت

$$\Rightarrow y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

Ex(10.1) A Finite impulse response system (FIR)



D : unit delay
تعمل shift للزمن للخلف بمقدار وحدة واحدة

It's a moving average filter

from the fig. → $y[n] = \dfrac{(x[n] + x[n-1] + x[n-2])}{3}$

Let $\begin{pmatrix} H[n] = y[n] \\ \delta[n] = x[n] \end{pmatrix} \Rightarrow H[n] = \dfrac{\delta[n] + \delta[n-1] + \delta[n-2]}{3}$

thus, $h[0] = \dfrac{\delta[0] + \cancel{\delta[-1]} + \cancel{\delta[-2]}}{3} = \dfrac{1+0+0}{3} = \dfrac{1}{3}$

$h[1] = \dfrac{\delta[1] + \delta[0] + \delta[-1]}{3} = \dfrac{0+1+0}{3} = \dfrac{1}{3}$

$h[2] = \dfrac{\delta[2] + \delta[1] + \delta[0]}{3} = \dfrac{0+0+1}{3} = \dfrac{1}{3}$

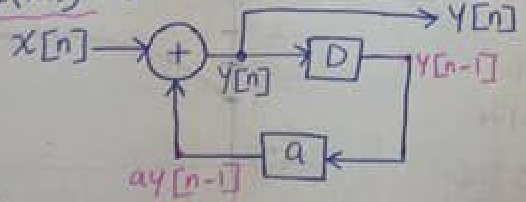
$\delta[n] = 0$, all n unless n = 0 / $\delta[0] = 1$

$h[n] \neq 0$ all value of n

∴ the impulse response contains a finite no. of nonzero terms

Initially, the numbers stored in the two delays must be zero,
واذا كانت قيمة ال delay صفر
the output includes an initial condition response, in addition to the impulse response, by superposition.

Ex(10.2) Calculation of the impulse response of a discrete system



From this figure :-

$\boxed{y[n] = ay[n-1] + x[n]}$

Let $x[0] = 1$

$x[n] = 0 \ , n \neq 0$

(the system is causal ، لأن ال o/p بيعتمد على القيم الحالية و السابقة من ال i/p)

* the value stored in the ideal delay at $n=0$ is zero

$$Y[0] = h[0] = aY[-1] + x[0] = a(0) + 1 = 1$$
$$Y[1] = h[1] = aY[0] + x[1] = a(1) + 0 = a$$
$$Y[2] = h[2] = aY[1] + x[2] = a(a) + 0 = a^2$$
$$Y[3] = h[3] = aY[2] + x[3] = a(a^2) + 0 = a^3$$
$$\vdots$$

$$h[n] = \begin{cases} a^n , & n \geq 0 \\ 0 , & n < 0 \end{cases} \quad \text{or } h[n] = a^n u[n]$$

it consists of unbounded no. of terms, so this sys. is called: infinite impulse response (IIR) system.

Ex(10.4) step response of a discrete system

For the system of the previous example, Let $\boxed{a = 0.6}$

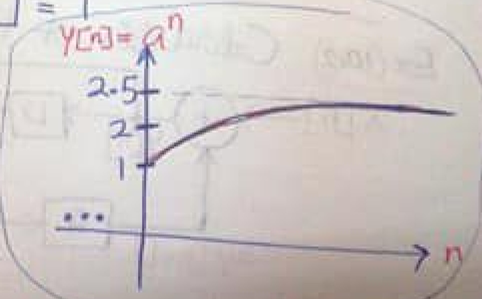
the impulse response $\rightarrow$ $h[n] = Y[n] = 0.6^n u[n]$

$$\therefore Y[n] = \sum_{k=0}^{n} (0.6)^k$$

$$Y[n] = \sum_{k=0}^{n} (0.6)^k = \frac{1 - (0.6)^{n+1}}{1 - 0.6} = 2.5\left[1 - (0.6)^{n+1}\right] , \quad n \geq 0$$

$\rightarrow$ so, the Calculations of values of $Y[n]$ yields:

$$Y[0] = 2.5\left[1 - 0.6^{0+1}\right] = 1$$
$$Y[1] = 1.6$$
$$Y[2] = 1.96$$
$$Y[3] = 2.176$$
$$Y[4] = 2.306$$
$$\vdots$$



$\boxed{Y[\infty] = 2.5}$ $\Rightarrow$ the steady state value of $y[n]$

the system is BIBO, if the output remains bounded for any bounded input.

→ the boundedness of the input :- $|x[n]| < M$ (real constant)

then, $|y[n]| = \left|\sum_{k=-\infty}^{\infty} x[n-k]h[k]\right| \leq \sum_{k=-\infty}^{\infty} |x[n-k]h[k]|$

$$= \sum_{k=-\infty}^{\infty} |x[n-k]||h[k]| \leq \sum_{k=-\infty}^{\infty} M\,|h[k]| = M \underbrace{\sum_{k=-\infty}^{\infty} |h[k]|}_{\text{the impulse response}}$$

* $y[n]$ bounded → $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$, $h[n]$ absolutely summable

If the o/p is unbounded for bounded i/p ⇒ the system is unstable.

For an LTI causal system, this condition reduces to :-

$$\boxed{\sum_{k=0}^{\infty} |h[k]| < \infty}$$

Ex (10.6) Stability of an LTI discrete system

Find the properties of $h[n] = \left(\frac{1}{2}\right)^n u[n]$ according to :-

① Memory
② Causality
③ Stability

$a < 1$ ← stable
$a > 1$ ← unstable

→ 1- memory system, with again k

2- Causal, current value of n

3- $\sum_{-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n] = \left[\sum_{0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{1-\frac{1}{2}} = \frac{1}{1/2} = 2\right]$ → بحسب القانون

$\frac{1}{1-a}$

ملاحظة لو كانت بدل الـ a ← عدد أكبر من (1)
بتكون (unstable) لأنه كل ما زاد n (power)
بتزيد لل ∞