The Hashemite University — Department of Mathematics — March 13, 2013

Calculus (3) — First Exam — Time: 1 hour

Name (in Arabic): [redacted] Student Number: [redacted] Serial Number: [redacted]

Section number or lecture time: 9 - 10 Instructor name: عمر الدولة

Please do all your work in this booklet and show all the steps. Write your final answer in the corresponding box. Calculators and notes are not allowed.

13

$\|u\times v\| = \|u\|\,\|v\|$ $\boxed{\cos\theta}$

$u\cdot v = \|u\|\,\|v\|$

$\cos\theta = \frac{u\cdot v}{\|u\|\,\|v\|}$

1. Consider the points $A(2, 1, 2)$, $B(3, 1, 4)$, $C(4, -1, 4)$ and $D(4, 1, 3)$

(a) (3 points) Find the COSINE of the angle between $\overrightarrow{AD}$ and $\overrightarrow{AB}$

$(\vec u)\ \overrightarrow{AD} = \langle 2, 0, 1\rangle$

$(\vec v)\ \overrightarrow{AB} = \langle 1, 0, 2\rangle$

1

$\cos\theta = \frac{\|\overrightarrow{AD}\times\overrightarrow{AB}\|}{\|\overrightarrow{AD}\|} \Rightarrow \frac{\|\vec u \times \vec v\|}{\|\vec u\|}$

$\Rightarrow \|\vec u\| = \sqrt5$

$u\times v = \begin{vmatrix} \hat i & \hat j & \hat k \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{vmatrix}$

$= \hat i\, 0 - \hat j(4-1) \hat k\, 0$

$= \boxed{-3}\hat j$

$\|u\times v\| = \sqrt9 = \boxed{3}$

$\Rightarrow \cos\theta = \frac{3}{\sqrt5}$

$\theta = \cos^{-1}\frac{3}{\sqrt5}$

Answer: $\theta = \cos^{-1}\frac{3}{\sqrt5}$

(b) (2 points) Find the orthogonal projection of $\overrightarrow{BC}$ on $\overrightarrow{AD}$

$\overrightarrow{BC} = \vec v = \langle 1, -2, 0\rangle$

$\overrightarrow{CD} = \vec b = \langle 0, 2, -1\rangle$

$\text{proj}_b^v = \frac{v\cdot b}{\|b\|^2}\cdot b$

$v\cdot b = (0 + -4 + 0) = \boxed{-4}$

$\|b\| = \sqrt5$

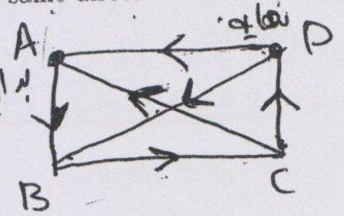
1

$\Rightarrow \text{proj}_b^v = \frac{-4}{\sqrt5}\cdot\langle 0, 2, -1\rangle$

$= \left(0, \frac{-8}{\sqrt5}, \frac{4}{\sqrt5}\right)$

Answer: $\langle 0, \frac{-8}{\sqrt5}, \frac{4}{\sqrt5}\rangle$

(c) (2 points) Find the vector of length 5 that has the same direction as $\overrightarrow{AD}$.



~~$\overrightarrow{AD} = \langle 2, 0, 1\rangle$~~

~~$\overrightarrow{AC} = \langle 2, -2, 2\rangle$~~

~~Length = 5~~

~~$\overrightarrow{DC} = \langle 0, 2, -1\rangle$~~

~~Length = $\|v - \text{proj}_b^v\|$~~

Answer:

0

5

$\frac{\overrightarrow{AD}}{\|\overrightarrow{AD}\|}$

Length = $\|v - \text{proj}_b^v\|$

point $\vec{u} = a\vec{v}$

2. (2 points) Find parametric equations of the line passing through (0,1,2) and parallel to the line $x = 1,\ y = 1 - 5t,\ z = 2 - 6t$.

x_0, y_0, z_0

point $= (0, 1, 2) = (x_0, y_0, z_0)$

$\vec{v} = \langle 0, -5, -6 \rangle$

$\langle a, b, c \rangle$

$x = x_0 + at = 0 + 0t$
$y = y_0 + bt = 1 - 5t$
$z = z_0 + ct = 2 - 6t$

$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$

Answer: $\boxed{\cancel{\frac{x-0}{0}} = \frac{y-1}{-5} = \frac{z-2}{-6} \Rightarrow \frac{y-1}{5} = \frac{z-2}{-6}}$

2

3. Given that the plane P: $2x + 2y + z = 11$ and the line L: $x = 1 + 2t,\ y = -3 + t,\ z = 4 + 5t$.

(a) (2 points) Find the possible intersection point of the line L with the plane P (if any).

P: $2x + 2y + z - 11 = 0 \Rightarrow n_1 = \langle 2, 2, 1 \rangle$

L: $x = 1 + 2t,\ y = -3 + t,\ z = 4 + 5t$

point $(1, -3, 4)$, $\vec{v} = \langle 2, 1, 5 \rangle$

0

$n \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 2 & 1 & 5 \end{vmatrix}$

$= \oplus \hat{i}(10-1) \ominus \hat{j}(10-2) \oplus \hat{k}(2-4)$

$9\hat{i} - 8\hat{j} - 2\hat{k}$

Answer: $\boxed{9\hat{i} - 8\hat{j} - 2\hat{k}}$

(b) (3 points) Find the equation of the plane through the point (1, 1, −2) which is parallel to the line L and perpendicular to the plane P

point $(1, 1, -2)$

parallel $P \parallel L$

$\Rightarrow$ plane $: ax_0 + by_0 + cz_0 + d = 0$

$\Rightarrow a + b - 2 = 0$

$P \parallel L \Rightarrow \vec{u} = a\vec{v}$

1/2

$a\ b\ c$
$(1, 2, 3)$

$(1, 1, -1)$
$\boxed{(-1, 1, 2)}$

Answer:

4. (3 points) Find the distance between the line L: $x = -1 + t,\ y = 1 + t,\ z = 2 - t$ and the plane P: $x + 2y + 3z = -5$

$\vec{u}$ (point $(-1, 1, 2)$) for line.

p: $\vec{v} = \langle 1, 2, 3 \rangle$) for plane.

$\|\vec{u}\| = \sqrt{6} \Rightarrow \|u\|^2 = 6$

$\|\vec{v}\| = \sqrt{14} \Rightarrow \|v\|^2 = 14$

$\vec{u} \cdot \vec{v} = (-1 + 2 + 6) = (7)$

Answer: $\boxed{d = \sqrt{77}}$

$\Rightarrow \left\{ d = \sqrt{\|u\|^2 \|v\|^2 - (u \cdot v)} \right\}$ Pascal

$d = \sqrt{(6)(14) - (7)}$

$= \sqrt{77}$

0

$D(A, P) = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c}}$

2

5. (3 points) Identify the surface $2x^2 - y^2 + z^2 = 14$ (by the name). Determine intercepts with the coordinate axes (if any).

$$\frac{2x^2}{14} - \frac{y^2}{14} + \frac{z^2}{14} = \frac{14}{14}$$

(x - axis)

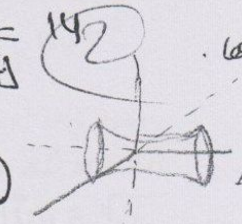
$$\frac{x^2}{7} - \frac{y^2}{14} + \frac{z^2}{14} = 1$$

$b^2 = c^2 = 14$

$a^2 = \boxed{7}$

2

[illegible]



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \text{ (Haygorpolied one sheet)}$$

Answer: $\frac{x^2}{7} - \frac{y^2}{14} + \frac{z^2}{14} = 1$ (x - axies)

6. (3 points) If $f(t) = e^t$, $\vec{u}(t) = e^{3t}\hat{i} + 3e^{4t}\hat{j} + 2e^{2t}\hat{k}$ and $\vec{v}(t) = e^{-3t}\hat{i} + e^{-2t}\hat{j} + e^{-4t}\hat{k}$.

Evaluate $\int f(t)\,[\,\vec{u}(t) \times \vec{v}(t)]dt$

$\vec{u} = \langle e^{3t}\hat{i}, 3e^{4t}\hat{j}, 2e^{2t}\hat{k} \rangle$

$\vec{v} = \langle e^{-3t}\hat{i}, e^{-2t}\hat{j}, e^{-4t}\hat{k} \rangle$

$\Rightarrow \int e^t - e^t - 2e^{2t} \cdot dt$

$= \cancel{e^t} - \cancel{e^t} - 2te^{2t} + C \quad = -2te^{2t} + C$

2

$$\vec{u} \times \vec{v} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ e^{3t} & 3e^{4t} & 2e^{2t} \\ e^{-3t} & e^{-2t} & e^{-4t} \end{vmatrix} = \oplus\, \hat{i}(3-2) \ominus \hat{j}(e^t - e^{-t}) \oplus \hat{k}(e^t - 3e^t)$$

$= \hat{i} - \hat{j} + 2e^t\hat{k}$

$= \langle 1, -1, -2e^t \rangle * e^t = \boxed{e^t - e^t - 2e^{2t}}$

Answer: $-2te^{2t} + C$

7. (2 points) A particle is traveling along the curve $\vec{r}(t) = < \sqrt{3}\cos(t), \sqrt{3}\sin(t), \sqrt{6}\,t >$.

Find the distance traveled by the particle from $t = 0$ to $t = \pi$

Helex

$a = 0$

$b = \pi$

$\vec{r}(t) = \langle \sqrt{3}\cos(t), \sqrt{3}\sin(t), \sqrt{6}t \rangle$

$\vec{r}'(t) = \langle \sqrt{3}\sin(t), -\sqrt{3}\cos(t), \sqrt{6} \rangle$

$\vec{r}'(0) = \langle 0, \sqrt{3}, \sqrt{6} \rangle$

$\vec{r}'(\pi) = \langle 0, \sqrt{3}, \sqrt{6} \rangle$

$\Rightarrow \|\vec{r}'(t)\| = \sqrt{0+3+} = \sqrt{9}$

$\sqrt{9} = \boxed{3}$

$\Rightarrow L = \int_a^b \|\vec{r}'(t)\| \cdot dt$

$\Rightarrow \|\vec{r}'(t)\| = \boxed{3}$

2

Answer: $L = \boxed{3\pi}$

$= \int_0^\pi 3 \cdot dt$

$= 3t \Big]_0^\pi$

$3(\pi) - (3 \times 0)$

$3\pi - 0 = \boxed{3\pi}$

3

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The Hashemite University Department of Mathematics June 20, 2013

Calculus (3) First Exam Time: 1 hour

Name (in Arabic): زفال نوفل رضا Student Number: ١٢٣٤٦٦١ Serial Number: ٤٩

Section number or lecture time: 10:30 – 11 Instructor name: د. عمر حرز الله

Please do all your work in this booklet and show all the steps. Calculators and notes are not allowed.

7 7 9

1. (5 points) Consider the vectors $\vec{u} = <1, -2, 0>$ and $\vec{v} = <0, 3, 1>$.

(a) Find the directional COSINES of the vector $\vec{u} + \vec{v}$.

$\vec{u} + \vec{v} = <1, -2, 0> + <0, 3, 1> = <1, +1, 1>$

$|\vec{u} + \vec{v}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$

$\cos\alpha = \frac{a}{|u+v|} = \frac{1}{\sqrt{3}}$

$\cos\beta = \frac{b}{|u+v|} = \frac{1}{\sqrt{3}}$, $\cos\gamma = \frac{c}{|u+v|} = \frac{1}{\sqrt{3}}$

(b) Find the area of the triangle determined by $\vec{u}$ and $\vec{v}$.

The area of the triangle is $= \frac{1}{2} |\vec{u} \times \vec{v}|$

$\vec{u} \times \vec{v} = \begin{vmatrix} \cancel{0, 3, 2} & & \\ i & j & k \\ 1, & -2, & 0 \\ 0, & 3, & 1 \end{vmatrix} = (-2*1 - 3\times 0)\hat{i} - (1\times 1)\hat{j} + (3 - 2*0)\hat{k}$

$= (-2)\hat{i} - (1)\hat{j} + 3\hat{k}$

$|\vec{u} \times \vec{v}| = \sqrt{(2)^2 + (1)^2 + (3)^2} = \sqrt{4+1+9} = \sqrt{14}$

the area is equal to $\frac{\sqrt{14}}{2}$

2. (2 points) Find parametric equations of the line passing through (0,1,2) and perpendicular to the plane $-x + z = 3$.

* the line is passing the point $(x_0, y_0, z_0) = \cancel{(0, 1, 2)}\ (0, 1, 2)$

~~* we should know the a vector that parallel to the point, and~~

* the line is prependicular to the plane, so that the line is parallel to the normal of the plane.

(1) a point $(0, 1, 2)$

(2) a parallel vector $\vec{v} = (-1, 0, 1)$

$x = x_0 + vt = 0 - t$

$y = y_0 + vt = 1$

$z = z_0 + vt = 2 + t$

3. (6 points) Consider the following lines.

$L_1 : x = 2 - t,\ y = -2 + t,\ z = 1 - t \qquad L_2 : x = s,\ y = 3 - 4s,\ z = -1 + s.$

(a) Find the possible intersection point of L_1 and L_2 (if any).

L_1 :
$x = 2 - t$ — (1)
$y = -2 + t$ — (2)
$z = 1 - t$ — (3)

L_2 :
$x = s$ — (4)
$y = 3 - 4s$ — (5)
$z = -1 + s$ — (6)

$s = 2 - t$ — (7)
$-2 + t = 3 - 4s$ (8)
$1 - t = -1 + s$ (9)

تعويض (7) في (8)

$-2 + t = 3 - 4(2 - t) \Rightarrow -2 + t = \underbrace{3 - 8}_{-5} + 4t \Rightarrow 3t = 3$

$\boxed{t = 1}$

(b) Find the equation of the plane that contains L_1 and L_2.

$s = 2 - t = 2 - 1 = 1 \Rightarrow \boxed{s = 1}$

$-1 + 1 \overset{??}{=} -1 + 1$ تعويض (6)

$\Rightarrow 0 = 0 \Rightarrow$ the point is $(1, -$

$L_1 \parallel V_1$; $V_1 = (-1, 1, -1)$

$L_2 \parallel V_2$; $V_2 = (1, -4, 1)$

to know the equation of a plane we need a point on the plane and the prependicular on the plane. (1) a point $(1, -1, 0)$

(2) $\vec{V_1} \times \vec{V_2} = \begin{vmatrix} -1, & 1, & -1 \\ 1, & -4, & 1 \end{vmatrix} = (1 - 4)\hat{i} - (-1 + 1)\hat{j} + (4 - 1)\hat{k}$
$= -3\hat{i} + 3\hat{k}$

the equation :

$\boxed{-3(x - 1) + 0(y + 1) + 3(z - 0) = 0}$

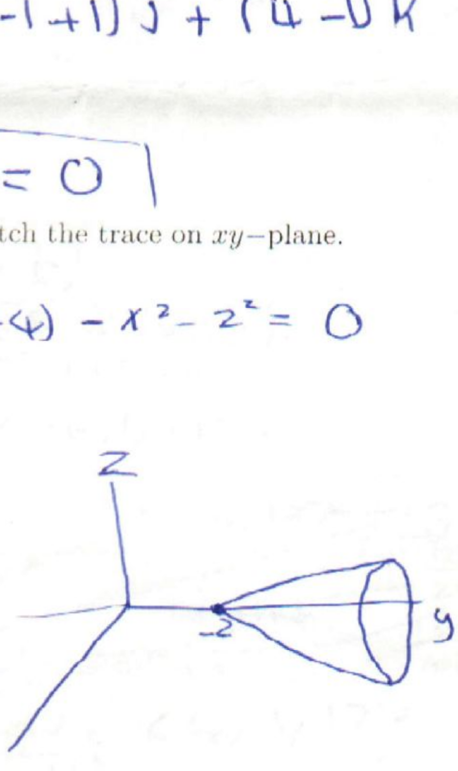
4. (3 points) Identify the surface $y^2 - x^2 - z^2 + 4y = -4$ (by the name). Sketch the trace on xy−plane.

$(y^2 + 4y) - z^2 - x^2 = -4 \Rightarrow (y^2 + 4y + 4) - x^2 - z^2 = 0$

$(y + 2)^2 - x^2 - z^2 = 0$

$(y + 2)^2 = x^2 + z^2 \Rightarrow$ Cone.

symmetric about the y-axis.



1

$P_1: x+3y-2z-5=0$

$P_2: +x+3y-2z+4=0$

5. (2 points) Compute the distance between the parallel planes $x+3y-2z=5$ and $-2x-6y+4z=8$.

* the distance between the parallel planes:

$d = \frac{|d_1 - d_2|}{|\vec{n}|}$, $\vec{n}: (1, 3, -2)$, $|\vec{n}| = \sqrt{4+9+1} = \sqrt{14}$

$= \frac{9}{\sqrt{14}} = \boxed{\frac{9}{\sqrt{14}}}$ 2

6. (2 points) Find the natural domain for $\vec{r}(t) = \sqrt{6-t}\, i + \ln(t-4)\, j + e^t\, k$.

* let $f(t) = \sqrt{6-t}$, $g(t) = \ln(t-4)$, $h(t) = e^t$

$D_f = (-\infty, 6]$, $D_g = (4, \infty)$, $D_h = R$ 2

the domain is $D_f \cap D_g \cap D_h = (4, 6]$

7. (2 points) Find the parametric equations for the tangent line to the curve $\vec{r}(t) = < \sqrt{6t}, t-1, e^{5t} >$ at the point $(\sqrt{6}, 0, e^5)$.

$\cancel{\frac{dr}{dt} = e\sqrt{6t}}$ $\frac{dr}{dt} = < \frac{\cancel{6}\,3}{\cancel{2}\sqrt{6t}}, 1, 5e^{5t} > = < \frac{3}{\sqrt{6t}}, 1, 5e^{5t} >$

$\cancel{\frac{dr}{dt}}$ at the point $(\sqrt{6}, 0, e^5)$, $t-1=0 \Rightarrow \boxed{t=1}$

$\left.\frac{dr}{dt}\right|_{t=1} = < \frac{3}{\sqrt{6}}, 1, 5e^5 >$

$x = x_0 + at = \boxed{\sqrt{6} + \frac{3}{\sqrt{6}} t}$, $\boxed{y = y_0 + at = 0 + t}$

2 $\boxed{z = z_0 + at = e^5 + 5e^5 t}$

8. (3 points) If $f(t) = \vec{u}(t).\vec{v}(t)$ such that $\vec{v}(t) = < t^2, t+1, t^3 >$, $\vec{u}(2) = < -1, -1, 1 >$, and $\frac{d\vec{u}}{dt}(2) = < -2, 3, 4 >$. Find $f'(2)$.

$f'(t) = \vec{u}(t) \cdot \left.\frac{dv}{dt}\right|_{t=2} + \vec{v}(t) \cdot \frac{du}{dt}$

$f'(2) = < -1, -1, 1 > \cdot < 4, 1, 12 > +$

$< 4, 3, 8 > \cdot < -2, 3, 4 >$

3

$= (-4 - 1 + 12) + (-8 + 9 + 32)$

$7 + (1 + \cancel{24}\,32)$

$7 + \cancel{28}\,33 = \boxed{\cancel{32}}$

40

$\frac{dv}{dt} = < 2t, 1, 3t^2 >$

$\cancel{\left.\frac{dv}{dt}\right|_{t=2} = < 4, 1, 12 >}$

$\cancel{16 + 1 + 144}$

$\cancel{\left|\frac{dv}{dt}\right| = \sqrt{16+1+}}$

$\left.\frac{dv}{dt}\right|_{t=2} = < 4, 1, 12 >$

$\vec{v}(2) = < 4, 3, 8 >$

12
12
24
120
144

-15 10--

The Hashemite University
Department of Mathematics
Calculus III

First Exam

Time: One Hour

Name: ......علي محمد الجولاني...... Number: ...938030...

Select the best correct answer and fill it in the following table: (2 points each)

	1	2	3	4	5	6	7	8	9	10	11	12	13
(a)													
(b)													
(c)													
(d)													

24

1. The distance between the point $(1,1,1)$ and the plane $2x+2y+z=6$ is:

a. $\frac{5}{3}$ b. $\frac{5}{9}$ c. $\frac{1}{3}$ d. $\frac{1}{9}$

2. Let L be a line in the 3-space where A is a point on L and U is a vector parallel to L. Then the distance between a point P ad the line L is given by:

a. $\|\overrightarrow{AP}-proj_U\overrightarrow{AP}\|$ b. $\|proj_U\overrightarrow{AP}\|$ c. $\|\overrightarrow{AP}\|$ d. $\|U\|$

3. Let U and V be two vectors in the 3-space such that $U\cdot V=-\frac{5}{13}$. The the angle between U and V is:

a. acute b. obstute c. right d. None

4. Let $U=\langle 1,1,1\rangle$, $V=\langle 2,-1,1\rangle$, $W=\langle 1,2,1\rangle$. Then $(v\times w).u=$

a. 1 b. -1 c. 3 d. -3

5. The area of the triangle ABC, where $\overrightarrow{AB}=\langle 2,-1,1\rangle$ and $\overrightarrow{AC}=\langle 1,2,1\rangle$ is:

a. $\sqrt{35}$ b. $\frac{1}{2}\sqrt{35}$ c. 35 d. $\frac{35}{2}$

6. The equation $r=2$ given in cylindrical coordinates represents: in 3-space

a. a sphere b. a plane c. a cylinder d. None

7. The rectangular coordinates of a point P are $(1,-1,-\sqrt{2})$. Then its spherical coordinates are:

a. $\left(2,\frac{\pi}{4},\frac{7\pi}{4}\right)$ b. $\left(2,\frac{\pi}{4},\frac{3\pi}{4}\right)$ c. $\left(2,\frac{7\pi}{4},\frac{7\pi}{4}\right)$ d. $\left(2,\frac{7\pi}{4},\frac{3\pi}{4}\right)$

8. Let U, V be two unit vectors in 3-space such that $U \bullet V = \frac{-5}{13}$. Then $\|U \times V\| =$

a. $\frac{5}{13}$ b. $-\frac{5}{13}$ c. $\frac{12}{13}$ d. $-\frac{12}{13}$

9. The equation $\rho = 2$ given in spherical coordinates represents:

a. an upper circular cone b. a cylinder
c. a sphere d. a lower circular cone

10. The lines $L_1: \frac{x-2}{2} = \frac{y-1}{4} = \frac{z}{-2}$ and $L_2: \frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{-1}$ are:

a. the same b. parallel c. orthogonal d. none of the previous

11. The distance between the the line $L: \frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{-1}$ and the line $L: \frac{x+1}{1} = \frac{y}{2} = \frac{z-1}{-1}$ equals:

a. 5 b. $\sqrt{3}$ c. $\sqrt{5}$ d. 3

12. Consider the two planes P_1: $2x+4y+6z-2=0$ and P_2: $x-z+1=0$. One of the following is an equation of the line passing through the point $(3,5,0)$ that is parallel to each of the planes P_1 and P_2:

a. $\frac{x-3}{2} = \frac{y-5}{4} = \frac{z}{-2}$ b. $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z}{1}$

c. $\frac{x-3}{2} = \frac{y-5}{4} = \frac{z}{2}$ d. $\frac{x+3}{1} = \frac{y+5}{2} = \frac{z}{1}$

13. Let u, v two orthogonal vectors in 3-space such that $\|u\| = 2, \|v\| = 3$. Then $(u \times v) \times v =$

a. 0 b. $6u$ c. $18u$ d. $-18u$

The Hashemite University
Department of Mathematics
Calculus III

First Exam
Date: 7 / 3/ 2010
Time: One Hour

Name: .... Number:..................................

Select the best correct answer and fill it in the following table: (2 points each)

	1	2	3	4	5	6	7	8	9	10	11	12	13
(a)													
(b)													
(c)													
(d)													

1. Let U, V be two unit vectors in 3-space such that $U \cdot V = \frac{12}{13}$. Then $\|U \times V\| =$

a. $\frac{5}{13}$ b. $-\frac{5}{13}$ c. $-\frac{12}{13}$ d. $\frac{12}{13}$

2. The equation $\theta = \frac{\pi}{6}$ given in cylindrical coordinates represents:

a. an upper circular cone b. a cylinder
c. a part of a plane d. a sphere

3. The lines $L_1 : \frac{x-2}{2} = \frac{y-1}{4} = \frac{z}{-2}$ and $L_2 : \frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{-1}$ are:

a. parallel b. the same c. orthogonal d. none of the previous

4. Let u, v two orthogonal vectors in 3-space such that $\|u\| = 2, \|v\| = 4$. Then $(u \times v) \times v =$

a. $-32u$ b. $8u$ c. $32u$ d. 0

* Consider the two planes $P_1: 2x + 4y + 6z - 2 = 0$ and $P_2: x - z + 1 = 0$. Use this to answer problems 5 and 6.

5. The equation of the plane passing through the point $(3,5,0)$ and perpendicular to each of the planes P_1 and P_2 is:

a. $2(x-3) + 4(y-5) + 2z = 0$ b. $(x-3) - 2(y-5) + z = 0$
c. $(x+3) + 2(y+5) + z = 0$ d. $2(x-3) + 4(y-5) - 2z = 0$

6. One of the following is an equation of the line passing through the point $(3,5,0)$ that is parallel to each of the planes P_1 and P_2:

a. $\dfrac{x-3}{2}=\dfrac{y-5}{4}=\dfrac{z}{-2}$ b. $\dfrac{x-3}{2}=\dfrac{y-5}{4}=\dfrac{z}{2}$

c. $\dfrac{x-3}{1}=\dfrac{y-5}{-2}=\dfrac{z}{1}$ d. $\dfrac{x+3}{1}=\dfrac{y+5}{2}=\dfrac{z}{1}$

7. The distance between the the line $L:\dfrac{x-2}{1}=\dfrac{y-1}{2}=\dfrac{z}{-1}$ and the line $L:\dfrac{x+1}{1}=\dfrac{y}{2}=\dfrac{z-1}{-1}$ equals:

a. 5 b. 3 c. $\sqrt{5}$ d. $\sqrt{3}$

8. The equation $r = 4\sin\theta$ given in cylindrical coordinates represents:

a. a sphere b. a plane c. an elliptic paraboloid d. a cylinder

9. The equation $\rho = 4\sin\phi$ given in spherical coordinates represents:

a. a plane b. a sphere c. a cylinder d. an elliptic paraboloid

10. The acute angle that the plane $x+y-z=2$ makes with the plane $x-y+z=1$ is:

a. $\cos^{-1}\dfrac{1}{\sqrt{3}}$ b. $\cos^{-1}\dfrac{1}{3}$ c. $\dfrac{\pi}{3}$ d. $\dfrac{\pi}{6}$

* Let u, v and w be three vectors in 3-space such that $(u\times w)\cdot v=-6$. Use this to answer 11, 12 and 13.

11. $(v\times w).2u=$

a. -12 b. 6 c. -6 d. 12

12. $(u\times 2u).w=$

a. -12 b. 12 c. 6 d. 0

13. The volume of the parallelpiped whose adjacent sides are u, v and w is:

a. 12 b. 6 c. 36 d. 0

14

الاسم: .... الرقم الجامعي: 938147

مدرس المادة: د. ماجد عبيدات وقت المحاضرة: 11 - 12 رقم التسلسل: 46

The Hashemite University | **Calculus 3** | **1st Semester 2010-2011**

Department of Math. | **101201** | **First Exam**

(Q1) 16 points: Select the best correct answer and fill it in the following table.

	1	2	3	4	5	6	7	8
a	✗			✓			✓	
b		✓	✓			✓		
c	✓	✗		✗	✓			
d								✓

= 10

1. The point of intersection of the line $x = 5 - 4t$, $y = 7 + 3t$, $z = 2 + t$ and the plane parallel to the xy-plane and containing the point $(0,0,1)$ is:
a. $(9,4,1)$ b. $(13,1,1)$ c. $(-7,16,1)$ d. $(-11,19,1)$

2. The two lines $L_1 : x = -3 + t$, $y = 7 + 3t$, $z = 5 - 2t$ and $L_2 : x = -5 - 2t$, $y = 3 - 6t$, $z = 7 + 4t$ are:
a. intersecting at a point but not perpendicular b. skew c. parallel d. perpendicular

3. The natural domain of $\vec{r}(t) = \sin \pi t \ \hat{i} + \sqrt{8 - 2t} \ \hat{j} + \ln(t - 3) \ \hat{k}$ is:
a. $(2,3]$ b. $(3,4]$ c. $[3,4)$ d. $[2,3)$

4. If the distance between the point $(1,-2,k)$ and the xy-plane is $2k$, then $k =$
a. 1 b. -2 c. 0 d. 2

$D = \frac{|0+0+k+0|}{\sqrt{1}}$

$2k = k$

5. The equation $4x^2 + 4y^2 - z^2 - 2z - 1 = 0$ represents:
a. a sphere b. a paraboloid c. a cone d. a point

6. The rectangular coordinates of the point $(\rho, \theta, \phi) = (4, \frac{\pi}{6}, \frac{\pi}{2})$ are given by $(x, y, z) =$
a. $(\sqrt{3},1,0)$ b. $(2\sqrt{3},2,0)$ c. $(3\sqrt{3},3,0)$ d. $(4\sqrt{3},4,0)$

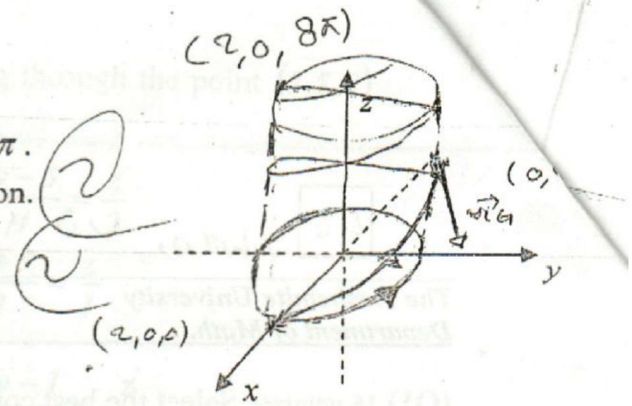
7. $\int_0^1 \left(\frac{1}{1+t^2} \hat{i} - e^t \ \hat{j} + 2t \ \hat{k} \right) dt =$
a. $\left\langle \frac{\pi}{4}, 1-e, 1 \right\rangle$ b. $\left\langle \frac{\pi}{4}, 1-e, \frac{1}{2} \right\rangle$ c. $\left\langle \frac{\pi}{2}, 1-e, 1 \right\rangle$ d. $\left\langle \frac{1}{2}, 1-e, \frac{1}{2} \right\rangle$

8. Let $\vec{u} = \langle 2,1,-1 \rangle$, $\vec{v} = \langle -4, m, 2 \rangle$, $\vec{w} = \langle 1,0,n \rangle$. The values of m and n such that $\vec{u}$ is parallel to $\vec{v}$, and $\vec{v}$ is perpendicular to $\vec{w}$ are:
a. $m = -3$ and $n = 3$ b. $m = 2$ and $n = -2$ c. $m = 3$ and $n = -3$ d. $m = -2$ and $n = 2$

(Q2) 9 points: Let C be the curve given by:

$$C:\ \vec{r}(t) = \langle 2\cos t, 2\sin t, 4t \rangle, \qquad 0 \le t \le 2\pi .$$

(a) *(2 points)* Sketch the graph of the curve C with its orientation.



(b) *(2 points)* Show that the parametric equations of the tangent line L_T to the curve C at $t = \dfrac{\pi}{2}$ are:

$$L_T:\ x = -2t,\ y = 2,\ z = 4t + 2\pi$$

helix equt:

$\vec{r}(t) = (2\cos t, 2\sin t, 4t)$

at $t = \frac{\pi}{2} \Rightarrow (0, 2, 2\pi)$ point

$\vec{r}'(t) = \langle -2\sin t, 2\cos t, 4 \rangle$

at $t = \frac{\pi}{2} \Rightarrow \vec{v} = \langle -2, 0, 4 \rangle$ vector

$L_T \Rightarrow x = -2t,\ y = 2,\ z = 4t + 2\pi$

(c) *(3 points)* Find the equation of the plane containing the two endpoints of the curve C and parallel to the line L_T in part *(b)* [write the details, not only the final answer.]

L_T: $x = -2t,\ y = 2,\ z = 4t + 2\pi$ $(2, 0, 0), (2, 0, 8\pi)$

(d) *(2 points)* In general, for given two points A, B and a given line L, how many planes in 3-space does the following sentence determine? Consider all cases.

"The plane containing two points A, B and parallel to a line L."

line // line

; there are ~~one~~ infinite planes

becaus every plane has at least 3 point

~~two~~ A, B and the last from the line

and the line have infinit point

The Hashemite University
Department of Mathematics
Calculus III

First Exam
Date: 15 / 11/ 2009
Time: One Hour

Name: ....[redacted].... Number:....29.......................

23

Select the best correct answer and fill it in the following table: (2 points each)

	1	2	3	4	5	6	7	8	9	10	11	12
(a)		✓									✓	
(b)	✓			✓								✓
(c)	✓	✓		✓		✓						✓
(d)			✓		✓		✓	✓	✓	✓		

1. The equation $r = 5$ given in cylindrical coordinates represents:

a. an upper circular cone b. a cylinder

c. a lower circular cone d. an elliptic paraboloid

2. The lines $L_1 : \dfrac{x-2}{2} = \dfrac{y-1}{-2} = \dfrac{z}{-2}$ and $L_2 : \dfrac{x-2}{2} = \dfrac{y-1}{4} = \dfrac{z}{-2}$ are:

a. orthogonal ~~b.~~ parallel $\boxed{\text{c. the same}}$ ~~d.~~ none of the previous

3. The distance between the the point $(2,1,0)$ and the line $L : \dfrac{x+1}{1} = \dfrac{y}{2} = \dfrac{z-1}{-1}$ equals:

a. 3 b. $\sqrt{3}$ c. 5 d. $\sqrt{5}$

* Let u, v and w be three vectors in 3-space such that $v.(u \times w) = -6$. Use this to answer 4, 5 and 6.

4. $(v \times w).2u =$

a. -6 b. 12 c. 6 d. -12

5. $(u \times 2u).w =$

a. -10 b. 10 c. 5 d. 0

6. The volume of the parallelpiped whose adjacent sides are u, v and w is:

a. 36 b. 12 c. 6 d. 0

* Consider the two planes P_1: $2x + 4y + 6z - 2 = 0$ and P_2: $x - z + 1 = 0$. Use this to answer problems 7 and 8.

$\vec{n}_1 (2,4,6)$
$n_2 (1, 0, -1)$

$\begin{vmatrix} 2 & 4 & 6 \\ 1 & 0 & -1 \end{vmatrix} \Rightarrow -4\hat{i} - 8\hat{j} - 4\hat{k}$ $(2,1,0)$

$\Rightarrow 2(x-2) + 4(y-1)$
$2z = 0.0$

$(x-2)\dfrac{(y-1)}{2}$

7. The equation of the plane passing through the point $(2,1,0)$ and perpendicular to each of the planes P_1 and P_2 is:

a. $2(x-2)+4(y-1)-2z=0$ b. $(x-2)+2(y-1)+z=0$

c. $2(x-2)+4(y-1)+2z=0$ d. $(x-2)-2(y-1)+z=0$

8. One of the following is an equation of the line passing through the point $(2,1,0)$ that is parallel to each of the planes P_1 and P_2:

a. $\dfrac{x-2}{2}=\dfrac{y-1}{4}=\dfrac{z}{2}$ b. $\dfrac{x+2}{1}=\dfrac{y+1}{2}=\dfrac{z}{1}$

c. $\dfrac{x-2}{2}=\dfrac{y-1}{4}=\dfrac{z}{-2}$ d. $\dfrac{x-2}{1}=\dfrac{y-1}{-2}=\dfrac{z}{1}$

9. The acute angle that the plane $x+2y-z=2$ makes with the plane $2x-y+z=1$ is:

a. $\dfrac{\pi}{6}$ b. $\cos^{-1}\dfrac{1}{\sqrt{6}}$ c. $\dfrac{\pi}{3}$ d. $\cos^{-1}\dfrac{1}{6}$

10. The equation $\rho\cos\phi=5$ given in spherical coordinates represents:

a. a sphere b. an elliptic paraboloid c. a cylinder d. a plane

11. Let U, V be two unit vectors in 3-space such that U and V make an acute angle and $\|U\times V\|=\dfrac{12}{13}$. Then $U\cdot V=$

a. $\dfrac{5}{13}$ b. $-\dfrac{5}{13}$ c. $\dfrac{12}{13}$ d. $-\dfrac{12}{13}$

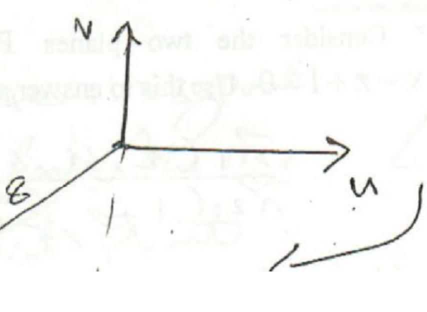
12. Let u, v two orthogonal vectors in 3-space such that $\|u\|=2, \|v\|=4$. Then $u\times(u\times v)=$

a. 0 b. $\boxed{-16v}$ c. $16v$ d. $8v$

$u\times(u\times v)$

$u\times 8$

$=16$



٩

٦٥، قرش

The Hashemite University	First Exam	November 8, 2012
Department of Mathematics	Calculus (3)	Time: One Hour.

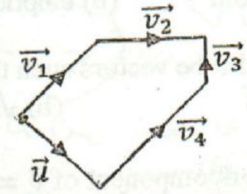
اسم الطالب: [redacted] الرقم الجامعي: [redacted] رقم التسلسل: 3

مدرس المادة: د. عز حرز الله . وقت المحاضرة: 10-11

Question One (12 points): Complete each of the following sentences by filling your answer in the box $\boxed{}$

2

1. Let $\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}$, and $\vec{u}$ be the vectors given in the figure. Then the vector $\vec{u}$ can be expressed in terms of the vectors $\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}$ as



$$\boxed{\vec{u} = \vec{v_4} + \vec{v_3} - \vec{v_2} - \vec{v_1}}$$

2. Let $\vec{u}$ and $\vec{v}$ be two vectors such that $\vec{u} = i - j$, $\|\vec{v}\| = 1$ and $\vec{u}\cdot\vec{v} = -\sqrt{2}$. Then

$$\boxed{\vec{v} = \frac{-1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j}$$

3. The spherical coordinates of the point with cylindrical coordinates $(r, \theta, z) = (1, \frac{7\pi}{4}, 1)$ are

$$\boxed{(\rho, \theta, \varphi) = \left(1, \frac{7\pi}{4}, \cos^{-1}(\ldots)\right)}$$

4. The equation of the plane that is tangent to the sphere $(x-1)^2 + y^2 + z^2 = 20$ at the point $A(1, -4, -2)$ is

$$\boxed{}$$

5. The area A of the parallelogram that has $\vec{u} = i - j + 2k$ and $\vec{v} = 3j + k$ as adjacent sides is

$$\boxed{A = }$$

6. The cylindrical coordinates of the point with rectangular coordinates $(x, y, z) = (5\sqrt{3}, -5, 6)$ are

$$\boxed{(r, \theta, z) = \left(\frac{-1}{10\sqrt{3}}, \frac{5\pi}{3}, 6\right)}$$

7

Question Two (14 points): Choose the best correct answer and fill it in the following table.

	1	2	3	4	5	6	7
a							
b							
c							
d							

1. The distance D between the point $P(1,3)$ and the line through $A(2,1)$ and $B(0,2)$ is
   (a) $\frac{1}{\sqrt{5}}$ (b) $\frac{4}{\sqrt{5}}$ (c) $\frac{6}{\sqrt{5}}$ (d) $\frac{3}{\sqrt{5}}$

2. The surface $z = \frac{x^2}{4} + \frac{y^2}{2}$ is
   (a) ellipsoid (b) elliptic paraboloid (c) elliptic cone (d) a sphere

3. Let $\vec{u}$ and $\vec{v}$ be vectors such that $\|\vec{u}\| = 2$, $\|\vec{v}\| = 1$ and $\vec{u} \cdot \vec{v} = -3$. Then $\|\vec{u} - 3\vec{v}\| =$
   (a) $\sqrt{103}$ (b) $\sqrt{63}$ (c) $\sqrt{31}$ (d) $\sqrt{58}$

4. The vector component of $\vec{v} = i + j + k$ orthogonal to $\vec{b} = 2i + 2j$ is
   (a) k (b) $i - 2k$ (c) $i + 2j - k$ (d) $3j + k$

5. Let L be the line through the point $(1,1,0)$ and parallel to the line:
   $$x = t, y = 2 - t, z = 3.$$
   Then the parametric equations of the line L are
   (a) $x = 1 + t, y = 1 - t, z = 3$ (b) $x = 1 + t, y = -1 + t, z = 3$
   (c) $x = 1 + t, y = 1 - t, z = 0.$ (d) $x = 1 + t, y = -1 + t, z = 0$

6. If the surface $\varphi = \frac{5\pi}{4}$ is given in spherical coordinates, then the equation of this surface can be expressed in rectangular coordinates as
   (a) $z = -\sqrt{x^2 + y^2}$ (b) $z^2 = x^2 + y^2$ (c) $z = \sqrt{x^2 + y^2}$ (d) $z = x^2 + y^2$

7. Let $\vec{r}(t) = (t-1)e^t i - \frac{4t}{t^2+1} j$. Then
   $$\int_1^2 \vec{r}(t)dt =$$
   (a) $ei - 3\ln(\frac{5}{2})j$ (b) $ei - 4\ln(\frac{5}{2})j$ (c) $ei - 5\ln(\frac{5}{2})j$ (d) $ei - 2\ln(\frac{5}{2})j$

End of Exam

Good Luck

HashemiteUniversity
ath. Department

Calculus(3) , FirstExam
First semester 2010/2011

26

Date: OCT 24, 2011
Time: 1 hour

			1	0	3	3	6	2	0

الرقم الجامعي:

اسم الطالب: جنين أيمن إبراهيم ابو[illegible] ....

الرقم المتسلسل: 3 موعد المحاضرة: 12-1

Warning: You MAY NOT use a calculator on this exam.

For questions 1-13, Select the correct answer and fill it in the following table: **(2 points each)**.

	1	2	3	4	5	6	7	8	9	10	11	12	13
a	✓		✓										
b				✓					✓	✓	✓	✓	
c		✓			✓	✓	✗	✓					✓
d		✗					✓						

$\sqrt{(2)(3)-(2)(4)}$

1. In 3−Space, the equation: $x^2+y^2=2$ represents a

$x^2+y^2=2$

(a) Cylindrical surface (b) Quadric surface

(c) Line → one space (d) Circle. $(x-x_0)^2+\ldots$ in 2-Space.

$\frac{18}{4^3}-\frac{16}{3}$

$\ldots + u_2v_2 + u_3v_3 = 4^3$

2. Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors with $\|\mathbf{u}\|=2$, $\|\mathbf{v}\|=3$ and $\mathbf{u}\cdot\mathbf{v}=4$. Then $\|\mathbf{u}+\mathbf{v}\|=$ $\sqrt{9+4}=$

$\|u+v\|^2=(u+v)\cdot(u+v)$

(a) 13 (b) $\sqrt{21}$ (c) $\sqrt{5}$ (d) 21

$=\sqrt{5}$

$\frac{4}{6}$

$u \times a = 13$ $\sqrt{9-2(4)}$

$9+8$

3. The natural domain of the function $\mathbf{r}(t)=\sqrt{t}\mathbf{i}+\ln t\mathbf{j}+\frac{\sin t}{t}\mathbf{k}$ is

(a) $(0,\infty)$ (b) $[0,\infty)$ (c) $(0,1)$ (d) $[0,1)$

4. The intersection between the line **L**: $x=2-2t$, $y=2t$, $z=5t$ and the plane **P**: $x+y+z=-8$ is

(a) $(2,0,0)$ (b) $(6,-4,-10)$ (c) $(0,-4,-10)$ (d) $(0,0,-8)$

5. The distance between the planes $\mathbf{P_1}$: $2x+2y+z=8$ and $\mathbf{P_2}$: $2x+2y+z=-4$ is;

(a) 12 (b) $\frac{4}{3}$ (c) $\frac{12}{\sqrt{5}}$ (d) 4

6. In 3−Space, the equation: $x^2+y^2+z^2-2x+4y=-6$ represents

$(x-1)^2+(y+2)^2+z^2=-6+1+4=(-1)$

(a) Sphere (b) Point (c) Has no graph (d) an elliptic paraboloid

7. The equation of the trace of the surface $z=x^2-y^2$ in the plane $y=4$ is

(a) A circle (b) An ellipse (c) A Parabola (d) A Hyperbola

$9+2(4)+4$

e equation $x^2+y^2=4$ in spherical coordinates is

$= 4\csc\phi$ (b) $\rho = 2\csc\theta$

$= 2\csc\phi$ (d) $\rho = 4\csc\theta$

t $\mathbf{v}=\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{b}=\mathbf{j}-\mathbf{k}$. Then $\mathbf{proj_b v} =$

$\mathbf{j}-\mathbf{k}$ (b) $-\mathbf{j}+\mathbf{k}$ (c) $-\sqrt{2}\mathbf{i}-\sqrt{2}\mathbf{k}$ (d) $-2\mathbf{j}-2\mathbf{k}$.

The unit vector that has the same direction as the vector $\mathbf{v} = 2\mathbf{i}-2\mathbf{j}+\mathbf{k}$ is $\mathbf{u} =$

$\mathbf{i}+\frac{1}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}$ (b) $\frac{2}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{1}{3}\mathbf{k}$

$\mathbf{i}+\frac{2}{3}\mathbf{j}+\frac{1}{3}\mathbf{k}$ (d) $\frac{2}{3}\mathbf{i}-\frac{1}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$.

Let $\mathbf{v}=\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{u}=\mathbf{i}+\mathbf{j}-\mathbf{k}$. Then $\cancel{\mathbf{u}\times\mathbf{v}} =$ $v \times u$

$\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ (b) $-2\mathbf{i}-2\mathbf{j}$ (c) $2\mathbf{i}+2\mathbf{j}$ (d) $-2\mathbf{i}+2\mathbf{j}$

The vector equation of the tangent line $\mathbf{L_T}$ to the graph of the circular helix $\mathbf{C}$: $\mathbf{r}(t) = \cos t\mathbf{i}+\sin t\mathbf{j}+t\mathbf{k}$ $= \pi$ is

$\mathbf{L_T}$: $\mathbf{r}(t) = -t\mathbf{i}+\mathbf{j}+(\pi+t)\mathbf{k}$ (b) $\mathbf{L_T}$: $\mathbf{r}(t) = -\mathbf{i}-\mathbf{j}+(\pi+t)\mathbf{k}$

$\mathbf{L_T}$: $\mathbf{r}(t) = -t\mathbf{i}+t\mathbf{j}+(\pi+t)\mathbf{k}$ (d) $\mathbf{L_T}$: $\mathbf{r}(t) = -\mathbf{i}+t\mathbf{j}+(\pi+t)\mathbf{k}$

Let $\mathbf{C}$: $\mathbf{r}(t) = -2t\mathbf{i}-\cos\pi t\mathbf{j}+\frac{1}{1+t^2}\mathbf{k}$. Then $\int_0^1 \mathbf{r}(t)dt =$

$+\frac{1}{\pi}\mathbf{j}+\frac{\pi}{4}\mathbf{k}$ (b) $\mathbf{i}+\mathbf{j}+\mathbf{k}$ (c) $-\mathbf{i}+\frac{\pi}{4}\mathbf{k}$ (d) $\mathbf{i}+\frac{\pi}{4}\mathbf{k}$

e HashemiteUniversity
Math. Department

Calculus(3) FirstExam
~~First semester~~ 2010/2011

Date: OCT 24, 2011
Time: 1 hour

الرقم الجامعي:

		١	٠	٣	٣	٦	٥	٨

اسم الطالب: جيبس يونس كساد ميجك.........

الرقم المتسلسل: 3... موعد المحاضرة: 11 - 12

Warning: You MAY NOT use a calculator on this exam.

For questions 1-13, Select the correct answer and fill it in the following table: **(2 points each).**

	1	2	3	4	5	6	7	8	9	10	11	12	13
a	////			////			////		////				
b		~~////~~	////		////	////							
c		////						////		////	////	////	
d							X						////

23

1. In 3−Space, the equation: $x + y = 2$ represents a

(a) Cylindrical surface (b) Quadric surface
(c) Line (d) Circle.

2. Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors with $\|\mathbf{u}\| = 2$, $\|\mathbf{v}\| = 3$, and $\mathbf{u}\cdot\mathbf{v} = 4$. Then $\|\mathbf{u}-\mathbf{v}\| =$

(a) 5 (b) 3 (c) $\sqrt{5}$ (d) 9

$\|u-v\|$ $\sqrt{13}$ $\sqrt{5}$

3. The natural domain of the function $\mathbf{r}(t) = \sqrt{t}\,\mathbf{i} + \ln t\,\mathbf{j} + \dfrac{\sin t}{t}\mathbf{k}$ is

(a) $[0,\infty)$ (b) $(0,\infty)$ (c) $[0,1)$ (d) $(0,1)$

4. The intersection between the line **L:** $x = 1-2t$, $y = 2t$, $z = 5t$ and the plane **P:** $x+y+z = 6$ is

(a) $(-1,2,5)$ (b) $(1,0,0)$ (c) $(3,-2,-5)$ (d) $(0,0,6)$

5. The distance between the planes $\mathbf{P_1}$: $2x+2y+z = 4$ and $\mathbf{P_2}$: $2x+2y+z = -4$ is;

(a) 8 (b) $\dfrac{8}{3}$ (c) $\dfrac{8}{\sqrt{5}}$ (d) 4

6. In 3−Space, the equation: $x^2+y^2+z^2-2x+4y = -5$ represents

(a) Sphere (b) Point (c) Has no graph (d) an elliptic paraboloid

7. The equation of the trace of the surface $z = x^2-y^2$ in the plane $z = 4$ is

(a) A circle (b) An ellipse (c) A Parabola (d) A Hyperbola

$$\|u-v\| = \sqrt{(u-v)\cdot(u-v)}$$
$$= \sqrt{\|u\|^2 - u\cdot v - u\cdot v + \|v\|^2}$$

8. The equation $x^2+y^2=1$ in spherical coordinates is

(a) $\rho = \csc\theta$ (b) $\rho = \sec\phi$

(c) $\rho = \csc\phi$ (d) $\rho = \sec\theta$

$(\rho\sin\phi)^2 = 1$

$\rho^2 \sin^2\phi = 1$

$\rho^2 = \frac{1}{\sin^2\phi}$

$\rho^2 = \csc^2\phi$

$\rho = \csc\phi$

9. Let $\mathbf{v}=\mathbf{i}-\mathbf{j}-\mathbf{k}$ and $\mathbf{b}=\mathbf{i}-\mathbf{k}$. Then $\mathbf{proj_b v} =$

(a) $\mathbf{i}-\mathbf{k}$ (b) $\mathbf{i}+\mathbf{k}$ (c) $\sqrt{2}\mathbf{i}-\sqrt{2}\mathbf{k}$ (d) $\frac{2}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$.

10. The unit vector that has the same direction as the vector $\mathbf{v} = 4\mathbf{i}+4\mathbf{j}+2\mathbf{k}$ is $\mathbf{u} =$

(a) $\frac{2}{3}\mathbf{i}+\frac{1}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}$ (b) $\frac{2}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{1}{3}\mathbf{k}$

(c) $\frac{2}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}+\frac{1}{3}\mathbf{k}$ (d) $\frac{2}{3}\mathbf{i}-\frac{1}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$.

$\frac{v}{\sqrt{16+16+4}}$ $\frac{4i+4j+2k}{\sqrt{36}}$ $\frac{2}{3}i + \frac{2}{3}j + \frac{1}{3}k$

11. Let $\mathbf{v}=\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{u}=\mathbf{i}+\mathbf{j}-\mathbf{k}$. Then $\mathbf{u}\times\mathbf{v} =$

(a) $2\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ (b) $\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ (c) $2\mathbf{j}+2\mathbf{k}$ (d) $-2\mathbf{j}+2\mathbf{k}$

12. The vector equation of the tangent line $\mathbf{L_T}$ to the graph of the circular helix $\mathbf{C}$: $\mathbf{r}(t) = \cos t\mathbf{i}+\sin t\mathbf{j}+$ at $t=\frac{\pi}{2}$ is

(a) $\mathbf{L_T}$: $\mathbf{r}(t) = -t\mathbf{i}+t\mathbf{j}+(\frac{\pi}{2}+t)\mathbf{k}$ (b) $\mathbf{L_T}$: $\mathbf{r}(t) = -t\mathbf{i}+\mathbf{j}+(\frac{\pi}{2}+1)\mathbf{k}$

(c) $\mathbf{L_T}$: $\mathbf{r}(t) = -t\mathbf{i}+\mathbf{j}+(\frac{\pi}{2}+t)\mathbf{k}$ (d) $\mathbf{L_T}$: $\mathbf{r}(t) = -\mathbf{i}+t\mathbf{j}+(\frac{\pi}{2}+t)\mathbf{k}$

$r'(t) = -\sin t\, i + \ldots + k$

$r'(\frac{\pi}{2})$ $-i+k$

13. Let $\mathbf{C}$: $\mathbf{r}(t) = 2t\mathbf{i}-\cos\pi t\mathbf{j}+\frac{1}{1+t^2}\mathbf{k}$. Then $\int_0^1 \mathbf{r}(t)dt =$

(a) $\mathbf{i}+\frac{1}{\pi}\mathbf{j}+\frac{\pi}{4}\mathbf{k}$ (b) $\mathbf{i}+\mathbf{j}+\mathbf{k}$ (c) $-\mathbf{i}+\frac{\pi}{4}\mathbf{k}$ (d) $\mathbf{i}+\frac{\pi}{4}\mathbf{k}$

$L_T = j + \frac{\pi}{2}k + t(-i \ldots$

$= -ti + j + (\frac{\pi}{2} + t)\ldots$

$\langle 2t, -\cos\pi t, \frac{1}{1+t^2} \rangle$

$\langle t^2]_0^1, -\sin\pi t]_0^1, \tan^{-1}]_0^1 \rangle$

$\langle (1-0), 0, \frac{\pi}{4} \rangle$

$i + \frac{\pi}{4}k$

4

3

HashemiteUniversity

... Department

Calculus(3) , FirstExam

First semester 2010/2011

Date: OCT 24, 2011

Time: 1 hour

اسم الطالب: [redacted] ....

الرقم الجامعي: ١٠٣٣٦٣٩

الرقم المتسلسل: ...٣٦... موعد المحاضرة: ...٩ ـ ١٠...

Warning: You MAY NOT use a calculator on this exam.

For questions 1-13, Select the correct answer and fill it in the following table: **(2 points each)**.

	1	2	3	4	5	6	7	8	9	10	11	12	13
a	✓			✓		✓			✓				
b			✓		✓								
c		✓						✓		✓	✓	✓	
d							✓						✓

23

1. In 3−Space, the equation: $x + y = 2$ represents a

(a) Cylindrical surface (b) Quadric surface
(c) Line (d) Circle.

2. Let **u** and **v** be two vectors with $||\mathbf{u}|| = 2$, $||\mathbf{v}|| = 3$, and $\mathbf{u} \cdot \mathbf{v} = 4$. Then $||\mathbf{u} - \mathbf{v}|| =$

(a) 5 (b) 3 (c) $\sqrt{5}$ (d) 9

3. The natural domain of the function $\mathbf{r}(t) = \sqrt{t}\mathbf{i} + \ln t\mathbf{j} + \frac{\sin t}{t}\mathbf{k}$ is

(a) $[0, \infty)$ (b) $(0, \infty)$ (c) $[0, 1)$ (d) $(0, 1)$

4. The intersection between the line **L:** $x = 1 - 2t$, $y = 2t$, $z = 5t$ and the plane **P:** $x + y + z = 6$ is

(a) (-1, 2, 5) (b) $(1, 0, 0)$ (c) $(3, -2, -5)$ (d) $(0, 0, 6)$

5. The distance between the planes $\mathbf{P_1}$: $2x + 2y + z = 4$ and $\mathbf{P_2}$: $2x + 2y + z = -4$ is;

(a) 8 (b) $\frac{8}{3}$ (c) $\frac{8}{\sqrt{5}}$ (d) 4

6. In 3−Space, the equation: $x^2 + y^2 + z^2 - 2x + 4y = -5$ represents

(a) Sphere (b) Point (c) Has no graph (d) an elliptic paraboloid

7. The equation of the trace of the surface $z = x^2 - y^2$ in the plane $z = 4$ is

(a) A circle (b) An ellipse (c) A Parabola (d) A Hyperbola

8. The equation $x^2+y^2=1$ in spherical coordinates is

(a) $\rho=\csc\theta$ (b) $\rho=\sec\phi$
(c) $\rho=\csc\phi$ (d) $\rho=\sec\theta$

9. Let $\mathbf{v}=\mathbf{i}-\mathbf{j}-\mathbf{k}$ and $\mathbf{b}=\mathbf{i}-\mathbf{k}$. Then $\mathbf{proj_b v}=$

(a) $\mathbf{i}-\mathbf{k}$ (b) $\mathbf{i}+\mathbf{k}$ (c) $\sqrt{2}\mathbf{i}-\sqrt{2}\mathbf{k}$ (d) $\frac{2}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$.

10. The unit vector that has the same direction as the vector $\mathbf{v}=4\mathbf{i}+4\mathbf{j}+2\mathbf{k}$ is $\mathbf{u}=$

(a) $\frac{2}{3}\mathbf{i}+\frac{1}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}$ (b) $\frac{2}{3}\mathbf{i}-\frac{2}{3}\mathbf{j}+\frac{1}{3}\mathbf{k}$
(c) $\frac{2}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}+\frac{1}{3}\mathbf{k}$ (d) $\frac{2}{3}\mathbf{i}-\frac{1}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$.

~~X 11 خطأ~~

11. Let $\mathbf{v}=\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{u}=\mathbf{i}+\mathbf{j}-\mathbf{k}$. Then $\mathbf{u}\times\mathbf{v}=$

(a) $2\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ (b) $\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ (c) $-2\mathbf{j}\mp2\mathbf{k}$ (d) $-2\mathbf{j}+2\mathbf{k}$

12. The vector equation of the tangent line $\mathbf{L_T}$ to the graph of the circular helix $\mathbf{C}$: $\mathbf{r}(t)=\cos t\mathbf{i}+\sin t\mathbf{j}+$ at $t=\frac{\pi}{2}$ is

(a) $\mathbf{L_T}$: $\mathbf{r}(t)=-t\mathbf{i}+t\mathbf{j}+(\frac{\pi}{2}+t)\mathbf{k}$ (b) $\mathbf{L_T}$: $\mathbf{r}(t)=-t\mathbf{i}+\mathbf{j}+(\frac{\pi}{2}+1)\mathbf{k}$
(c) $\mathbf{L_T}$: $\mathbf{r}(t)=-t\mathbf{i}+\mathbf{j}+(\frac{\pi}{2}+t)\mathbf{k}$ (d) $\mathbf{L_T}$: $\mathbf{r}(t)=-\mathbf{i}+t\mathbf{j}+(\frac{\pi}{2}+t)\mathbf{k}$

13. Let $\mathbf{C}$: $\mathbf{r}(t)=2t\mathbf{i}-\cos\pi t\mathbf{j}+\frac{1}{1+t^2}\mathbf{k}$. Then $\int_0^1\mathbf{r}(t)dt=$

(a) $\mathbf{i}+\frac{1}{\pi}\mathbf{j}+\frac{\pi}{4}\mathbf{k}$ (b) $\mathbf{i}+\mathbf{j}+\mathbf{k}$ (c) $-\mathbf{i}+\frac{\pi}{4}\mathbf{k}$ (d) $\mathbf{i}+\frac{\pi}{4}\mathbf{k}$

*ملاحظة دائرة 11 يوجد خطأ"
لازم يطلع الجواب $\boxed{-2\mathbf{j}-2\mathbf{k}}$

The Hashemite University **First Exam**
Department of Mathematics
Calculus III **Time: One Hour**

Name: ……………… …… ……… Number:……………………………

Select the best correct answer and fill it in the following table: (2 points each)

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
(a)						X		/	/			/	
(b)	X				/		X	✓	✓		/		/
(c)	/		✓		X	/	/			/	✓	X	✓
(d)		✓		✓						X			

1. Let $U=\langle 3,3,3\rangle$, $V=\langle 2,-1,1\rangle$, $W=\langle 1,2,1\rangle$. Then $(v\times w).u=$

a. 1 b. 3 c. -1 d. -3

2. The equation $\rho=2$ given in spherical coordinates represents:

a. an upper circular cone b. a cylinder
c. a lower circular cone d. a sphere

3. The rectangular coordinates of a point P are $(1,-1,-\sqrt{2})$. Then its spherical coordinates are:

a. $\left(2,\frac{\pi}{4},\frac{7\pi}{4}\right)$ b. $\left(2,\frac{\pi}{4},\frac{3\pi}{4}\right)$ c. $\left(2,\frac{7\pi}{4},\frac{3\pi}{4}\right)$ d. $\left(2,\frac{7\pi}{4},\frac{7\pi}{4}\right)$

$z=\rho\cos\phi$ $x=\rho\sin\phi\cos\theta$ $y=\rho\sin\phi\sin\theta$ 315 135

4. Consider the two planes P_1: $2x+4y+6z-2=0$ and P_2: $x-z+1=0$. One of the following is an equation of the line passing through the point $(3,5,0)$ that is parallel to each of the planes P_1 and P_2:

a. $\frac{x-3}{2}=\frac{y-5}{4}=\frac{z}{-2}$ b. $\frac{x+3}{1}=\frac{y+5}{2}=\frac{z}{1}$

c. $\frac{x-3}{2}=\frac{y-5}{4}=\frac{z}{2}$ d. $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z}{1}$

5. The lines $L_1:\frac{x-2}{2}=\frac{y-1}{4}=\frac{z}{-2}$ and $L_2:\frac{x-2}{1}=\frac{y-1}{2}=\frac{z}{-1}$ are: توازي

a. orthogonal b. parallel c. the same d. none of the previous

6. Let u, v two orthogonal vectors in 3-space such that $\|u\|=2,\|v\|=3$. Then $(u\times v)\times v=$

a. $-18u$ b. $6u$ c. $18u$ d. 0

The Hashemite University
Department of Mathematics
Calculus III

First Exam
Time: One Hour

Name: .... Number:....................................

Select the best correct answer and fill it in the following table: (2 points each)

	1	2	3	4	5	6	7	8	9	10	11	12	13
(a)						X		✓	✓			/	
(b)	X				/		X				✓		✓
(c)	/		✓		X	/	/			/		X	
(d)		✓		✓						X			

1\. Let $U = \langle 3,3,3\rangle$, $V = \langle 2,-1,1\rangle$, $W = \langle 1,2,1\rangle$. Then $(v \times w).u =$

a. 1 b. 3 c. -1 d. -3

2\. The equation $\rho = 2$ given in spherical coordinates represents:

a. an upper circular cone b. a cylinder
c. a lower circular cone d. a sphere

3\. The rectangular coordinates of a point P are $(1,-1,-\sqrt{2})$. Then its spherical coordinates are:

a. $\left(2,\frac{\pi}{4},\frac{7\pi}{4}\right)$ b. $\left(2,\frac{\pi}{4},\frac{3\pi}{4}\right)$ c. $\left(2,\frac{7\pi}{4},\frac{3\pi}{4}\right)$ d. $\left(2,\frac{7\pi}{4},\frac{7\pi}{4}\right)$

$x=\rho \sin\phi \cos\theta$
$y=\rho \sin\phi \sin\theta$
$z=\rho\cos\phi$

4\. Consider the two planes P_1: $2x+4y+6z-2=0$ and P_2: $x-z+1=0$. One of the following is an equation of the line passing through the point $(3,5,0)$ that is parallel to each of the planes P_1 and P_2:

a. $\frac{x-3}{2}=\frac{y-5}{4}=\frac{z}{-2}$ b. $\frac{x+3}{1}=\frac{y+5}{2}=\frac{z}{1}$

c. $\frac{x-3}{2}=\frac{y-5}{4}=\frac{z}{2}$ d. $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z}{1}$

5\. The lines $L_1 : \frac{x-2}{2}=\frac{y-1}{4}=\frac{z}{-2}$ and $L_2 : \frac{x-2}{1}=\frac{y-1}{2}=\frac{z}{-1}$ are:

a. orthogonal b. parallel c. the same d. none of the previous

6\. Let u, v two orthogonal vectors in 3-space such that $\|u\|=2, \|v\|=3$. Then $(u\times v)\times v =$

a. $-18u$ b. $6u$ c. $18u$ d. 0

7. The equation $r = 2$ given in cylindrical coordinates represents:

a. a sphere b. a cylinder c. a plane d. None

8. Let U, V be two unit vectors in 3-space such that $\|U \times V\| = \frac{+12}{13}$. Then $U \cdot V$ =

a. $-\frac{5}{13}$ b. $\frac{5}{13}$ c. $\frac{12}{13}$ d. $-\frac{12}{13}$

9. Let U and V be two vectors in the 3-space such that $U \cdot V = -\frac{5}{13}$. The the angle between U and V is:

a. obstute b. acute c. right d. None

10. The distance between the the line $L: \frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{-1}$ and the line $L: \frac{x+1}{1} = \frac{y}{2} = \frac{z-1}{-1}$ equals:

a. 5 b. $\sqrt{3}$ c. 3. d. $\sqrt{5}$

11. The distance between the point $(1,1,1)$ and the plane $2x+2y+z=6$ is:

a. $\frac{5}{3}$ b. $\frac{1}{3}$ c. $\frac{5}{9}$ d. $\frac{1}{9}$

12. The area of the triangle ABC, where $\overrightarrow{AB} = \langle 2,-1,1\rangle$ and $\overrightarrow{AC} = \langle 1,2,1\rangle$ is:

a. $\sqrt{35}$ b. 35 c. $\frac{1}{2}\sqrt{35}$ d. $\frac{35}{2}$

13. Let L be a line in the 3-space where A is a point on L and U is a vector parallel to L. Then the distance between a point P ad the line L is given by:

a. $\|proj_U \overrightarrow{AP}\|$ b. $\|\overrightarrow{AP} - proj_U \overrightarrow{AP}\|$ c. $\|\overrightarrow{AP}\|$ d. $\|U\|$

cos φ = c — plane   r = c — cylinder

ρ = c   φ = c — cone

The Hashemite University
Department of Mathematics
Calculus III

First Exam
Date: 7 / 3/ 2010
Time: One Hour

Name: ……. … … ……………   Number: ……………………

Select the best correct answer and fill it in the following table: (2 points each)

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
(a)	✓			✓					✓	✓	✓	✓	
(b)	X			X	✓	✓	✓				X		✓
(c)		✓	✓		X			✓					X
(d)		X											

$r = \frac{2y}{r}$ $r^2 = 2y$ $\frac{y}{r} = r\sin\theta$

14

16

1. The equation $r = 2\sin\theta$ given in cylindrical coordinates represents:

a. a sphere   b. a plane   c. a cylinder   d. an elliptic paraboloid

$x^2 + y^2 = 2y$

2. The equation $\rho = 2\sin\phi$ given in spherical coordinates represents:

a. a sphere   b. a plane   c. a cylinder   d. an elliptic paraboloid

$\rho^2 = 2r$ $r = \rho\sin\phi$

$x^2 + y^2 - 2y + 1$

$x^2 + (y-1)^2$

3. Let U, V be two unit vectors in 3-space such that $U \cdot V = \frac{5}{13}$. Then $\|U \times V\|$ =

a. $\frac{5}{13}$ b. $-\frac{5}{13}$ c. $\frac{12}{13}$ d. $-\frac{12}{13}$

4. The equation $\theta = \frac{\pi}{3}$ given in cylindrical coordinates represents:

a. an upper circular cone   b. a part of a plane
c. a cylinder   d. a sphere

5. The lines $L_1 : \frac{x-2}{2} = \frac{y-1}{4} = \frac{z}{-2}$ and $L_2 : \frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{-1}$ are:

a. the same   b. parallel   c. orthogonal   d. none of the previous

6. The distance between the the line $L : \frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{-1}$ and the line $L : \frac{x+1}{1} = \frac{y}{2} = \frac{z-1}{-1}$ equals:

a. 5   b. $\sqrt{5}$ c. 3.   d. $\sqrt{3}$

$\|\vec{AP} \times u\|$

$A(2,1,0)$ $\vec{AP} = \langle$

$P(-1, 0, 1)$

* Let u, v and w be three vectors in 3-space such that $(u \times w) \cdot v = -7$. Use this to answer 7,8 and 9.

7. $(v \times w).2u =$

a. -14 b. 14 c. 7 d. -7

8. $(u \times 2u).w =$

a. -14 b. 14 c. 0 d. 7

9. The volume of the parallelpiped whose adjacent sides are u, v and w is:

a. 7 b. 14 c. 49 d. 0

* Consider the two planes P_1: $2x + 4y + 6z - 2 = 0$ and P_2: $x - z + 1 = 0$. Use this to answer problems 10 and 11.

10. The equation of the plane passing through the point $(3,5,0)$ and perpendicular to each of the planes P_1 and P_2 is:

a. $(x-3)-2(y-5)+z=0$ b. $2(x-3)+4(y-5)+2z=0$

c. $(x+3)+2(y+5)+z=0$ d. $2(x-3)+4(y-5)-2z=0$

11. One of the following is an equation of the line passing through the point $(3,5,0)$ that is parallel to each of the planes P_1 and P_2:

a. $\dfrac{x-3}{2}=\dfrac{y-5}{4}=\dfrac{z}{-2}$ b. $\dfrac{x-3}{1}=\dfrac{y-5}{-2}=\dfrac{z}{1}$

c. $\dfrac{x-3}{2}=\dfrac{y-5}{4}=\dfrac{z}{2}$ d. $\dfrac{x+3}{1}=\dfrac{y+5}{2}=\dfrac{z}{1}$

12. The acute angle that the plane $x + y - z = 2$ makes with the plane $x - y + z = 1$ is:

a. $\cos^{-1}\dfrac{1}{3}$ b. $\cos^{-1}\dfrac{1}{\sqrt{3}}$ c. $\dfrac{\pi}{3}$ d. $\dfrac{\pi}{6}$

13. Let u, v two orthogonal vectors in 3-space such that $\|u\| = 2, \|v\| = 3$. Then $(u \times v) \times v =$

a. 0 b. $6u$ c. $18u$ d. $-18u$

The Hashemite University
Department of Mathematics
Calculus III

First Exam
Time: One Hour

Name: .... Number: ....

Select the best correct answer and fill it in the following table: (2 points each)

	1	2	3	4	5	6	7	8	9	10	11	12	13
(a)						X		/	/			/	
(b)	X				/		X	✓	✓		/		/
(c)	/		✓		X	/	/			/	✓	X	✓
(d)		✓		✓						X			

1. Let $U = \langle 3,3,3 \rangle$, $V = \langle 2,-1,1 \rangle$, $W = \langle 1,2,1 \rangle$. Then $(v \times w).u =$

a. 1 b. 3 c. -1 d. -3

$\begin{pmatrix} i & j & k \\ 2 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$

$(-1-2)-(2-1)+(4+1)$

$-3 - 1 + 5$

$= 1$

$(-3,-1,5)$

2. The equation $\rho = 2$ given in spherical coordinates represents:

a. an upper circular cone
b. a cylinder
c. a lower circular cone
d. a sphere

3. The rectangular coordinates of a point P are $(1,-1,-\sqrt{2})$. Then its spherical coordinates are:

$x\ y\ z$

$\rho\ \theta\ \phi$ $z=\rho\cos\phi$

$x=\rho\sin\phi\cos\theta$

$y=\rho\sin\phi\sin\theta$

a. $\left(2,\frac{\pi}{4},\frac{7\pi}{4}\right)$ b. $\left(2,\frac{\pi}{4},\frac{3\pi}{4}\right)$ c. $\left(2,\frac{7\pi}{4},\frac{3\pi}{4}\right)$ d. $\left(2,\frac{7\pi}{4},\frac{7\pi}{4}\right)$

315 135

4. Consider the two planes P_1: $2x+4y+6z-2=0$ and P_2: $x-z+1=0$. One of the following is an equation of the line passing through the point $(3,5,0)$ that is parallel to each of the planes P_1 and P_2:

a. $\frac{x-3}{2}=\frac{y-5}{4}=\frac{z}{-2}$
b. $\frac{x+3}{1}=\frac{y+5}{2}=\frac{z}{1}$
c. $\frac{x-3}{2}=\frac{y-5}{4}=\frac{z}{2}$
d. $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z}{1}$

5. The lines $L_1 : \frac{x-2}{2}=\frac{y-1}{4}=\frac{z}{-2}$ and $L_2 : \frac{x-2}{1}=\frac{y-1}{2}=\frac{z}{-1}$ are:

a. orthogonal b. parallel c. the same d. none of the previous

6. Let u, v two orthogonal vectors in 3-space such that $\|u\|=2, \|v\|=3$. Then $(u\times v)\times v =$

a. $-18u$ b. $6u$ c. $18u$ d. 0

. The equation $r = 2$ given in cylindrical coordinates represents:

a sphere b. a cylinder c. a plane d. None

Let U, V be two unit vectors in 3-space such that $\|U \times V\| = \frac{+12}{13}$. Then $\cancel{U \times V}$ $U.V$ =

$-\frac{5}{13}$ b. $\frac{5}{13}$ c. $\frac{12}{13}$ d. $-\frac{12}{13}$

Let U and V be two vectors in the 3-space such that $U \cdot V = -\frac{5}{13}$. The the angle
tween U and V is:

obstute b. acute c. right d. None

). The distance between the the line $L: \frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{-1}$ and the line
: $\frac{x+1}{1} = \frac{y}{2} = \frac{z-1}{-1}$ equals:

5 b. $\sqrt{3}$ c. 3. d. $\sqrt{5}$

. The distance between the point $(1,1,1)$ and the plane $2x + 2y + z = 6$ is:

$\frac{5}{3}$ b. $\frac{1}{3}$ c. $\frac{5}{9}$ d. $\frac{1}{9}$

. The area of the triangle ABC, where $\overrightarrow{AB} = \langle 2,-1,1 \rangle$ and $\overrightarrow{AC} = \langle 1,2,1 \rangle$ is:

$\sqrt{35}$ b. 35 c. $\frac{1}{2}\sqrt{35}$ d. $\frac{35}{2}$

. Let L be a line in the 3-space where A is a point on L and U is a vector parallel to
Then the distance between a point P ad the line L is given by:

$\left\|proj_U \overrightarrow{AP}\right\|$ b. $\left\|\overrightarrow{AP} - proj_U \overrightarrow{AP}\right\|$ c. $\left\|\overrightarrow{AP}\right\|$ d. $\|U\|$

$r\cos\theta =$ [illegible] plane $\quad r =$ [illegible] $\Rightarrow$ cylinder

$z =$ — $\quad \rho =$ [illegible] sphere $\quad \phi =$ [illegible] cone

The Hashemite University
Department of Mathematics
Calculus III

First Exam
Date: 7 / 3/ 2010
Time: One Hour

Name: .... Number:....AKS.7.S................

Select the best correct answer and fill it in the following table: (2 points each)

	1	2	3	4	5	6	7	8	9	10	11	12	13
(a)	✓			✓					✓	✓	✓	✓	
(b)	X			X	✓ X	✓	✓				X		
(c)	✓	✓	✓					✓					✓
(d)		X											X

$r = \frac{2y}{r} \qquad r^2 = 2y \qquad \cancel{x = r\cos\theta} \qquad \frac{y}{r} = \sin\theta$

1. The equation $r = 2\sin\theta$ given in cylindrical coordinates represents:
   a. a sphere b. a plane c. a cylinder d. an elliptic paraboloid

$x^2 + y^2 =$

2. The equation $\rho = 2\sin\phi$ given in spherical coordinates represents:
   a. a sphere b. a plane c. a cylinder d. an elliptic paraboloid

$\rho^2 = 2r \quad \rho = \frac{2r}{\rho} \qquad r = \rho\sin\phi \qquad x^2 + y^2 - 2y + 1 \qquad x^2 + (y-$

3. Let U, V be two unit vectors in 3-space such that $U \cdot V = \frac{5}{13}$. Then $\|U \times V\| =$

a. $\frac{5}{13}$ b. $-\frac{5}{13}$ c. $\frac{12}{13}$ d. $-\frac{12}{13}$

4. The equation $\theta = \frac{\pi}{3}$ given in cylindrical coordinates represents:
   a. an upper circular cone b. a part of a plane
   c. a cylinder d. a sphere

5. The lines $L_1 : \frac{x-2}{2} = \frac{y-1}{4} = \frac{z}{-2}$ and $L_2 : \frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{-1}$ are:
   a. the same b. parallel c. orthogonal d. none of the previous

6. The distance between the the line $L : \frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{-1}$ and the line $L : \frac{x+1}{1} = \frac{y}{2} = \frac{z-1}{-1}$ equals:

a. 5 b. $\sqrt{5}$ c. 3. d. $\sqrt{3}$

$\|\vec{AP} \times u\| \qquad A(2,1,0) \quad \vec{AP} = \langle$

Let u, v and w be three vectors in 3-space such that $(u\times w)\cdot v=-7$. Use this to answer 7,8 and 9.

7. $(v\times w).2u=$

a. -14 b. 14 c. 7 d. -7

8. $(u\times 2u).w=$

a. -14 b. 14 c. 0 d. 7

9. The volume of the parallelpiped whose adjacent sides are u, v and w is:

a. 7 b. 14 c. 49 d. 0

Consider the two planes $P_1: 2x+4y+6z-2=0$ and $P_2: x-z+1=0$. Use this to answer problems 10 and 11.

2 4 6
1 0 −1

10. The equation of the plane passing through the point $(3,5,0)$ and perpendicular to each of the planes P_1 and P_2 is:

a. $(x-3)-2(y-5)+z=0$ b. $2(x-3)+4(y-5)+2z=0$

c. $(x+3)+2(y+5)+z=0$ d. $2(x-3)+4(y-5)-2z=0$

11. One of the following is an equation of the line passing through the point $(3,5,0)$ that is parallel to each of the planes P_1 and P_2:

a. $\dfrac{x-3}{2}=\dfrac{y-5}{4}=\dfrac{z}{-2}$ b. $\dfrac{x-3}{1}=\dfrac{y-5}{-2}=\dfrac{z}{1}$

c. $\dfrac{x-3}{2}=\dfrac{y-5}{4}=\dfrac{z}{2}$ d. $\dfrac{x+3}{1}=\dfrac{y+5}{2}=\dfrac{z}{1}$

12. The acute angle that the plane $x+y-z=2$ makes with the plane $x-y+z=1$ is:

a. $\cos^{-1}\frac{1}{3}$ b. $\cos^{-1}\frac{1}{\sqrt{3}}$ c. $\frac{\pi}{3}$ d. $\frac{\pi}{6}$

13. Let u, v two orthogonal vectors in 3-space such that $\|u\|=2, \|v\|=3$. Then $(u\times v)\times v=$

a. 0 b. $6u$ c. $18u$ d. $-18u$

The Hashemite University | Department of Mathematics | Feb 19, 2014
Calculus (3) | First Exam | Time: 1 hour
Name (in Arabic): ________ Student Number: ________ Serial Number: ________
Section number or lecture time: ________ Instructor name: ________

> Please do all your work in this booklet and show all the steps. Calculators and notes are not allowed.

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1. Consider the vectors $\vec{u} = <1, 0, -2>$, $\vec{v} = <0, 1, 3>$ and $\vec{w} = <1, 2, 0>$.

(a) (2 points) Find $3\vec{u} + \vec{v} - 2\vec{w}$

$3\vec{u} = <3, 0, -6>$ $\quad +2\vec{w} = <2, 4, 0>$

$3\vec{u} + \vec{v} - 2\vec{w} = <1, -3, -3>$

(b) (3 points) Find the area of the parallelogram determined by $\vec{u}$ and $\vec{w}$.

area of parallelogram $= |u \times w|$ $\Rightarrow$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & -2 \\ 1 & 2 & 0 \end{vmatrix} = i(4) - j(2) + k(2 = 4i - 2j + 2k$$

$|u \times w| = \sqrt{4^2 + (-2)^2 + (2)^2}$

$= \sqrt{24}$

(c) (2 points) Find the vector projection of $\vec{u}$ onto $\vec{v}$.

$\text{Proj}_{\vec{v}} \vec{u} = \frac{u \cdot v}{|v|^2} \vec{v} \Rightarrow = \frac{0 + 0 - 6}{10} <0, 1, 3>$

$= -\frac{3}{5} <0, 1, 3> = <0, \frac{-3}{5}, \frac{-9}{5}>$

2. (3 points) Let $\vec{u}$ and $\vec{v}$ be non-zero vectors. Show that

$$(\vec{u} . \vec{v})^2 = |\vec{u}|^2 |\vec{v}|^2 - |\vec{u} \times \vec{v}|^2$$

$(\vec{u} . \vec{v})^2 = (|\vec{v}||\vec{u}| \cos\theta)^2 \Rightarrow (|\vec{v}|^2 |\vec{u}|^2 \cos^2\theta)$ but $\cos^2\theta = 1 - \sin^2\theta$

$|v|^2 |u|^2 (1 - \sin^2\theta) \Rightarrow |v|^2 |u|^2 - |v|^2 |u|^2 \sin^2\theta$

but $|v||u| \sin\theta = |v \times u|$ then $|u|^2 |v|^2 - |v|^2 |u|^2 \sin^2\theta = |\vec{u}|^2 |\vec{v}|^2 - |\vec{u} \times \vec{v}|^2$

#

19 . 2 . 2014

3. (2 points) Find parametric equations of the line through the point (1,-1,-2) and parallel to the line passing through the points A(2,1,0) and B(0,1,2).

L_1

$\vec{AB} = \langle -2, 0, 2 \rangle \Rightarrow \vec{AB} \parallel L_1$

L_1: $x = 1 \cancel{+} 2t$, $y = -1$, $z = -2 + 2t$ } parametric equations

4. Given that the plane **P**: $2x - y + z = 5$ and the line **L**: $x = 2 + 2t,\ y = 2 + t,\ z = 3 + 5t$.

(a) (2 points) Find the possible intersection point of the line **L** with the plane **P** (if any).

$2(2+2t) - \cancel{y}(2+t) + (3+5t) = 5$

$4+4t - 2 - t + 3 + 5t = 5$

$8t = 0 \Rightarrow t = 0$

~~The~~

$x|_{t=0} = 2$ $y|_{t=0} = 2$ $z|_{t=0} = 3$

The intersection point (2,2,3)

(b) (3 points) Find the equation of the plane through the point (1,1,1) which is parallel to the plane **P**

P_1

$P_1 \parallel P$, ~~$n_P \perp P \Rightarrow n_P \perp P_1 = \langle 2, \ldots \rangle$~~ $n_P \parallel n_{P_1} \Rightarrow n_P = c\, n_{P_1}$

Let $c = 2$

the equation of $P_1 = 4(x-1) - 2(y-1) + 2(z-1) = 0$

$n_1(x - x_0) + n_2(y - y_0) + n_3(z - z_0) = 0$

$P_1 = 4x - 2y + 2z - 4$ ~~$= 2$~~

5. (3 points) Identify the following surfaces (by the name).

(a) $x^2 + y^2 = 16$

~~cylinder~~ cylender

(b) $y = x + 4$

plane

(c) $z = x^2 + y^2 - 2y$

$1 + z = x^2 + y^2 - 2y + 1$

$1 + z = x^2 + (y-1)^2$

~~Eliptic cone hyperbola probolo~~

Eliplic probolaiD

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6. (2 points) Compute the distance between the parallel planes $x + 3y - 2z = 5$ and $-x - 3y + 2z = 18$.

$P_1 = x+3y-2z=5$ point at $P_1 = (5, 0, 0)$

$P_2 = -x-3y+2z-18=0$

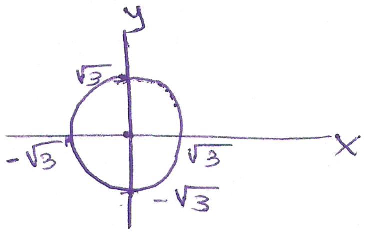
$$D = \frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}} = \frac{|-5+0+0-18|}{\sqrt{(-1)^2+(-3)^2+2^2}}$$

$$\frac{23}{\sqrt{14}}$$

7. (3 points) Sketch the region in R^3 which is represented by the following inequalities $z \geq x^2 + y^2$, $z < 3$

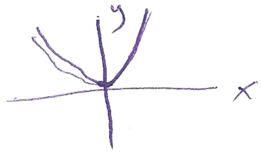
when $z=3$

$x^2+y^2=3 \Rightarrow$

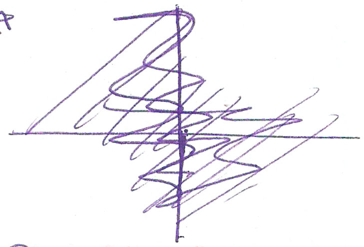


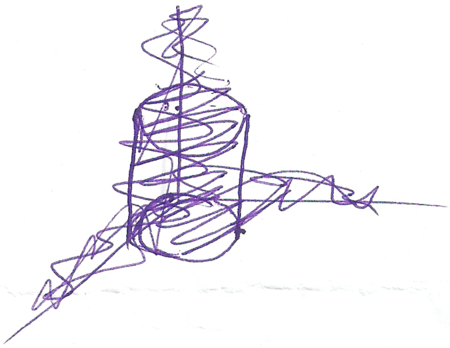
when $y=0$

$x^2 \leq z$



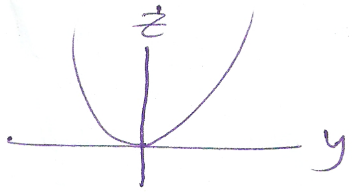
~~the sketch~~



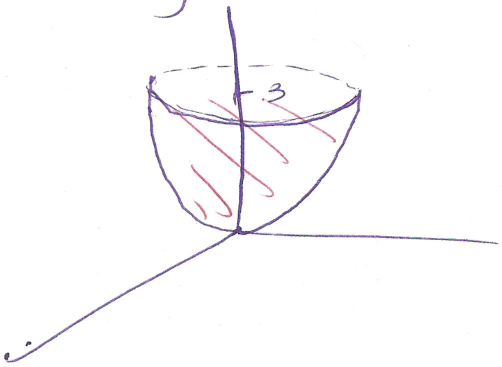


when $x=0$

$z \geq y^2$



the sketch



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